香港電腦奧林匹克競賽
Hong Kong Olympiad in Informatics

G263 - Screws

Casper Wong {gasbug}

2026-02-14

Background

Problem Idea by `gasbug`

Preparation by `yellowtoad`

The Problem

There are N screws to be tightened in order, numbered from 1 to N. Screw i has diameter D_i.

There are 10^9 screwdrivers, each with a unique size ranging from 1 to 10^9. A screw of diameter x can be tightened with a screwdriver of size y if and only if $x - K \leq y \leq x + K$, where K is a given constant.

Find the minimum number of times he needs to swap his current screwdriver with another in order to tighten all screws.

Statistics

Task	Attempts	Max	Mean	Std Dev	Subtasks					
G263 - Screws	100	100	24.8	33.092	7: 59	9: 32	17: 42	24: 17	23: 19	20: 11

First solved by **dgs221195** at **46m 26s**

Subtasks

For all subtasks, $1 \le N \le 10^5$, $0 \le K \le 10^9$, $1 \le D_i \le 10^9$ for all $1 \le i \le N$.

Subtask	Points	Constraints
1	7	$D_i \le K$ for all $1 \le i \le N$
2	9	$N = 2$
3	17	$K = 0$
4	24	The answer must be either 0 or 1
5	23	$D_{i-1} \le D_i$ for all $2 \le i \le N$
6	20	No additional constraints

Subtask 1: $D_i \leq K$ for all $1 \leq i \leq N$

Note that a screwdriver of size K can be used to tighten all N screws ($0 \leq D_i \leq 2K$).

So, we can choose screwdriver K at first and use it to tighten all screws without requiring any screwdriver swaps.

Hence, the output is always 0.

Expected Score: 7

Subtask 2: $N = 2$

Note that the upper bound for the output is 1. (First use screwdriver of size D_1 to tighten screw 1, then swap it to the screwdriver of size D_2 and use it to tighten screw 2.)

So, we only have to determine whether we can do it with 0 swaps (i.e. whether there exists a screwdriver that can tighten both screw 1 and 2).

Subtask 2: $N = 2$

If there exists a screwdriver of size x that is able to tighten both screw 1 and 2, then x must satisfy both $D_1 - K \leq x \leq D_1 + K$ and $D_2 - K \leq x \leq D_2 + K$.

So, x exists if and only if $\max(D_1 - K, D_2 - K) \leq \min(D_1 + K, D_2 + K)$.

Rearranging the terms in the above inequality:

$$\max(D_1 - K, D_2 - K) \leq \min(D_1 + K, D_2 + K)$$
$$\max(D_1, D_2) - K \leq \min(D_1, D_2) + K$$
$$\max(D_1, D_2) - \min(D_1, D_2) \leq 2K$$
$$|D_1 - D_2| \leq 2K$$

Subtask 2: $N = 2$

So, we can use a screwdriver to tighten both screws if and only if the difference between D_1 and D_2 is less than or equal to $2K$.

Use an if-clause to check whether the above inequality is true. If the inequality is true, output 0, otherwise output 1.

Expected Score: 9

Subtask 3: $K = 0$

$K = 0$ implies that a screwdriver can only tighten screws that have diameter equal to the screwdriver's size.

It means that whenever the diameter of a screw differs from the diameter of the last screw tightened, we must swap the screwdriver.

So, the answer is equal to the number of i where $D_i \neq D_{i-1}$, for all $2 \leq i \leq N$.

Expected Score: 17

Subtask 4: The answer must be either 0 or 1

It is guaranteed that the upper bound for the answer is 1.

So, we only have to determine whether we can do it with 0 swaps (i.e. whether there exists a screwdriver that can tighten all screws).

Subtask 4: The answer must be either 0 or 1

If there exists a screwdriver of size x that is able to tighten all screws then x must satisfy $D_i - K \leq x \leq D_i + K$ for all $1 \leq i \leq N$.

So, x exists if and only if $\max_{i=1}^{N}(D_i - K) \leq \min_{i=1}^{N}(D_i + K)$.

Rearranging the terms in the above inequality:

$$\max_{i=1}^{N}(D_i - K) \leq \min_{i=1}^{N}(D_i + K)$$

$$\max_{i=1}^{N}(D_i) - K \leq \min_{i=1}^{N}(D_i) + K$$

$$\max_{i=1}^{N}(D_i) - \min_{i=1}^{N}(D_i) \leq 2K$$

Subtask 4: The answer must be either 0 or 1

Use for loop to find $\max_{i=1}^{N}(D_i)$ and $\min_{i=1}^{N}(D_i)$. Then, check if the inequality is true. If it is true, the answer is 0. Otherwise, the answer must be 1.

Expected Score: 40

Subtask 5: $D_{i-1} \leq D_i$ for all $2 \leq i \leq N$

From subtask 4, we know that it is possible to tighten all screws from l to r with a single screwdriver if and only if $\max_{i=l}^{r}(D_i) - \min_{i=l}^{r}(D_i) \leq 2K$.

In this subtask, D is non-decreasing. So, the inequality $\max_{i=l}^{r}(D_i) - \min_{i=l}^{r}(D_i) \leq 2K$ can be reduced to $D_r - D_l \leq 2K$.

Note that using the same screwdriver to tighten as many screws as possible 'never makes things worse'.

So, we can then use a greedy approach to find the optimal positions to swap the screwdrivers.

Subtask 5: $D_{i-1} \le D_i$ for all $2 \le i \le N$

Greedy Algorithm: For each $1 \le i \le N$, let l be the position where we perform the last swap (initially, $l = 1$). We maintain the value of l in the following way:

- If $D_i - D_l \le 2K$, we know that we can tighten all screws from l to i using the same screwdriver, so we keep l unchanged and move on.
- If $D_i - D_l > 2K$, we cannot tighten all screws from l to i using the same screwdriver. So, we tighten all screws from l to $i - 1$ with a screwdriver, then add 1 to our answer, indicating the swap that must be performed between tightening screw $i - 1$ and screw i. Then, set l to i.

Expected Score: 23

Subtask 6: No additional constraints

We can no longer conclude whether it is possible to tighten all screws from l to r with a single screwdriver simply by checking if the inequality $D_r - D_l \leq 2K$ is satisfied.

However, we can still check this using the inequality $\max_{i=l}^{r}(D_i) - \min_{i=l}^{r}(D_i) \leq 2K$.

A greedy approach still works here because using the same screwdriver to tighten as many screws as possible 'never makes things worse'.

Subtask 6: No additional constraints

When we loop through $1 \leq i \leq N$ in our greedy algorithm, in addition to l, we maintain 2 more values $mx = \max_{j=l}^{i} D_j$ and $mn = \min_{j=l}^{i} D_j$.

Use mx and mn to check whether a swap needs to be performed (check if $mx - mn \leq 2K$).

Expected Score: 100