S224 - Connectors

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Background

Author: gabrielliu2001

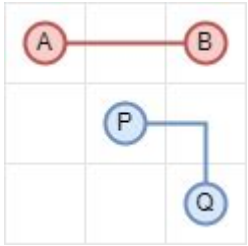
Setters: gabrielliu2001, trashcan, cmli

Pictures: gabrielliu2001, trashcan

Problem Restatement

In a R x C grid, connect two red connectors and two blue connectors respectively without overlapping, or conclude that it's impossible

Sample 1 Input	Output
3 3 1 1 1 3 2 2 3 3	Possible 3 1 1 1 2 1 3 3 2 2 2 3 3 3



Sample 4 Input	Output
4 4 2 2 3 3 3 2 2 3	Impossible

Subtasks

For all cases:

$1 \le R, C \le 100$

$1 \le R_A, R_B, R_P, R_Q \le R$

$1 \le C_A, C_B, C_P, C_Q \le C$

$R \times C \ge 4$

PARTIAL SCORE PROVIDED

100%: All correct

40%: Determine Possible/Impossible correctly

	Points	Constraints
1	16	$R = 2$ $C = 3$
2	25	All four connectors are located on the boundary of the grid Boundary of the grid: $(1,1), (1,2), \ldots, (1,C), (2,C), \ldots, (R,C), (R,C-1), \ldots (R,1), (R-1,1), \ldots (2,1)$
3	21	Exactly three connectors are located on the boundary of the grid
4	38	No additional constraints

Statistics

Task	Attempts	Max	Mean	Std Dev	Subtasks			
S224 - Connectors	41	100	17.497	33.346	16: 8 6.4: 7	25: 7 10: 3	21: 6 8.4: 1	38: 5 15.2: 1

First solved by **mlwong** at **1:37:52**

Subtask 1

R = 2

C = 3

Exhaustion / Case handling (very annoying in this task)

Perhaps get some insights from small cases (?)

Subtask	Score	Max Score
1	16	16
2	0	25
3	0	21
4	0	38
Total	16	100

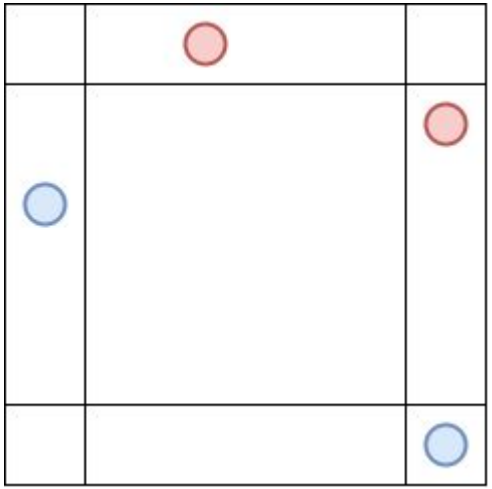
Subtask 2

All four connectors are located on the boundary of the grid

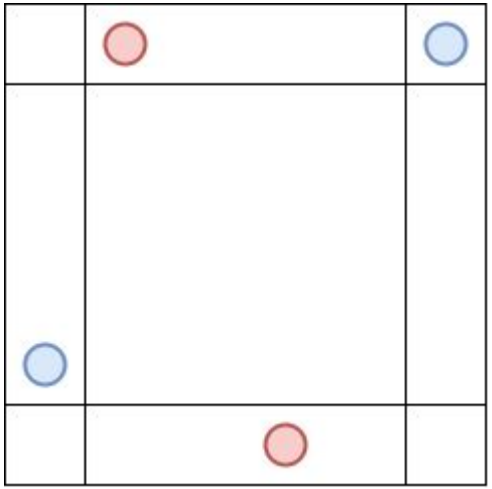
What does this constraint tell us?

Subtask 2

Possible Case:

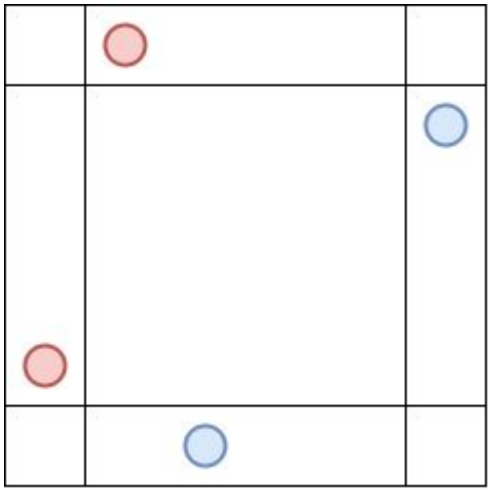


Impossible Case:



Implementation

Start from any cell of the boundary, traverse the boundary in clockwise/anti-clockwise direction and record the order of the connectors



Be careful about cases like “RBBR”

What’s the answer for:

- R = 1 or C = 1?
- R > 1 and C > 1?

Subtask 2

If you take a look at the judge results page, or think a bit about this subtask, you will notice that Subtask 2 includes Subtask 1

Subtask	Score	Max Score
1	16	16
2	25	25
3	0	21
4	0	38
Total	**41**	**100**

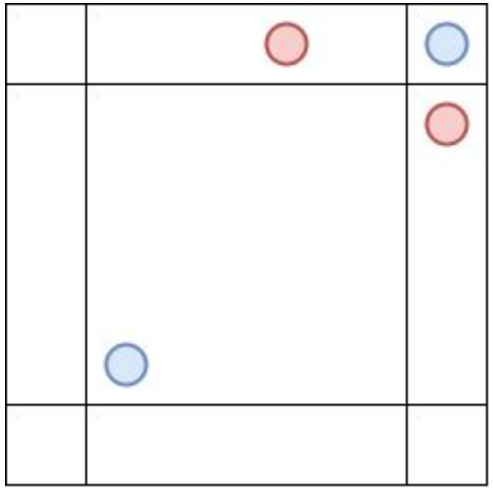
Subtask 3

Exactly three connectors are located on the boundary of the grid

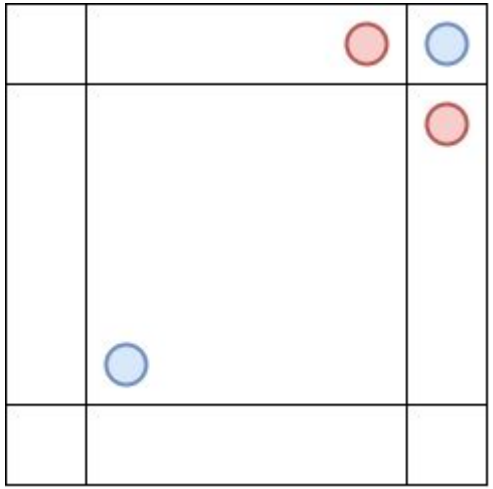
What does this constraint tell us this time?

Subtask 3

Possible Case:



Impossible Case:



Implementation

If the answer is “Possible”, the pair of connectors on the boundary doesn’t need to use any cells that is not on the boundary

Try both clockwise/anti-clockwise path for the pair of connectors on the boundary

For the remaining pair, use your favourite path finding algorithm (DFS/BFS/whatever) to check if there’s a path to connect them

Subtask 3

If you implement the solution carefully, you can get the first 3 subtasks correct

Subtask	Score	Max Score
1	16	16
2	25	25
3	21	21
4	0	38
Total	**62**	**100**

Full Solution

The idea from Subtask 3 can be extended

For one pair of connectors, use only the boundary and all necessary cells for reaching the boundary

Exhaust all possible directions to make life easier

For the remaining pair, again use your favourite path finding algorithm to check if there's a path connecting them

If it doesn't work for the first chosen pair, swap the pairs and try again

Full Solution 2

Lemma: if a solution exists, there exists a solution which at least one pair of connectors can use any of its shortest path

Using this lemma, a BFS solution can be implemented

Full Solution 2

The BFS solution:

1. BFS to find a path for the red pair connectors
2. BFS to find a path for the blue pair connectors without crossing Step 1's path
3. If a solution is found, output; else
4. BFS to find a path for the blue pair connectors
5. BFS to find a path for the red pair connectors without crossing Step 4's path
6. If a solution is found, output; else it's impossible

Full Solution 3

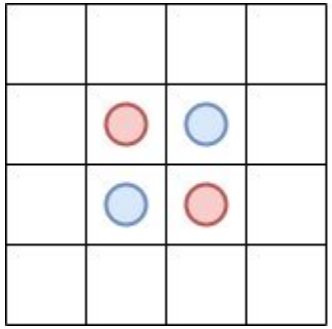
Lemma: if a solution exists, there exists a solution which at least one pair of connectors can use a path with at most 3 turns

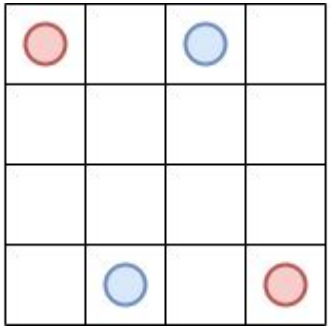
Using this lemma, a solution using only DFS twice can also be implemented

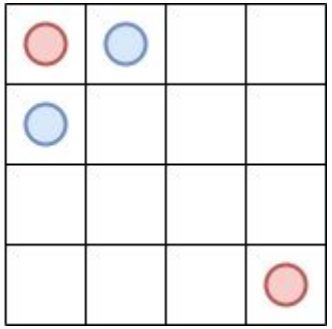
Other Notes

There are only 3 types of impossible cases, which can be found in Sample 4, Subtask 2 and Subtask 3

You can obtain partial score if you try to determine them in your code



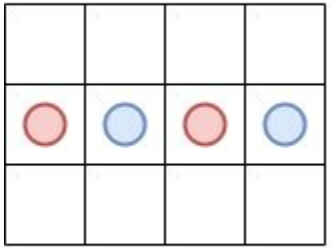


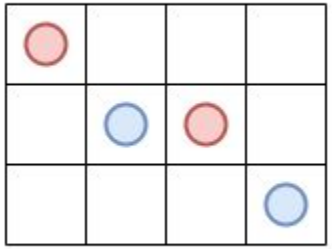


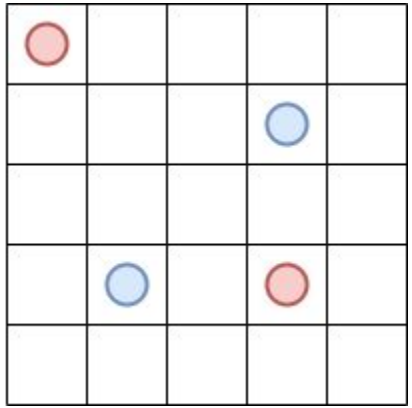
Other Notes

You can even check “Possible/Impossible” first, then for finding the first path, using some heuristic strategies to simulate BFS in Full Solution 2, or at most 3 turns in Full Solution 3

Depends on your implementation, you can pass various subtasks, or even get Accepted







Other Notes

If you attempt full score for any subtasks, partial score also helps you check if you determine “Possible/Impossible” correctly or not, which can further help you check if you find the solution paths correctly or not