香港電腦奧林匹克競賽
Hong Kong Olympiad in Informatics

S264 - Triangular Territories

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Background

Problem Idea by `yellowtoad`

Solution by `QwertyPi`, `firewater`

Preparation by `yellowtoad`, `QwertyPi`, `firewater`

Problem Restatement

You are to place flags of two colors (Red and Blue) on a triangular grid.

When you place a flag on a cell, all cells on the sub-triangle starting at the flag's cell and ending at the bottom row will be coloured in the flag's colour (overwriting previous colours).

Given N, A, B, X, Y, find a flag placement sequence on a triangular grid of size N with at most A red flags and B blue flags such that the resulting grid has X red-coloured cell and Y blue-coloured cell, or report that it is impossible.

Statistics

Task	Attempts	Max	Mean	Std Dev	Subtasks							
S264 - Triangular Territories	57	57	18.561	13.873	6: 49	7: 44	18: 19	11: 9	15: 1	24: 0	12: 0	7: 0

First solved by no one.

Highest score = **57** by **WYK23F32**

Subtasks

For all subtasks, $1 \le N \le 500$, $0 \le A, B, X, Y \le \frac{N(N+1)}{2}$, $1 \le A + B, X + Y \le \frac{N(N+1)}{2}$.

Subtask	Points	Constraints
1	6	$A = 0, B = 1$
2	7	$A = 0, B = \frac{N(N+1)}{2}$
3	18	$A = 0, B \ge 2$
4	11	$A = B = 1$
5	15	$A, B \ge 2$
6	24	$A = 1, B \ge 3$
7	12	$N \le 100$
8	7	No additional constraints

Subtask 1: $A = 0, B = 1$

Since no red flags can be placed, if $X > 0$, we can immediately conclude that it is impossible.

Otherwise, it is possible if and only if Y is a **triangular number** (numbers that have the form $\frac{k(k+1)}{2}$), since only one blue flag can be placed.

Subtask 1: $A = 0, B = 1$

Check whether there exists an integer solution to the equation $\frac{k(k+1)}{2} = Y$ using a for loop.

If there exists a solution k, then placing a blue flag on cell $(N - k + 1, 1)$ always results in Y blue-coloured cells.

Special handle the case where $Y = 0$ (no flag placements are required).

Time Complexity: $O(N)$
Expected Score: 6

Subtask 2: $A = 0, B = \frac{N(N+1)}{2}$

In this subtask, we have enough blue flags to fill up the whole grid.

So, we can use Y blue flags to fill up Y cells starting from bottom to top, from left to right.

Time Complexity: $O(N^2)$
Expected Score: 7

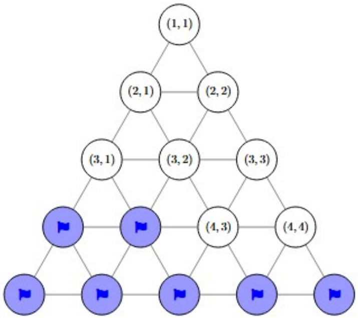


Figure: An example with $N = 5$ and $Y = 7$

Subtask 3: $A = 0, B \geq 2$

$B \geq 2$ is a strange constraint (usually upper bound is given instead of lower bound).

Could this be a hint that we do not have to handle something in this case?

Is it always possible to construct a flag sequence?

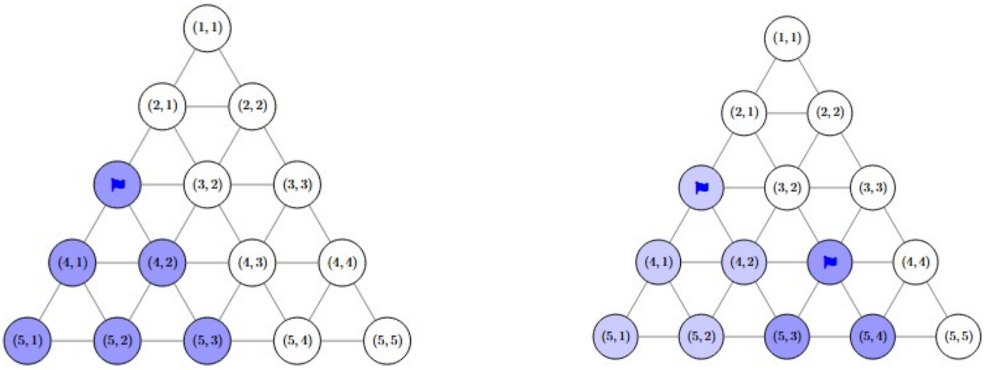
Subtask 3: $A = 0, B \geq 2$

Observation: It is always possible to colour any number of cells using 2 flags of that colour.

Construction: Let T_k be the k-th triangular number (i.e. $T_k = \frac{k(k+1)}{2}$). Let T_y be the maximum triangular number that satisfies $T_y \leq Y$. First, colour T_y cells in blue with one flag. Then, place another flag on row $(N - Y + T_y + 1)$, right next to the sub-triangle that is just being coloured (special handle if $Y = T_y$).

Subtask 3: $A = 0, B \geq 2$

Visualization of the construction of $Y = 8$ on a grid of size $N = 5$:



Subtask 3: $A = 0, B \geq 2$

Proof: Since T_y is the maximum triangular number that is less than or equal to Y, we have $Y < T_{y+1}$. Subtracting T_y from both sides, we have:

$$Y - T_y < T_{y+1} - T_y$$
$$Y - T_y < y + 1$$
$$Y - T_y \leq y$$

That means $Y - T_y$ is always smaller than the length of the sub-triangle we coloured using the first flag. So, we can always place the second flag right next to the sub-triangle on the correct row to obtain Y coloured cell.

Time Complexity: $O(1)$
Expected Score: 18 (Cumulative: 25)

Subtask 4: $A = B = 1$

WLOG, assume a red flag is placed before a blue flag.

Then, the conditions for a flag placement sequence to exist are:

1. Y is a triangular number (i.e. $Y = T_y$ for some y).
2. There exists triangular numbers T_x and T_k such that $T_k \le T_y$ and $T_x - T_k = X$. (T_x is the number of cells coloured by the first flag, and T_k is the number of cells that is overwritten by the second flag.)
3. $y + x - k \le N$. (Ensure that our solution fits inside the grid)

Subtask 4: $A = B = 1$

So, we only have to find whether there exists a solution x, y, k that satisfies all three conditions above.

To find a solution, exhaust all possible values of x $(0 \leq x \leq N)$. y and k can be derived accordingly.

Run the solution twice, one for placing a red flag first and one for placing a blue flag first.

Time Complexity: $O(N)$
Expected Score: 11

Subtask 5: $A, B \geq 2$

From subtask 3, we know that we can colour any number of cells with just 2 flags.

Can we use this fact to find a construction for any X and Y?

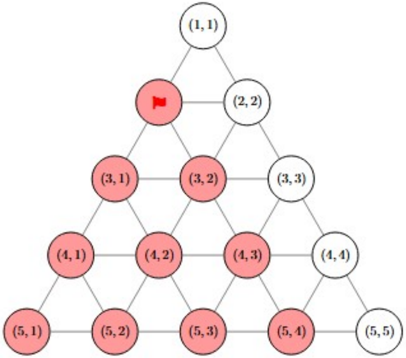
Subtask 5: $A, B \geq 2$

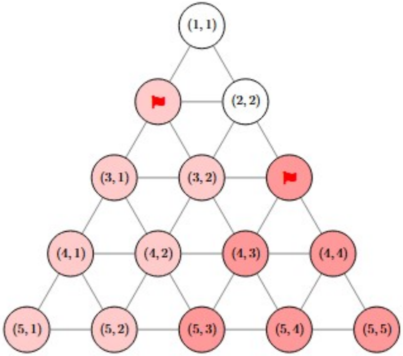
Observation: It is always possible to construct a flag placement sequence when $A, B \geq 2$.

Construction: First, colour $X + Y$ cells in red with 2 red flags. Then, overwrite Y previously-coloured cells in blue with 2 blue flags.

Subtask 5: $A, B \geq 2$

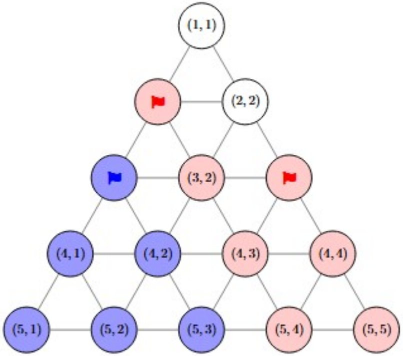
Visualization of the construction of $X = 5, Y = 8$ on a grid of size $N = 5$:

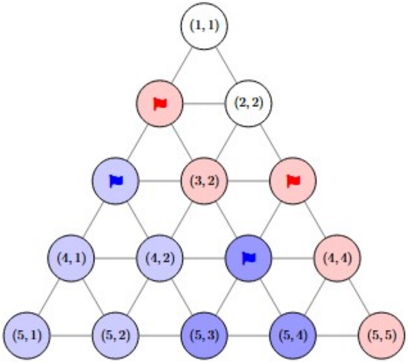




Subtask 5: $A, B \geq 2$

Visualization of the construction of $X = 5$, $Y = 8$ on a grid of size $N = 5$:





Subtask 5: $A, B \geq 2$

Is it always possible to fit the Y blue-coloured cells entirely inside the $X + Y$ red-coloured cells?

Yes! (Proof is left as exercise for readers)

Time Complexity: $O(1)$
Expected Score: 15

Subtask 6: $A = 1, B \geq 3$

We have only 1 red flag, but it is guaranteed that we have at least 4 flags in total.

Can we still use the idea of the construction from subtask 5?

In some cases, yes.

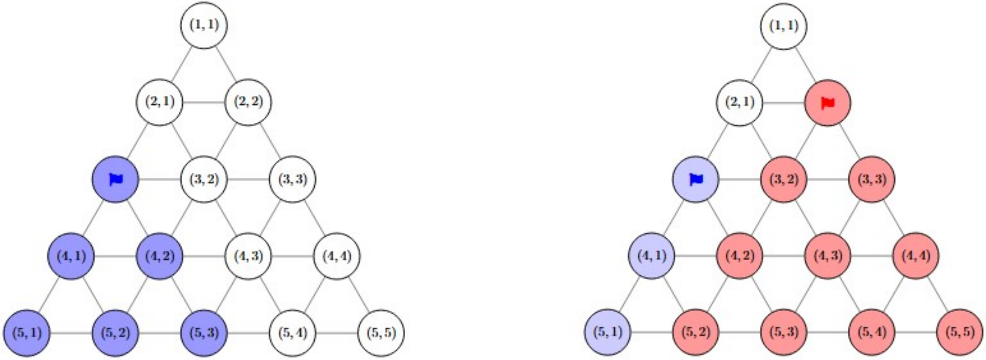
Subtask 6: $A = 1, B \geq 3$

Construction: Construct $X + Y$ with one blue flag (placed first) and one red flag (placed second), such that there are T_x cells coloured in red and $X + Y - T_x$ cells coloured in blue, where T_x is the maximum triangular number less than or equal to $X + Y$.

Then, overwrite $T_x - X$ red-coloured cells in blue. The resulting grid will have X red-coloured cells and Y blue-coloured cells.

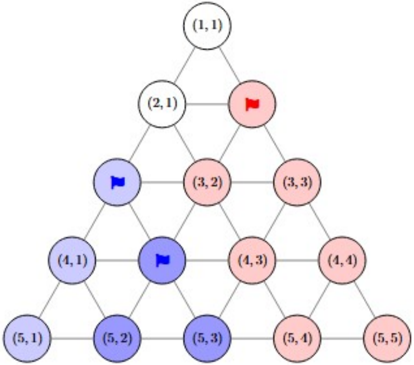
Subtask 6: $A = 1, B \geq 3$

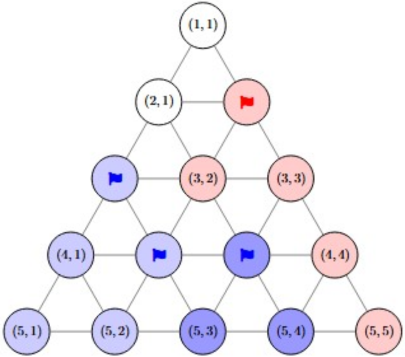
Visualization of the construction of $X = 5$, $Y = 8$ on a grid of size $N = 5$:



Subtask 6: $A = 1, B \geq 3$

Visualization of the construction of $X = 5$, $Y = 8$ on a grid of size $N = 5$:





Subtask 6: $A = 1, B \geq 3$

Is it always the possible to construct any X, Y with this construction? No!

If $X > T_x$, then we cannot 'overwrite $T_x - X$ red-coloured cells in blue', because $T_x - X$ is negative.

Subtask 6: $A = 1, B \geq 3$

Consider the case where $X > T_x$.

We must colour at least T_{x+1} cells in red with our only red flag, otherwise it is impossible to obtain X red cells.

Since T_x is the largest triangular number that is less than or equal to $X + Y$, we have $T_{x+1} > X + Y$.

We know that if a cell is coloured, it will always remain coloured in one of the colours.

So, after colouring T_{x+1} cells in red, it is impossible obtain a grid with $X + Y$ coloured cells.

Subtask 6: $A = 1, B \geq 3$

So, if $X > T_x$, we know that it is impossible to find a flag placement sequence.

Otherwise, our construction always works.

Time Complexity: $O(1)$
Expected Score: 24

Subtask 8: No additional constraints

Some implementation tricks we can use to facilitate our case handling:

- We don't need to fix the triangular grid at the beginning. We can translate them in the end to fit inside the grid.
- We can reflect the whole grid horizontally, so we can exhaust only half of the cases.
- When $A > B$, we can swap A and B, as well as X and Y to reuse the same code.

The only case we didn't solve yet is $A = 1, B = 2$, where we need to exhaust all possible orders of flags.

Subtask 8: No additional constraints

Case 1: Order of the flags is blue, blue, red.

X must be a triangular number.
We can directly run the solution of subtask 5, using only one red flag.

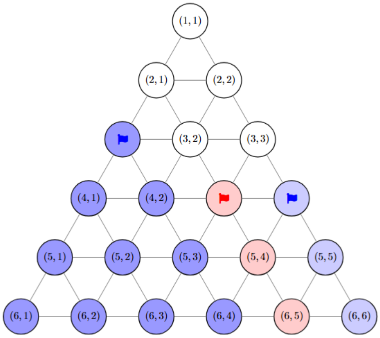
Subtask 8: No additional constraints

Case 2: Order of the flags is blue, red, blue.

Exhaust the following:

- Height (x-coordinate) of the red flag
- Coordinates of the first blue flag relative to the red flag

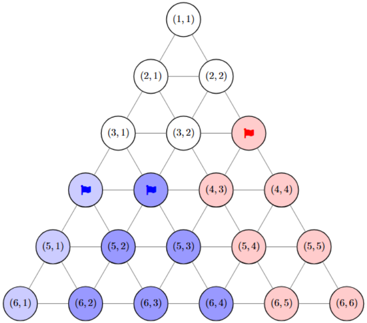
We can calculate the height of the second blue flag by subtracting Y from the number of cells colored by the first blue flag.
Then we can then shift it horizontally until the number of cells colored in red is X.

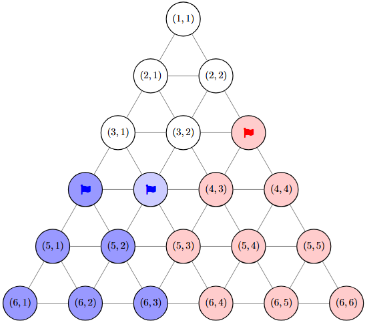


Subtask 8: No additional constraints

Case 2: Order of the flags is blue, red, blue.

Some other placement cases that you need to handle:

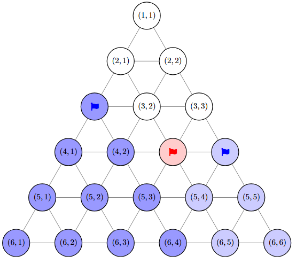




Subtask 8: No additional constraints

Case 3: Order of the flags is red, blue, blue, and the second blue flag is outside the red triangle.

The exhaustion method is the same as in case 2. Beware that the first blue triangle and the second blue triangle may overlap.

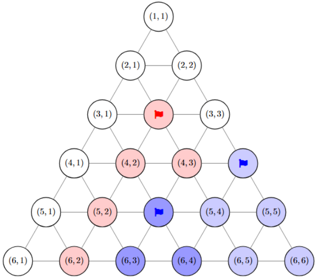


Subtask 8: No additional constraints

Case 4: Order of the flags is red, blue, blue, and the last flag is inside the red triangle.
Exhaust the following:

- Height of the red flag
- Height of the intersection of the red triangle and the second blue triangle
- Height of the intersection of two blue triangles

We can calculate the coordinates of the second blue flag by the number of extra red-coloured cells. Then, we can calculate the height of the second blue flag by the remaining number of blue-coloured cells.



Subtask 8: No additional constraints

If you exhausted *enough* cases, you will get Accepted!

Time Complexity: $O(N^3)/O(N^3 \log N)/O(N^4)$ (based on your implementation)

Expected Score: 100/93