香港電腦奧林匹克競賽
Hong Kong Olympiad in Informatics

Optimization

Casper Wong {gasbug}
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Acknowledgement

➢ A very large portion of this slide is based on this slide by Christy Cheng {christycty}.

Motivation

C++11	Time Limit Exceeded
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➢ We need to reduce time complexity by **avoiding repeated computation**

some rule-of-thumb in OI (assume time limit = 1s):

$O(N!)$ → $N \leq 10$

$O(2^N)$ → $N \leq 20$

$O(N^2)$ → $N \leq 5000$

$O(N \lg N)$ → $N \leq 10^6$

$O(N)$ → $N \leq 10^7$

Task List

Demonstrated in lecture

C200 Partial Sum

C201 2D Partial Sum

C202 Difference Array

M0652 Museum

C203 Discretization

Extra Exercises

T062 Tappy World

20108 Maximum Sum

I0111 Mobile Phones

J051 Pattern Matching

J134 Lucky Rainbow

J144 Fair Santa Claus

M1613 Fair Patrol

M1513 Advanced Optimization Coding

S164 Alice's Meal

T042 Game of Life IV

S032 Project

S072 Partners

S113 Gravity Game

S152 Apple Garden

S211 Skyscraperhenge

Feel free to head to the level A session if you know how to solve all these (and meet the entry criteria)!

Outline

- Partial Sum
- Difference Array
- Precomputation
- Sliding Window (Two Pointers)
- Batching Step
- Discretization

Outline

- **Partial Sum**
- Difference Array
- Precomputation
- Sliding Window (Two Pointers)
- Batching Step
- Discretization

Problem

Given an array of N integers and Q queries.

For each query, find the sum of the L^{th} to R^{th} element *(inclusive)* of the array.

i	1	2	3	4	5	6	7	8	9
a[i]	3	-2	6	1	0	5	-5	2	4

Example: sum of 3rd to 6th element = a[3] + a[4] + a[5] + a[6]

= 6 + 1 + 0 + 5

= 12

Naive Solution

```
for i ← 1 to N do
    input a[i]

for i ← 1 to Q do
    input L and R
    sum = 0
    for j ← L to R do
        sum += a[j]
    output sum
```

What is the problem of this solution?

It is too slow :(
Time complexity O(NQ)
(Imagine all queries ask for a[1] to a[N])

Naive Solution

i	1	2	3	4	5	6	7	8	9
a[i]	3	-2	6	1	0	5	-5	2	4

We can see that the sum of `a[4]` to `a[6]` is calculated many times...

Can we perform the calculation once only?

1D Partial Sum - Idea

`c[i]` = sum of the first i elements (`a[1]+a[2]+...+a[i]`)

i	1	2	3	4	5	6	7	8	9
a[i]	3	-2	6	1	0	5	-5	2	4
c[i]	3	1	7	8	8	13	8	10	14

e.g. `c[3] = a[1] + a[2] + a[3]`

`= 3 + (-2) + 6 = 7`

1D Partial Sum - Idea

`c[i]` = sum of the first i elements (`a[1]+a[2]+...+a[i]`)

i	1	2	3	4	5	6	7	8	9
a[i]	3	-2	6	1	0	5	-5	2	4
c[i]	3	1	7	8	8	13	8	10	14

Using array c only, how can we find the values of `a[4]`?

Hint

```
c[4] = a[1] + a[2] + a[3] + a[4]
c[3] = a[1] + a[2] + a[3]
```

1D Partial Sum - Idea

Using array c only, how can we find the values of a[4]?

i	1	2	3	4	5	6	7	8	9
a[i]	3	-2	6	1	0	5	-5	2	4
c[i]	3	1	7	8	8	13	8	10	14

```
c[4] = a[1] + a[2] + a[3] + a[4]
c[3] = a[1] + a[2] + a[3]
```

⇒ `a[4] = c[4] - c[3]`

We can generalize it to **`a[x] = c[x] - c[x-1]`** *(proof left as exercise :P)*

1D Partial Sum - Idea

How can we find the values of `a[L] + a[L+1] + a[L+2]`?

i	1	2	3	4	5	6	7	8	9
a[i]	3	-2	6	1	0	5	-5	2	4
c[i]	3	1	7	8	8	13	8	10	14

Let's use our previous formula `a[x] = c[x] - c[x-1]`

```
a[L]   = c[L] - c[L-1]
a[L+1] = c[L+1] - c[L]
a[L+2] = c[L+2] - c[L+1]
```

Sum them up and we will get

```
a[L] + a[L+1] + a[L+2]
= c[L+2] - c[L-1]
```

1D Partial Sum - Idea

What if we would like to find values of `a[L] + a[L+1] + ... + a[R]`?

i	1	2	3	4	5	6	7	8	9
a[i]	3	-2	6	1	0	5	-5	2	4
c[i]	3	1	7	8	8	13	8	10	14

We obtained: `a[L] + a[L+1] + a[L+2] = c[L+2] - c[L-1]`

Generalize it: `a[L] + a[L+1] + ... + a[R] = c[R] - c[L-1]`

1D Partial Sum - Idea

Apply `a[L] + ... + a[R] = c[R] - c[L-1]`, find sum of `a[3]` to `a[6]`

i	1	2	3	4	5	6	7	8	9
a[i]	3	-2	6	1	0	5	-5	2	4
c[i]	3	1	7	8	8	13	8	10	14

```
L = 3, R = 6
ans = c[6] - c[3-1]
    = 13 - 1
    = 12
```

Verify

```
a[3] + a[4] + a[5] + a[6]
= 6 + 1 + 0 + 5
= 12
```

1D Partial Sum - Implementation

```
for i ← 1 to N do
    c[i] = c[i-1] + a[i]
```

First, compute array `c`.

Why we use `c[i-1] + a[i]`, instead of `a[1] + a[2] + ... + a[i]`?

Looping through the first i elements each time is slow! Time complexity $O(N^2)$

Note that `c[i-1] = a[1] + ... + a[i-1]`,

```
c[i]   = a[1] + ... + a[i-1] + a[i]
       = c[i-1] + a[i]
```

1D Partial Sum - Implementation

```
for i ← 1 to N do
    c[i] = c[i-1] + a[i]

for i ← 1 to Q do
    input L and R
    output c[R]-c[L-1]
```

Time Complexity

O(N) partial sum precomputation

O(1) per query → total O(Q)

= O(N + Q)

Reminder:

- Use long long if value of `a[i]` is large
- Initialize `c[0]` first
- Be aware of 0/1-based issue

Demo - HKOJ C200 Partial Sum

Problem

Now, let's extend everything to 2-dimension

-3	1	2	0	2
2	0	4	-3	1
-1	3	0	1	4
5	-2	1	4	2

Given an array a with `N` rows and `M` columns and some queries.

In each query, you need to find the sum of elements from `(Sr, Sc)` to `(Er, Ec)`.

(r, c) = element at r^{th} row and c^{th} column

What is the sum from `(1, 2)` to `(2, 4)`?

```
1 + 2 + 0 + 0 + 4 + (-3) = 4
```

Naive Solution

```
for i ← 1 to Q do
    input Sr, Sc, Er, Ec
    sum = 0
    for j ← Sr to Er do
        for k ← Sc to Ec do
            sum += a[j][k]
    output sum
```

Loop over the required area in each query and sum up the numbers

Time Complexity?
O(NMQ)
Imagine all queries ask for (1, 1) to (N, M)

1D Partial Sum Solution

Can we applied what we learned just now?

-3	1	2	0	2
2	0	4	-3	1
-1	3	0	1	4
5	-2	1	4	2

1D Partial Sum Solution

Compute a 1D partial sum array for each row

-3	1	2	0	2
2	0	4	-3	1
-1	3	0	1	4
5	-2	1	4	2

-3	-2	0	0	2
2	2	6	3	4
-1	2	2	3	7
5	3	4	8	10

1D Partial Sum Solution

```
for i ← 1 to N do
    for ← 1 to M do
        c[i][j] = c[i][j-1] + a[i][j]

for i ← 1 to Q do
    input Sr, Sc, Er, Ec
    sum = 0
    for j ← Sr to Er do
        sum += c[j][Ec] - c[j][Sc-1]
    output sum
```

Compute array c

- Note that here we arrange the partial sum arrays horizontally (i.e. one row = one array)
- depends on range of N and M, you may arrange it vertically (i.e. one column = one array)

Perform query

1D Partial Sum Solution

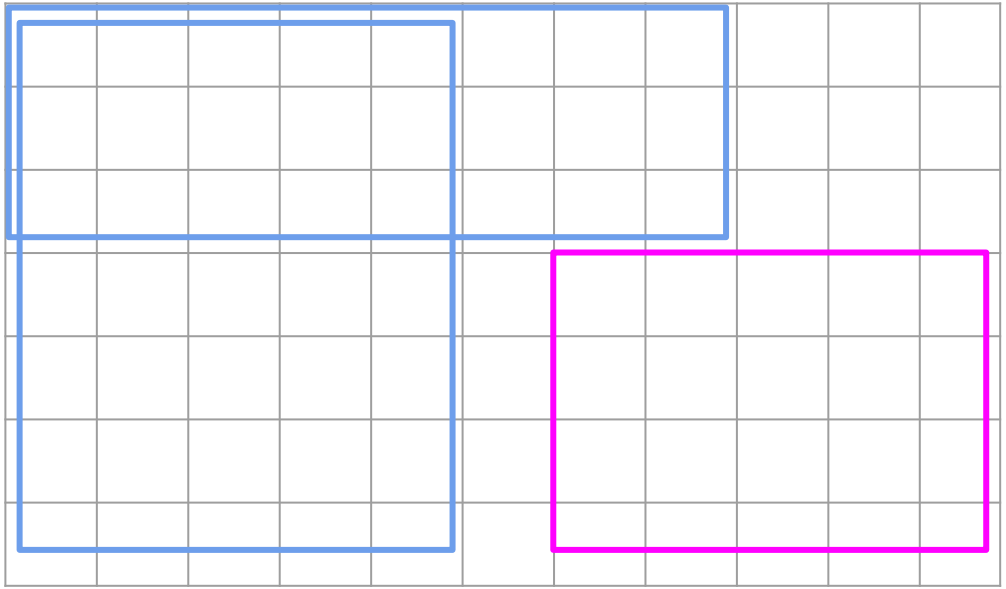
```
for i ← 1 to N do
    for ← 1 to M do
        c[i][j] = c[i][j-1] + a[i][j]

for i ← 1 to Q do
    input Sr, Sc, Er, Ec
    sum = 0
    for j ← Sr to Er do
        sum += c[j][Ec] - c[j][Sc-1]
    output sum
```

Time Complexity?

O(NM) precomputation
O(min(N, M)) per query
= O(NM + min(N,M) * Q)

Not bad, but can it be better?

2D Partial Sum - Idea

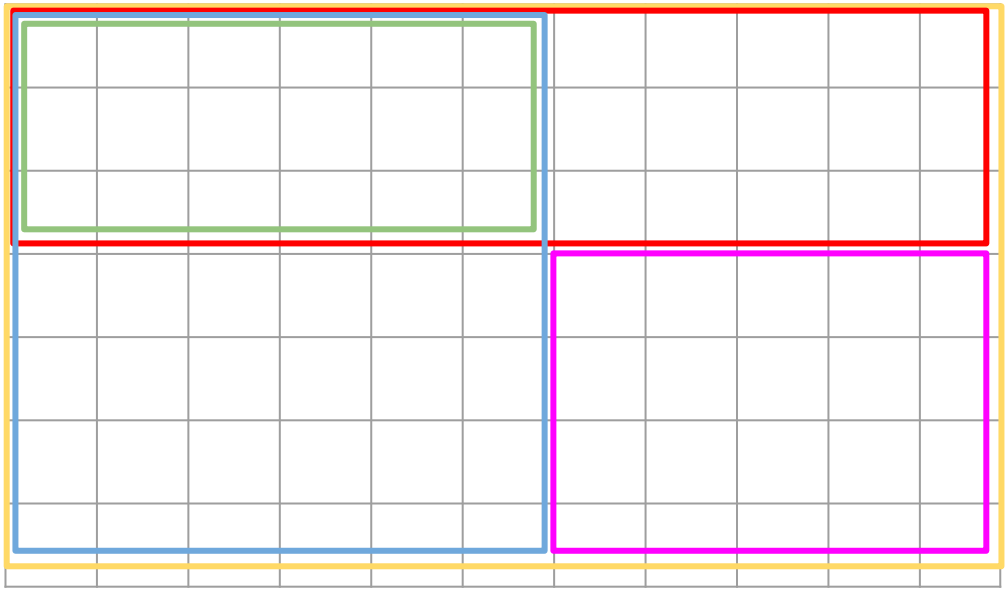


Given that we have area of all rectangles with top-left corner at (1, 1) *(such as the blue ones)*

How can we find the area of the magenta rectangle?

2D Partial Sum - Idea

Magenta = Yellow – Red – Blue + Green

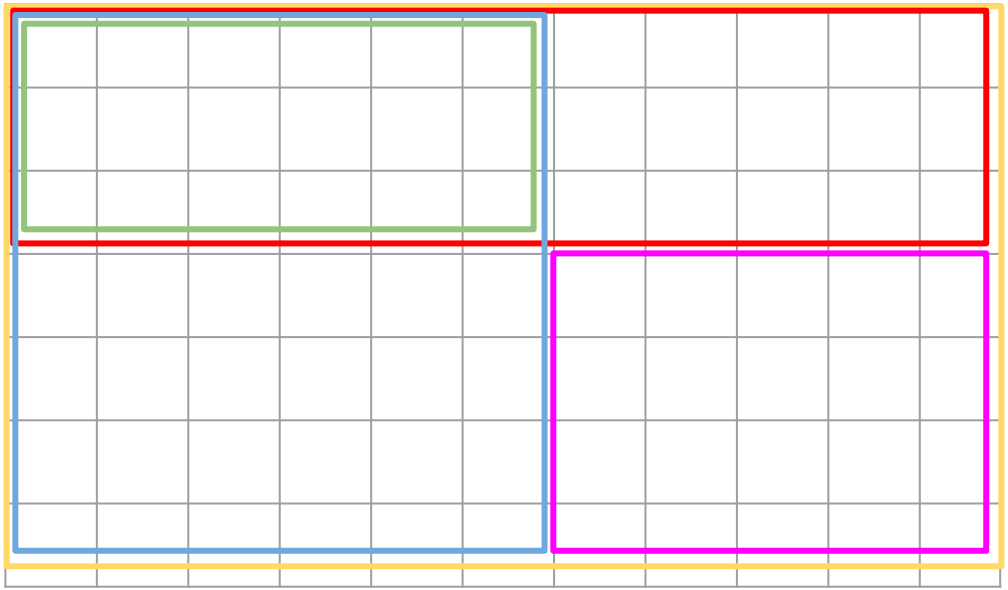


1	2
3	4

1 + 2 + 3 + 4
- 1 - 2
- 1 - 3
+ 1
= 4

2D Partial Sum - Idea

Magenta = **Yellow** – **Red** – **Blue** + **Green**



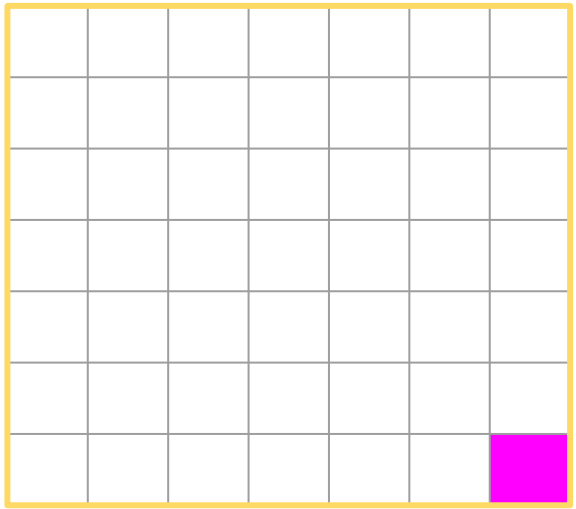
Let's verify that by calculation!

7 * 11 = 77
3 * 11 = 33
7 * 6 = 42
3 * 6 = 18
4 * 5 = 20

```
77 - 33 - 42 + 18 = 20
```

2D Partial Sum - Idea

How to precompute the area of each rectangle touching (1, 1)?



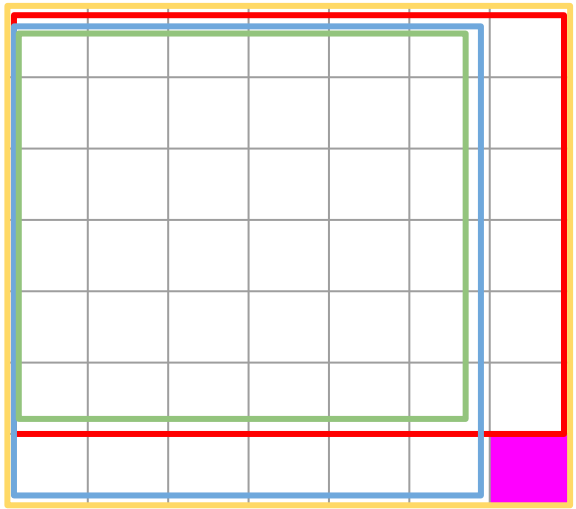
Given that we have areas of all rectangles touching `(1, 1)` within the table, except that at `(N, M)`.

We also have area of magenta grid.

How can we compute area of the table (yellow)?

2D Partial Sum - Idea

How to precompute the area of each rectangle touching (1, 1)?



Yellow = Magenta + Red + Blue – Green

1	2
3	4

4
+ 1 + 2
+ 1 + 3
- 1
= 1 + 2 + 3 + 4

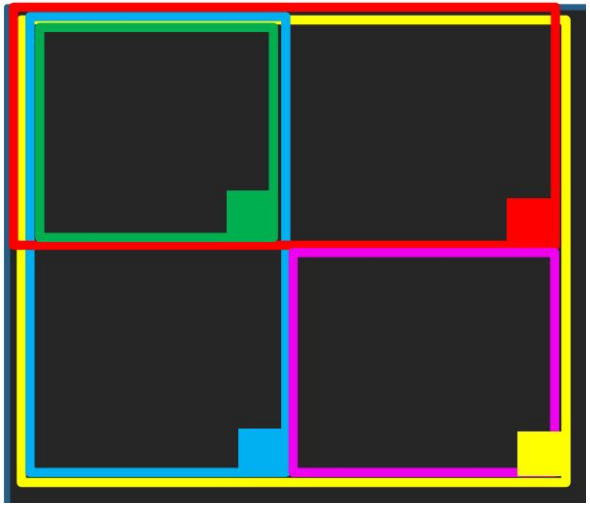
2D Partial Sum - Idea

Or mathematically...

Magenta = Yellow – Red – Blue + Green

after rearranging some terms, we have:

Yellow = Magenta + Red + Blue – Green

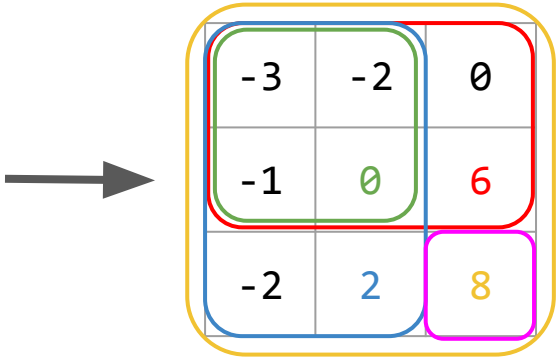


2D Partial Sum - Precomputation

Let c[i][j] = sum of elements from a[1][1] to a[i][j]

```
for i ← 1 to N do
for j ← 1 to M do
    c[i][j] = a[i][j] + c[i-1][j] + c[i][j-1] - c[i-1][j-1]
```

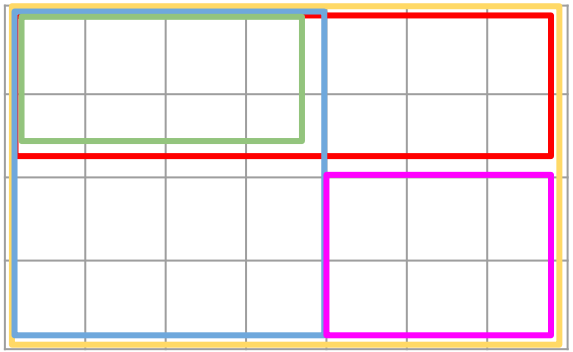
-3	1	2
2	0	4
-1	3	0



2D Partial Sum - Query

Magenta: (Sr, Sc) to (Er, Ec)

```
for i ← 1 to Q do
    input Sr, Sc, Er, Ec
    sum = c[Er][Ec] - c[Sr-1][Ec] - c[Er][Sc-1] + c[Sr-1][Sc-1]
    output sum
```



2D Partial Sum - Query

Time Complexity?

```
for i ← 1 to Q do
    input Sr, Sc, Er, Ec
    sum = c[Er][Ec] - c[Sr-1][Ec] - c[Er][Sc-1] + c[Sr-1][Sc-1]
    output sum
```

O(NM) precomputation and O(1) per query
= O(NM + Q)

2D Partial Sum - Example

Query (2, 3) to (4, 4)

-3	1	2	0	2
2	0	4	-3	1
-1	3	0	1	4
4	2	-2	1	3

2D Partial Sum - Example

Query (2, 3) to (4, 4)

```
c[4][4] - c[1][4] - c[4][2] + c[1][2]
= 11 - 0 - 8 + (-2)
= 1
```

-3	1	2	0	2
2	0	4	-3	1
-1	3	0	1	4
4	2	-2	1	3

-3	-2	0	0	2
-1	0	6	3	6
-2	2	8	6	13
2	8	12	11	21

2D Partial Sum - Example

Query (2, 2) to (3, 5)

-3	1	2	0	2
2	0	4	-3	1
-1	3	0	1	4
4	2	-2	1	3

2D Partial Sum - Example

Query (2, 2) to (3, 5)

```
c[3][5] - c[1][5] - c[3][1] + c[1][1]
= 13 - 2 - (-2) + (-3)
= 10
```

-3	1	2	0	2
2	0	4	-3	1
-1	3	0	1	4
4	2	-2	1	3

-3	-2	0	0	2
-1	0	6	3	6
-2	2	8	6	13
2	8	12	11	21

Demo - HKOJ C201 2D Partial Sum

Attendance Taking + 5-minute Break (Until 11:20)

check out 20108 Maximum Sum if you are finding sth to do :)

Outline

- Partial Sum
- **Difference Array**
- Precomputation
- Sliding Window (Two Pointers)
- Batching Step
- Discretization

Problem

Given P actions. In each action, add v_i to `a[L...R]` of array `a` (size `N`). Find the final value of array `a`.

Add 2 to `a[2..4]`

i	1	2	3	4	5	6
a[i]	0	2	2	2	0	0

Add 3 to `a[3..6]`

i	1	2	3	4	5	6
a[i]	0	2	5	5	3	3

Naive Solution

```
for i ← 1 to P do
    input L, R, V
    for j ← L to R do
        a[j] += V

for i ← 1 to N do
    output a[i]
```

Simulate the operation, adding v_i to all `a[j]` with `L <= j <= R` each action

Time Complexity?

O(PN)

(Imagine all actions act on a[1] to a[N])

1D Difference Array - Idea

Assume the actions are:

1. add **2** for a[2] to a[6]
2. add **3** for a[4] to a[8]
3. add **4** for a[7] to a[7]

Try to compute the final result

1D Difference Array - Idea

Assume the actions are:

1. add **2** for `a[2]` to `a[6]`
2. add **3** for `a[4]` to `a[8]`
3. add **4** for `a[7]` to `a[7]`

Try to compute the difference between adjacent values, any observations?

Final Result:

i	1	2	3	4	5	6	7	8	9
a[i]	0	2	2	5	5	5	7	3	0

difference **+2** 0 **+3** 0 0 +2 **-4** **-3**

(+2 = **+4** **-2**)

1D Difference Array - Idea

Lets mark the differences as an array d.

i	1	2	3	4	5	6	7	8	9
d[i]		+2		+3			+2	-4	-3
a[i]	0	2	2	5	5	5	7	3	0

- `d[i] = a[i] - a[i-1]`

1D Difference Array - Idea

for each action `i`, it adds v_i to `d[L]` and subtracts v_i from `d[R+1]`

i	1	2	3	4	5	6	7	8	9
d[i]		+2		+3			-2 +4	-4	-3
a[i]	0	2	2	5	5	5	7	3	0

- add **2** for a[2] to a[6]
- add **3** for a[4] to a[8]
- add **4** for a[7] to a[7]

1D Difference Array - Idea

Can we compute value of array `a` based on array `d`?

i	1	2	3	4	5	6	7	8	9
d[i]		+2		+3			-2 +4	-4	-3
a[i]									

1D Difference Array - Idea

Can we compute value of array `a` based on array `d`?

i	1	2	3	4	5	6	7	8	9
d[i]		+2		+3			+2	-4	-3
a[i]	0	2	2	5	5	5	7	3	0

Yes! `a[i] = a[i-1] + d[i]`

1D Difference Array - Idea

Instead of updating the whole contiguous section, we can just mark the value at start and end points using array `d`.

array `a` can be computed based on `d` after all actions are done.

i	1	2	3	4	5	6	7	8	9
d[i]		+2		+3			-2 +4	-4	-3
a[i]	0	2	2	5	5	5	7	3	0

- add **2** for a[2] to a[6]
- add **3** for a[4] to a[8]
- add **4** for a[7] to a[7]

1D Difference Array - Implementation

```
for i ← 1 to P do
    input L, R, V
    d[L] += V
    d[R+1] -= V

for i ← 1 to N do
    a[i] = a[i-1] + d[i]
    output a[i]
```

Time Complexity
= O(N+P)

Demo - HKOJ C202 Difference Array

Problem

Now, let's extend the question to 2-dimension

0	2	2	2	0
0	2	2	2	0
0	0	0	0	0
0	0	0	0	0

Given an array a with N rows and M columns and some actions.

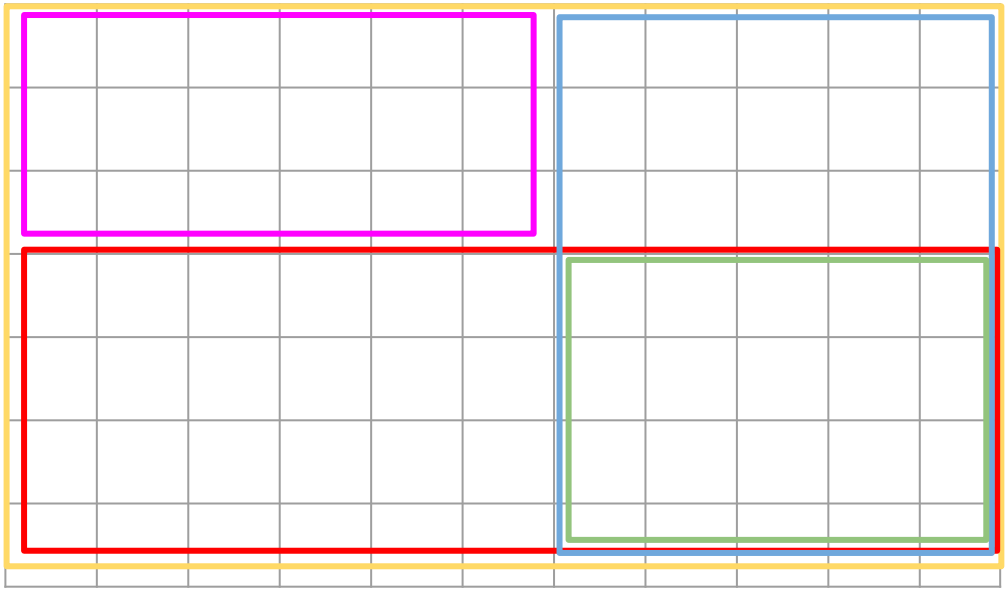
In each action, you need to add v_i all to elements from (Sr, Sc) to (Er, Ec).

example:

add 2 to all elements from (1, 2) to (2, 4)

2D Difference Array - Idea

This picture again... but this time rotated

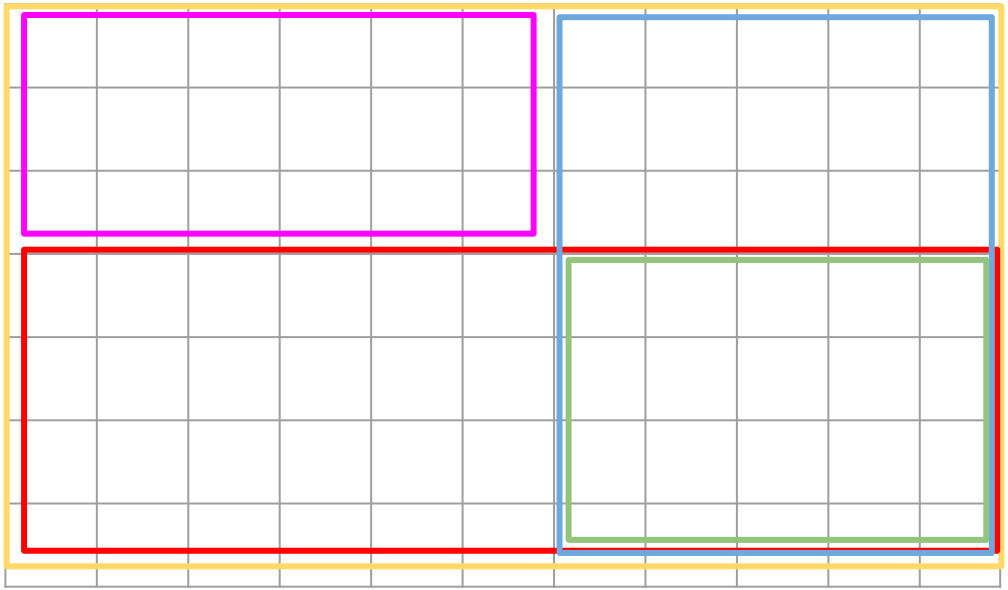


Assume we can get the “area” of each rectangle touching `(N, M)`

How can we get the “area” of magenta?

2D Difference Array - Idea

magenta = **yellow** - **red** - **blue** + **green**



4	3
2	1

1 + 2 + 3 + 4
- 1 - 2
- 1 - 3
+ 1
= 4

2D Difference Array - Idea

Now, let's define an array d such that

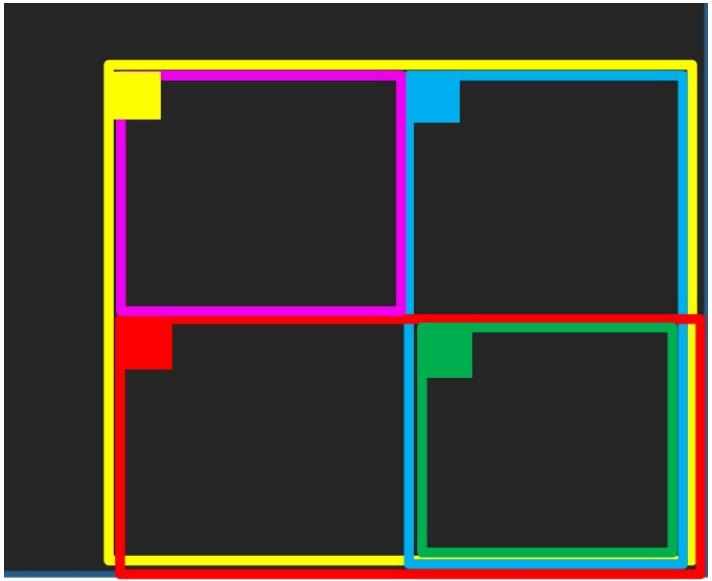
`d[i][j] = a[i][j] - a[i][j-1] - a[i-1][j] + a[i-1][j-1]`

0	2	2	4
2	5	2	3
2	6	1	5
3	4	0	1

0	2	0	2
2	1	-3	-1
0	1	-2	3
1	-3	1	-3

If you don't get the formula, just bear in mind that `d[i][j]` affects all value from `(i, j)` to `(N, M)`.

2D Difference Array - Idea



Imagine the small filled squares as `d[i][j]`, and the rectangle is the area affected by `d[i][j]`.

To update [Sr...Er, Sc...Ec], we first add v to **d[Sr][Sc]**.

Then, we block the increase outside the area by subtracting v from **d[Sr][Ec+1]** and **d[Er+1][Sc]**.

Since v is subtracted twice from [Er+1...N, Ec+1...M], we add v to **d[Er+1][Ec+1]**.

2D Difference Array - Idea

Now, let's see how we can compute array `a` from `d`.

In the previous slides, we defined:

```
d[i][j] = a[i][j] - a[i][j-1] - a[i-1][j] + a[i-1][j-1]
```

By rearranging the equation, we get:

```
a[i][j] = d[i][j] + a[i][j-1] + a[i-1][j] - a[i-1][j-1]
```

2D Difference Array - Action

```
for i ← 1 to P do
    Input Sx, Sy, Ex, Ey
    d[Sx][Sy] += v
    d[Sx][Ey+1] -= v
    d[Ex+1][Sy] -= v
    d[Ex+1][Ey+1] += v
```

What is the array d corresponding to this array?

0	0	0	0
5	5	5	0
5	5	5	0
0	0	0	0

0	0	0	0
+5	0	0	-5
0	0	0	0
-5	0	0	+5

2D Difference Array - Compute Result

```
for i ← 1 to N do
    for j ← 1 to M do
        a[i][j] = d[i][j] + a[i][j-1] + a[i-1][j] - a[i-1][j-1]
        output a[i][j]
```

Time Complexity = O(NM + P)

Outline

- Partial Sum
- Difference Array
- **Precomputation**
- Sliding Window (Two Pointers)
- Batching Step
- Discretization

Problem

Given a string that consists of ‘A’, 'B' and some queries.
For each query qi, output the closest ‘B’ whose index <= qi.

Example:
string = “AABAAABBA”
query 3 → output 3
query 6 → output 3
query 9 → output 8

Naive solution

```
for i ← 1 to Q do
    input qi
    for j ← qi downto 1 do
        if (s[j] = 'B')
            output j
            break
```

Notations:

s = given string

N = length of string

Time complexity = O(QN)

- Remember to handle the case where 'B' is not found

Precomputation - Solution

Build an array a where `a[i]` = the last “B” which index <= `i`

For example, string = “AABAAABBA”:

i	1	2	3	4	5	6	7	8	9
a[i]	-1	-1	3	3	3	3	7	8	8

- `-1` is used to indicate that there is no ‘B’ with index <= `i`.

Precomputation - Solution

What is the array a for string = “ABABABBAA”?

i	1	2	3	4	5	6	7	8	9
a[i]	-1	2	2	4	4	6	7	7	7

What is the algorithm used to compute `a[i]`?

Precomputation - Solution

Note that if `string[i] = 'B'`, then `a[i] = i`, else `a[i] = a[i-1]`

i	1	2	3	4	5	6	7	8	9
a[i]	-1	2	2	4	4	6	7	7	7

`string = "ABABABBAA"`

Precomputation - Solution

```
a[0] = -1
for i ← 1 to N do
    if (s[i] = ‘B’)
        a[i] = i
    else
        a[i] = a[i-1]

for i ← 1 to Q do
    input qi
    output a[qi]
```

Total time complexity = O(N + Q)

Precomputation

Some useful arrays to be precomputed

- Prefix / suffix sum (partial sum)
- Prefix / suffix max / min / special element
- Prefix / suffix XOR sum
- Prefix / suffix index of last special element
- Prefix / suffix count (e.g. number of odd numbers / number of “*”s)

Practice

20108 Maximum Sum
I0111 Mobile Phones
J051 Pattern Matching
J134 Lucky Rainbow
M1513 Advanced Optimization Coding
S032 Project
S113 Gravity Game
S164 Alice's Meal
T062 Tappy World

Outline

- Partial Sum
- Difference Array
- Precomputation
- **Sliding Window (Two Pointers)**
- Batching Step
- Discretization

Problem

Given two sorted array `a` and `b`,

find the number of pair (i, j) such that `a[i] + b[j] = C`

i	1	2	3	4	5	6	7	8	9
a[i]	1	5	8	10	12	14	15	18	25
b[i]	3	6	6	7	13	14	15	18	19

If C = 20, ans = 4 [(1, 9), (2, 7), (6, 2), (6, 3)]

Naive Solution

```
for i ← 1 to N do
    for j ← 1 to N do
        if (a[i] + b[j] = C)
            ans += 1
output ans
```

Exhaust all pairs of (i, j) and count how many of them satisfy `a[i] + b[j] = C`

Time Complexity = $O(N^2)$

Hint: the arrays are **sorted**

Binary Search Solution

For each element in a,
binary search the number of `b[j]` such that `b[j] = C - a[i]`

Two binary search (one for lower bound, one for upper bound) are needed if the elements in `b` are not distinct.

Time complexity = O(N log N)

Good, but can it be better?

Sliding Window - Observation

Find the number of pair (i, j) such that `a[i] + b[j] = C`

If C = 20, ans = 4 [(1, 9), (2, 7), (6, 2), (6, 3)]

i	1	2	3	4	5	6	7	8	9
a[i]	1	5	8	10	12	14	15	18	25
b[i]	3	6	6	8	13	14	15	18	19

Let's look at the array again, are there any pattern in the pairs' position?

When `a[i]` increases, its paired `b[j]` decreases!

Sliding Window - Observation

1	5	8	10	12	14	15	18	25
3	6	6	7	13	14	15	18	19

1	5	8	10	12	14	15	18	25
3	6	6	7	13	14	15	18	19

1	5	8	10	12	14	15	18	25
3	6	6	7	13	14	15	18	19

1	5	8	10	12	14	15	18	25
3	6	6	7	13	14	15	18	19

i increases
→ a[i] increases
→ C - a[i] decreases
→ b[j] decreases
→ j decreases

Sliding Window - Observation

from last slide, we know that when `i` increases, `j` decreases

⇒ when `a[i]` is selected as the xth pair, `b[j]` must be less than or equal to (x-1)th pair's `j` value

⇒ we can eliminate all possibility of `j` that is > last pair's `j` value!

Sliding Window - Implementation

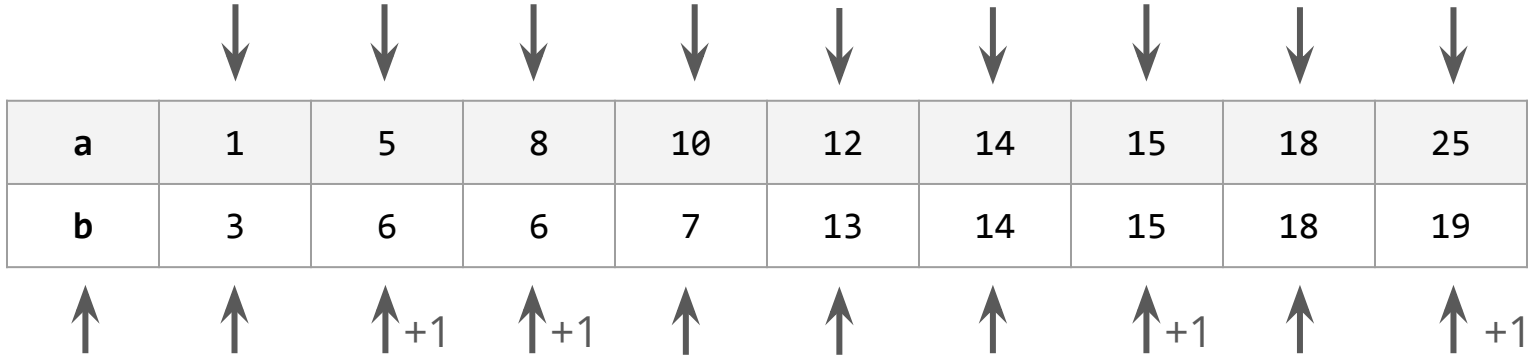
```
j = N
for i ← 1 to N do
    while (j > 0) and (a[i] + b[j] > C)
        j -= 1
    while (j > 0) and (a[i] + b[j] = C)
        ans += 1
output ans
```

Instead of storing each pair's j value, we let j be decreasing.

Time Complexity = O(N)

Sliding Window - Demonstration

Find number of pairs (i, j) such that a[i] + b[j] = C. Assume C = 20.




```
for i ← 1 to N do
    while (j > 0) and (a[i] + b[j] > C) j -= 1
    while (j > 0) and (a[i] + b[j] = C) ans++, j -= 1
```

Sliding Window - HKOJ M0652 Museum

Given N, M and an array of integers a.

Find the shortest consecutive segment that contains all numbers 1..M.

1 <= N <= 1000000, 1 <= M <= 2000

Example:

N = 12, M = 5

2	5	3	1	3	2	4	1	1	5	4	3

Sliding Window - M0652 Museum

We can set the start and end of the segment as our two pointers.

```
j = 1
for i ← 1 to N do
    while (a[i..j] does not contain all m integers) do
        j += 1
    ans = min(ans, j - i + 1)
```

- ➢ Counting array can be used to maintain if current segment contains all m integers.
- ➢ Remember to handle cases when `j >= N`.

- Note that `j` would not decrease when `i` increases.

Sliding Window - M0652 Museum

2	5	3	1	3	2	4	1	1	5	4	3

2	5	3	1	3	2	4	1	1	5	4	3

2	5	3	1	3	2	4	1	1	5	4	3

2	5	3	1	3	2	4	1	1	5	4	3

2	5	3	1	3	2	4	1	1	5	4	3

2	5	3	1	3	2	4	1	1	5	4	3

Sliding Window - Application

- Input is two array or one array that references itself

- the arrays are sorted, or
- the things we want to find have monotonicity (e.g. sum / count of sth)

Outline

- Partial Sum
- Difference Array
- Precomputation
- Sliding Window (Two Pointers)
- **Batching Step**
- Discretization

Batching Step

- In some simulation problems, some continuous steps can be performed at once.

Find the date of now + N days

➢ Instead of iterating over days, iterate over months
➢ Deduct Mi days from N

J184 Mysterious Area

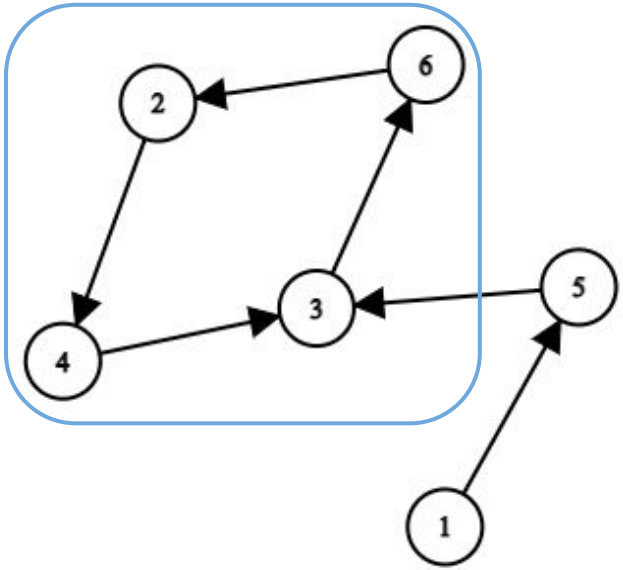
➢ Instead of moving grid by grid horizontally, directly move to the nearest pillar taller than or equal to current height

Batching Step - Find Cycles

In some simulation problems, we may encounter cycles that are traversed many times, causing TLE.

Walk X ($1 \le X \le 10^{18}$) steps starting from node 1, what is the final position?

If we simulate each step (O(X)),
we will get TLE :<



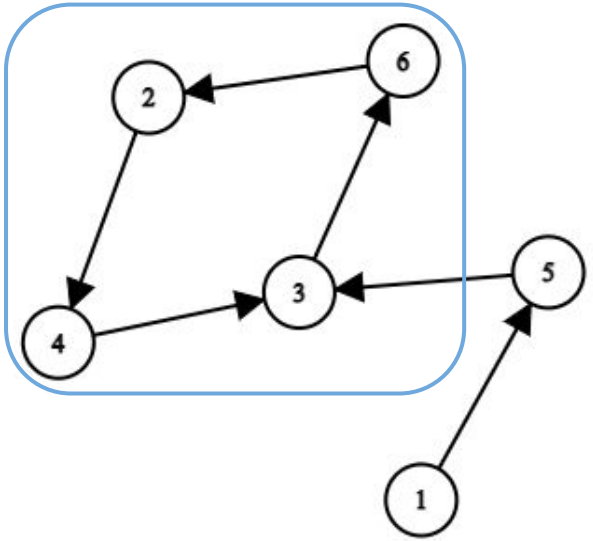
Batching Step - Find Cycles

Instead, we can skip the steps that traverse the whole cycle.

(The results won't be affected by the number of cycles traversed)

The simulation will then become:
2 steps (1 → 5 → 3)
\+ (X - 2) % 4 steps

The time complexity is now O(N)
(more details in graph sessions)



Batching Step - HKOJ T042 Game of Life IV

A terrorist has started to play a game with you, the best detective in the city, a game with life involved. A timed electronic bomb at the town center is set up to explode in M milliseconds. The bomb is equipped with a numeric display, showing a number between 1 and N inclusive at any moment. The number is 1 when the bomb is set up and it will change every millisecond.

The terrorist has left you a thread of hope. The bomb is equipped with a USB port. You can disarm the bomb, by plugging in a keyboard and typing in the number which would appear at the $(M+1)^{th}$ millisecond if the bomb does not explode. You don't have enough time to evacuate all people away from the town center, so what you can do is to crack the secret number.

A group of mathematicians have gathered to examine the numbers available on the display, and concluded that the number in the next millisecond only depends on the number appearing just before it. They have computed all such relationships and coded them in a table. You are going to write a program taking this table as input and giving the secret number as output. The bomb is about to explode. Act now!

INPUT

The first line of the input contains 2 space-separated integers, N and M, where $1 \leq N \leq 2^{21}-1$ and $1 \leq M \leq 2^{31}-1$. Each of the next N lines contains a single integer between 1 and N inclusive. The integer on the $(i+1)^{th}$ line indicates the number to which the display will change in the next millisecond if the current number is i.

OUTPUT

The output is a single number that should be typed in to save the lives.

Batching Step - HKOJ T042 Game of Life IV

- Let's simplify the statement first...
- We can view it as a directed graph with N nodes, each node has one outgoing edge.
- We need to go through M edges starting from node 1, where will we finally be at?

- How can we apply batching step to solve it?

Batching Step - HKOJ T042 Game of Life IV

- Note that there are a total of N edges, meaning that it must have a cycle
- When M is significantly larger than N, we must be stuck in a cycle for long

  ⇒ we can find out the cycle and “skip” it
  (again, implementation details in graph sessions)

Attendance Taking

Outline

- Partial Sum
- Difference Array
- Precomputation
- Sliding Window (Two Pointers)
- Batching Step
- **Discretization**

Problem

Count the number of occurrence of some numbers in array `a` (`a[i]` <= 10^9)

whereas `N` (size of `a`) <= `100000`

Naive Solutions

1. using a counting array.
   but... we probably cannot create a counting array of $[1..10^9]$:/

2. for each a[i], loop through a to count number of occurrences
   obviously, $O(N^2)$ gives TLE for N <= 100000

Discretization - Idea

- Discretization (離散法) is a technique that converts values (not necessarily integers) into integers, while maintaining their relative order

- Example: 7654321, 123456, 934602, 123456789 → 3, 1, 2, 4 (or 2, 0, 1, 3)

Discretization - Idea

- Put the values into an array, sort the array 123456, 934602, 7654321, 123456789
- For each value in the original array, find its rank using binary search *(or you can use map for this step, more in C++ STL / data structure)*
- Note: it's better for the same value to be converted into the same number *(by making the sorted array unique)*

Discretization - Solution

Count the number of occurrence of some numbers in array `a` (`a[i]` <= 10^9)

We can perform discretization and map the numbers to their rank.

A counting array can be built with size = number of **unique** numbers.

During counting/query, we can use the value's rank to access the counting array to modify/retrieve it.

Discretization - Solution

C++ Implementation

```
vector<int> v;
for (int i = 0; i < n; i++) v.push_back(a[i]);
sort(v.begin(), v.end());
v.resize(unique(v.begin(), v.end()) - v.begin());
for (int i = 0; i < n; i++)
    a[i] = lower_bound(v.begin(), v.end(), a[i]) - v.begin());
```

Demo - HKOJ C203 Discretization

More Practice

J144 Fair Santa Claus

M0652 Museum

M1613 Fair Patrol

S072 Partners

S152 Apple Garden

S211 Skyscraperhenge

T042 Game of Life IV