J193 Hyper Knight II

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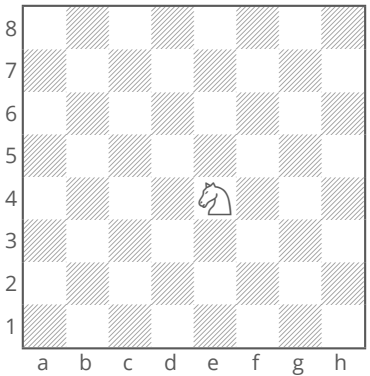
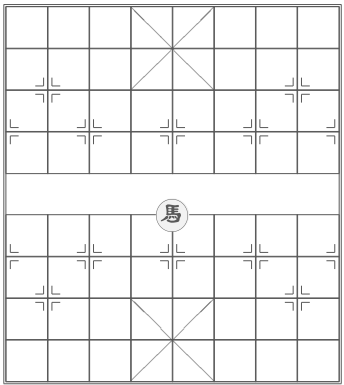
Introduction





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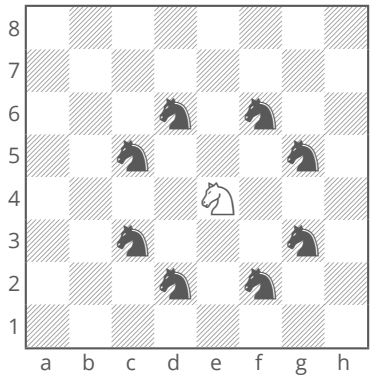

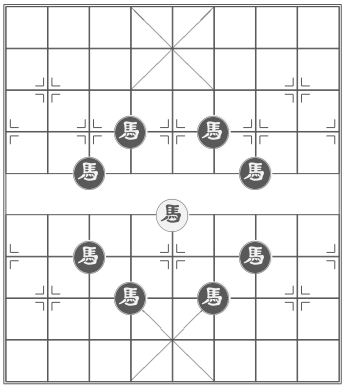

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- Generalize to 3D
  - 2D knight move: move 2 units along one axis, 1 unit along the other axis
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- The chessboard has no boundary
  - Every point with integral coordinates is a valid position

Sample

Input

1 1 -2
128000

Output

2
1 1 -2
1 2 0
0 0 0

Task

Find a path from $(1, 1, -2)$ to $(0, 0, 0)$ with at most 128000 steps

Solution

$(x_1, y_1, z_1) = (1, 1, -2)$
$(x_2, y_2, z_2) = (1, 2, 0)$ $\quad 0, +1, +2$
$(x_3, y_3, z_3) = (0, 0, 0)$ $\quad -1, -2, 0$
2 steps in total

Statistic

Subtask	Score	Number of Solves
#1	16	9
#2	19	18
#3	11	9
#4	12	6
#5	16	2
#6	26	2

Max. = 100

`hkoi19-lys` (0:59) `Pascal` 165 lines 3298 bytes

`mtyeung1` (2:21) `C++` 120 lines 2646 bytes

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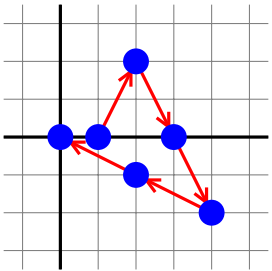
author `C++` 51 lines 1029 bytes

Subtask 0

$K = \infty$



An image from the problem statement

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  - $(0, Y, 0)$
  - $(X, 0, 0)$

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  - $(1, 0, 0)$
  - $(0, 0, Z)$
  - $(0, Y, 0)$
  - $(X, 0, 0)$
  - (X, Y, Z)

Subtask 0

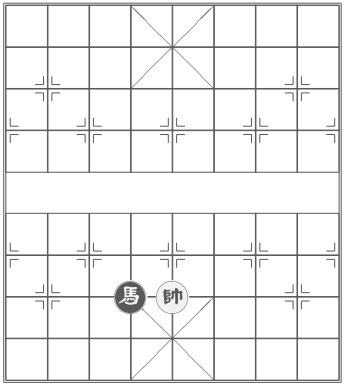
$K = \infty$

Number of small steps in each large step $= 5$

Number of steps $= 5(|X| + |Y| + |Z|) \leq 180000$

Subtask 1

$K = 128000$



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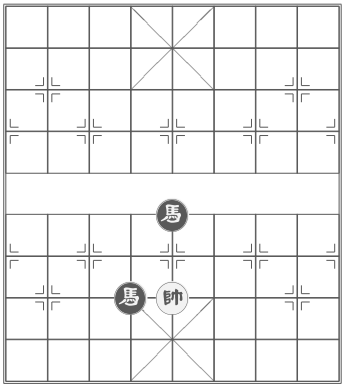

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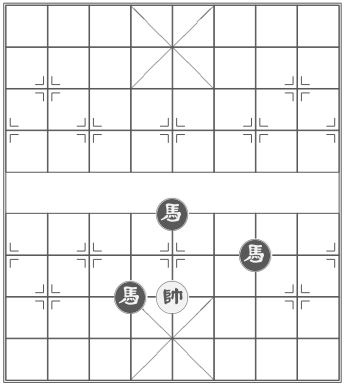

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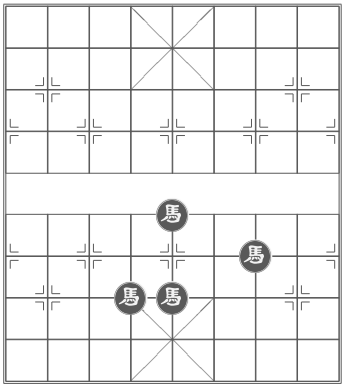

Subtask 1

$K = 128000$

Number of small steps in each large step $= 3$

Number of steps $= 3(|X| + |Y| + |Z|) \leq 108000$

Subtask 2

$K = 25600$

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- Then follow the original strategy

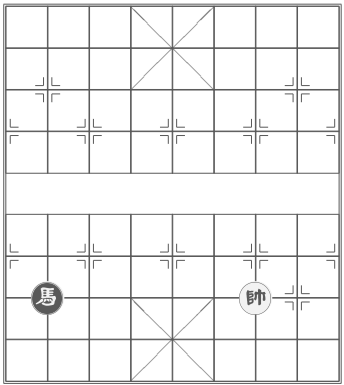
Subtask 2, 3, 4

$K = 25600$

- Note that we can move 2 units in one dimension in each step with a side effect in another dimension
- We can cancel out the side effect in the 2^{th} step (How?)
- Repeat them until we have less than 4 units left (for each dimension)
- Then follow the original strategy

Subtask 2, 3, 4

$K = 25600$



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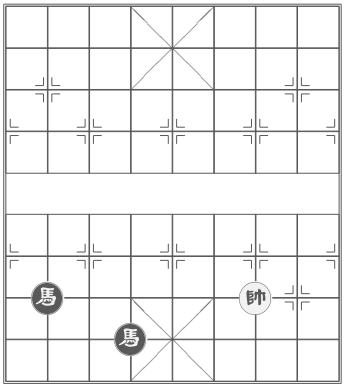

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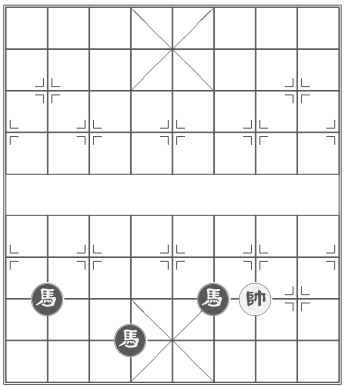

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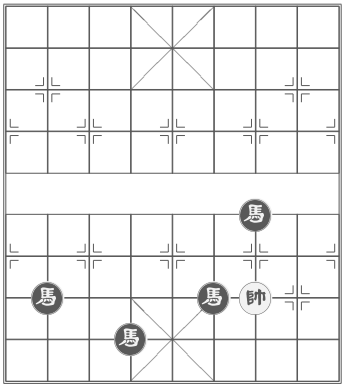

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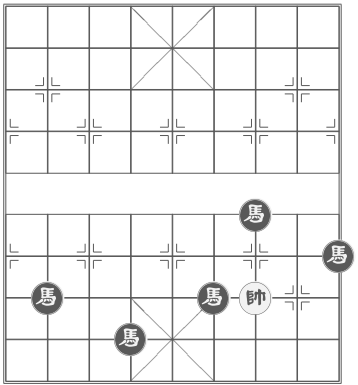

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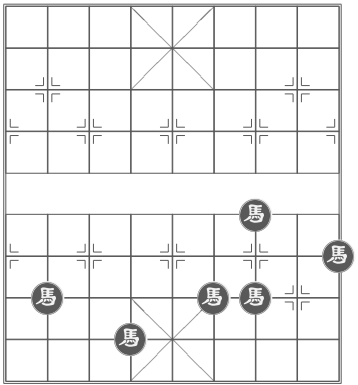

Subtask 2, 3, 4

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Upper bound of number of steps $\approx \frac{|X|+|Y|+|Z|}{2} = 18000$

Subtask 5

$K \geq 14400$

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- We cannot waste the side effect
- Choose another dimension with non-zero coordinate and reduce it
- Worst case: only one dimension left (only its coordinate has absolute value ≥ 2)
- Then fallback to the last strategy

Subtask 5

$K \geq 14400$

Upper bound of number of steps $\approx \frac{|X|+|Y|}{3} + \frac{|Z|}{2} = 14000$

Subtask 6

$K \geq 12800$

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- Greedy
  - dimension with largest absolute coordinate $\rightarrow \pm 2$
  - dimension with second largest absolute coordinate $\rightarrow \pm 1$

Subtask 6

$K \geq 12800$

- The worst case is too bad
- Improve our choice for the chosen dimensions for each step
- Greedy
  - dimension with largest absolute coordinate $\rightarrow \pm 2$
  - dimension with second largest absolute coordinate $\rightarrow \pm 1$
- If the absolute values of all coordinates are within a 2-unit range, then it remains after the above step

Subtask 6

$K \geq 12800$

Upper bound of number of steps $\approx \frac{|X|+|Y|+|Z|}{3} = 12000$

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Minimize the number of steps

- The number of steps by the last strategy is close to the theoretical lower bound

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- Hint: breadth-first search (BFS) (one of the possible solutions)
- (Prove your solution)