J211 - Paint Shop

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Background

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Task

7 colors of paints: cyan, magenta, yellow, black, red, green, and blue

Each paint price is an integer price per litre.

Obtain at least **N** litres of paint of a given color by buying and/or mixing the paints using the following formulas:

1L cyan + 1L magenta	→	2L blue
1L magenta + 1L yellow	→	2L red
1L cyan + 1L yellow	→	2L green

1L cyan + 1L magenta + 1L yellow	→	3L black
1L red + 1L green + 1L blue	→	3L black
1L cyan + 2L red	→	3L black
1L magenta + 2L green	→	3L black
1L yellow + 2L blue	→	3L black

Sample 1

	cyan	magenta	yellow	black	red	green	blue
Price	**10**	**10**	**10**	**15**	**15**	**15**	**15**

Required: **7** litres of red

1L cyan + 1L magenta	→	2L blue
1L magenta + 1L yellow	→	2L red
1L cyan + 1L yellow	→	2L green

3L magenta + 3L yellow → 6L red
1L red

1L cyan + 1L magenta + 1L yellow	→	3L black
1L red + 1L green + 1L blue	→	3L black
1L cyan + 2L red	→	3L black
1L magenta + 2L green	→	3L black
1L yellow + 2L blue	→	3L black

Answer = **3 * 10 + 3 * 10 + 15**
= **75**

Sample 2

	cyan	magenta	yellow	black	red	green	blue
Price	**10**	**10**	**10**	**25**	**25**	**25**	**25**

Required: **7** litres of red

1L cyan + 1L magenta	→	2L blue
1L magenta + 1L yellow	→	2L red
1L cyan + 1L yellow	→	2L green

4L magenta + 4L yellow → 8L red

Answer = **4 * 10 + 4 * 10**

= **80**

1L cyan + 1L magenta + 1L yellow	→	3L black
1L red + 1L green + 1L blue	→	3L black
1L cyan + 2L red	→	3L black
1L magenta + 2L green	→	3L black
1L yellow + 2L blue	→	3L black

Sample 3

	cyan	magenta	yellow	black	red	green	blue
Price	**1**	**2**	**3**	**4**	**3**	**2**	**1**

Required: **3** litres of black

1L cyan + 1L magenta	→	2L blue
1L magenta + 1L yellow	→	2L red
1L cyan + 1L yellow	→	2L green

1L yellow + 2L blue → 3L black

1L cyan + 1L magenta + 1L yellow	→	3L black
1L red + 1L green + 1L blue	→	3L black
1L cyan + 2L red	→	3L black
1L magenta + 2L green	→	3L black
1L yellow + 2L blue	→	3L black

Answer = **3** + **2** * **1**

= **5**

Subtasks

For all cases:

$1 \leq N \leq 100000$

$1 \leq$ price of any color per litre ≤ 100

	Points	Constraints
1	11	$1 \leq N \leq 10$ S is always `cyan`, `magenta` or `yellow`.
2	14	$1 \leq N \leq 10$ N is an even number. S is always `red`.
3	23	$1 \leq N \leq 10$ S is always `red`, `green` or `blue`.
4	19	$1 \leq N \leq 10$ N is a multiple of 3. S is always `black`.
5	27	$1 \leq N \leq 10$ S is always `black`.
6	6	No additional constraints

Statistics

Task	Attempts	Max	Mean	Std Dev	Subtasks					
J211 - Paint Shop	75	100	76.84	33.51	11: 69	14: 68	23: 61	19: 57	27: 48	6: 45

First solved by **ryanjz2024** at **0:14:39**

Last solved by **dbsic** at **2:29:52** (8 seconds before the contest ends)

Easiest task among all three junior tasks

More **Accepted**, More Happiness ^_^

Solutions

Subtask 1

1 ≤ N ≤ 10

The color is always cyan, magenta or blue

- We can only obtain these 3 colors by buying them from the shop
- Answer = price of the color * **N**

Subtask 2

1 ≤ N ≤ 10

N is an even number

The color is always red

- Option 1: buy **N** litres of red directly
- Option 2: buy **N** / **2** litres magenta + **N** / **2** litres yellow

- Answer = min(Option 1, Option 2)

1L cyan + 1L magenta	→	2L blue
1L magenta + 1L yellow	→	2L red
1L cyan + 1L yellow	→	2L green

Subtask 3

1 ≤ N ≤ 10

The color is always red, green or blue

Using red as example:

- Option 1: buy **N** litres of red directly
- Option 2: buy ceil(**N** / **2**) litres of magenta + ceil(**N** / **2**) litres of yellow

- Enough?

1L cyan + 1L magenta	→	2L blue
1L magenta + 1L yellow	→	2L red
1L cyan + 1L yellow	→	2L green

Subtask 3

- Of course not, sample 1 already reminds you ^_^

- For **N** is even, same as subtask 2
- For **N** is odd, we consider the followings:
  - First **N** - **1** litres: min((**N** - **1**) * **red**, (**N** - **1**) / **2** * (**magenta** + **yellow**))
  - Last **1** litre: min(**red**, **magenta** + **yellow**)

- Answer = answer for **N** - **1** litres + answer for **1** litre

- Same way to deal with green and blue

Subtask 4

1 ≤ N ≤ 10

N is a multiple of **3**

The color is always black

- How many options this time?

1L cyan + 1L magenta	→	2L blue
1L magenta + 1L yellow	→	2L red
1L cyan + 1L yellow	→	2L green

1L cyan + 1L magenta + 1L yellow	→	3L black
1L red + 1L green + 1L blue	→	3L black
1L cyan + 2L red	→	3L black
1L magenta + 2L green	→	3L black
1L yellow + 2L blue	→	3L black

Subtask 4

- First, undoubtedly, the 5 given formulas

1L cyan + 1L magenta	→	2L blue
1L magenta + 1L yellow	→	2L red
1L cyan + 1L yellow	→	2L green

- But then seems more complicated formulas exist!?
  - 1L magenta + 1L yellow → 2L red
  - Take 1L red from above, 1L red + 1L green + 1L blue → 3L black
  - ...

1L cyan + 1L magenta + 1L yellow	→	3L black
1L red + 1L green + 1L blue	→	3L black
1L cyan + 2L red	→	3L black
1L magenta + 2L green	→	3L black
1L yellow + 2L blue	→	3L black

Subtask 4

- In fact, the given formulas are sufficient to get optimal answer
- Do you know why?

Example

Let's prove that **magenta** + **yellow** + **green** + **blue** isn't optimal as an example

Denote **m** = **magenta**, **y** = **yellow**, **g** = **green**, **b** = **blue**

- Suppose **g** < **b**, we have (**m** + **y** + **g** + **b**) > (**m** + **y** + **g** + **g**) > (**m** + **g** + **g**)
- We can see that (**m** + **g** + **g**) is a better formula
- Similar proof can be done for **g** ≥ **b**

1L cyan + 1L magenta + 1L yellow	→	3L black
1L red + 1L green + 1L blue	→	3L black
1L cyan + 2L red	→	3L black
1L magenta + 2L green	→	3L black
1L yellow + 2L blue	→	3L black

Remaining proof is left as exercise

Subtask 4

- Knowing that the given formulas are sufficient to get optimal answer, we just need to consider all of them & consider buying black directly
  - Option 1: **N / 3** * (**cyan** + **magenta** + **yellow**)
  - Option 2: **N / 3** * (**red** + **green** + **blue**)
  - Option 3: **N / 3** * (**cyan** + **2** * **red**)
  - Option 4: **N / 3** * (**magenta** + **2** * **green**)
  - Option 5: **N / 3** * (**yellow** + **2** * **blue**)
  - Option 6: **N** * **black**

1L cyan + 1L magenta + 1L yellow	→	3L black
1L red + 1L green + 1L blue	→	3L black
1L cyan + 2L red	→	3L black
1L magenta + 2L green	→	3L black
1L yellow + 2L blue	→	3L black

- Answer = min(Option 1, Option 2, ... , Option 6)

Subtask 5

1 ≤ N ≤ 10

The color is always black

- **N** is not a multiple of **3** anymore
- What do we need to extra handle this time?

Subtask 5

- For the remainder of **N** / **3** = **0**, same as subtask 4

- For the remainder of **N** / **3** = **1**, what should we do?

Subtask 5

- Using similar idea from subtask 3, we consider two parts:
  - First (**N** - **1**) litres: same as subtask 4
  - Last **1** litre: min(Option 1, Option 2, ... , Option 6)
    - Option 1: **cyan** + **magenta** + **yellow**
    - Option 2: **red** + **green** + **blue**
    - Option 3: **cyan** + **2** * **red**
    - Option 4: **magenta** + **2** * **green**
    - Option 5: **yellow** + **2** * **blue**
    - Option 6: **black**

- Answer = answer for **N** - **1** litres + answer for **1** litre

Subtask 5

- The same idea can be applied when the remainder of **N** / **3** = **2**:
  - First (**N** - **2**) litres: same as subtask 4
  - Last **2** litre: min(Option 1, Option 2, ... , Option 6)
    - Option 1: **cyan** + **magenta** + **yellow**
    - Option 2: **red** + **green** + **blue**
    - Option 3: **cyan** + **2** * **red**
    - Option 4: **magenta** + **2** * **green**
    - Option 5: **yellow** + **2** * **blue**
    - Option 6: **2** * **black**

- Answer = answer for **N** - **2** litres + answer for **2** litre

Subtask 6

No additional constraints

- Combine subtask 1 to 5 to get the last 6 points!

Some Careless Mistakes

Pascal:

- Using **`integer`** but not **`longint`**, losing last 6 points :(
- **`read(N, Col)`** / **`readln(N, Col)`** leads to extra space
  - `Col = ‘ cyan’`

C++:

- Using **`double`** to output, resulting in wrong format
  - `2.9232e+06`

Implementation Tricks

For finding minimum, instead of writing brunches of **`if`**'s, we can make good use of **min** function and **arrays**

Example of subtask 5 when remainder of **N** / **3** = **1**:

```
opt[1] = c + m + y;
opt[2] = r + g + b;
opt[3] = c + 2 * r;
opt[4] = m + 2 * g;
opt[5] = y + 2 * b;

ans[1] = (N - 1) * k;
for (int i = 1; i <= 5; i++)
  ans[1] = min(ans[1], (N - 1) / 3 * opt[i]);

ans[2] = k;
for (int i = 1; i <= 5; i++)
  ans[2] = min(ans[2], opt[i]);
```