Graph IV

Ian

Things that we would talk about

- DFS
- Tree
- Connectivity

Useful website

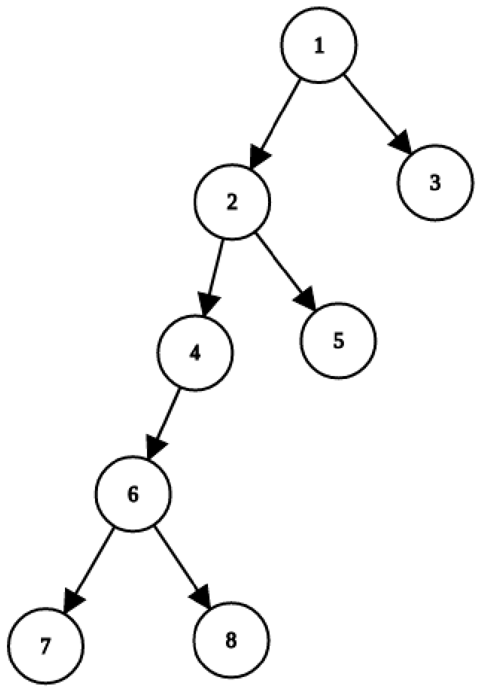
http://codeforces.com/blog/entry/16221

Recommended Practice Sites

- HKOJ
- Codeforces
- Topcoder
- Csacademy
- Atcoder
- USACO
- COCI

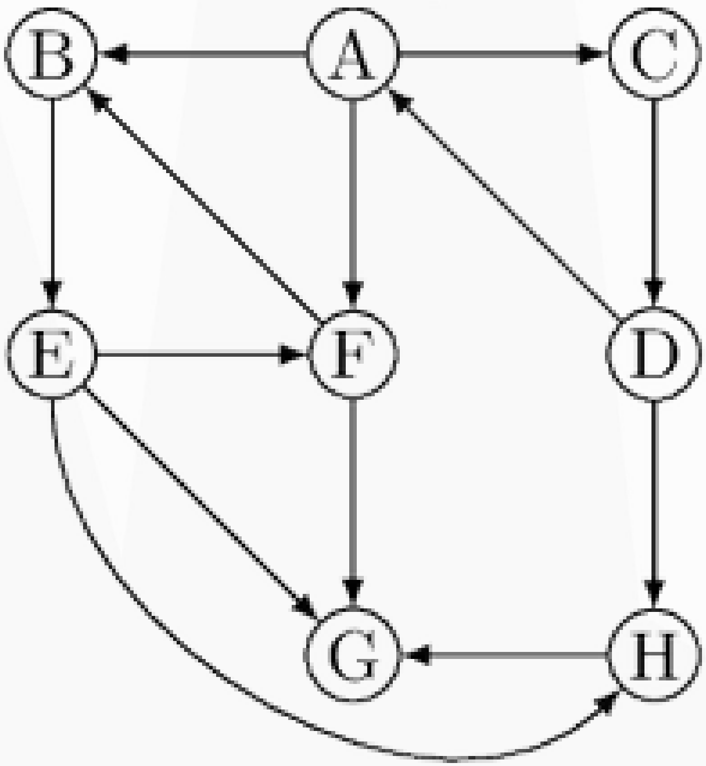
Term in Directed Tree

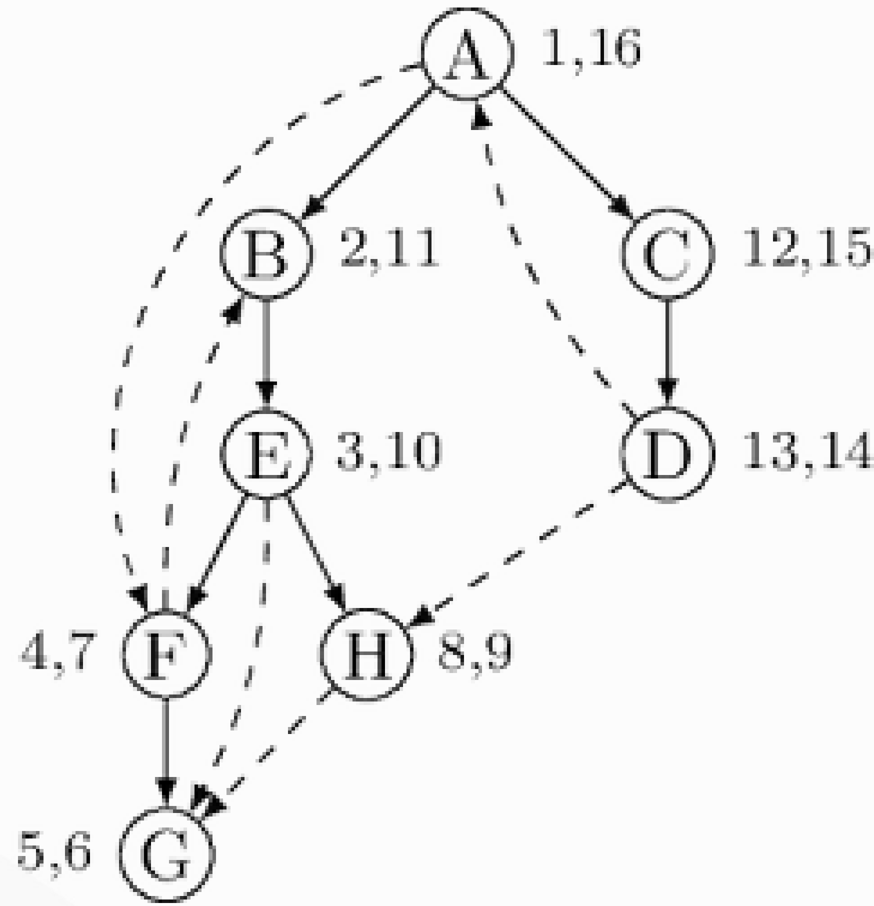
- **Consider node 4**
  - **Node 2 is its parent**
  - **Node 1, 2 is its ancestors**
  - **Node 5 is its sibling**
  - **Node 6 is its child**
  - **Node 6, 7, 8 is its descendants**
- **Node 1 is the root**



DFS Forest

- When we do DFS on a graph, we would obtain a DFS forest. Noted that the graph is not necessarily a tree.
- Some of the information we get through the DFS is actually very useful, such as
  - Starting time of a node
  - Finishing time of a node
  - Parent of the node


B
A
C
E
F
D
G
H


A
1,16
B
2,11
C
12,15
E
3,10
D
13,14
4,7
F
H
8,9
5,6
G

Some Tricks Using DFS Order

- Suppose vertex v is ancestor(not only parent) of u
  - Starting time of v < starting time of u
  - Finishing time of v > starting time of u
- st[v] < st[u] <= ft[u] < ft[v]
- O(1) to check if ancestor or not
- Flatten the tree to store subtree information(maybe using segment tree or other data structure to maintain)
- Super useful !!!!!!!!!!

Partial Sum on Tree

- Given queries, each time increase all node from node v to node u by 1
- Assume node v is ancestor of node u
- sum[u]++, sum[par[v]]--
- Run dfs in root

```
dfs(v)
    for all child u
        dfs(u)
        d[v] = d[v] + d[u]
```

Types of Edges

- **Tree edges**
  - **Edges that forms a tree**
- **Forward edges**
  - **Edges that go from a node to its descendants but itself is not a tree edge.**
- **Back edges**
  - **Edges that go from a node to its ancestor**
- **Cross edges**
  - **Any other edges, connect in the same tree or different tree.**

Determination of Edges

- Let's say now there is an directed edge (u, v).
  - It is tree edge
    - if parent of v is u
  - It is forward edge
    - if parent of v is not u(not a tree edge) and
    - Starting time of u < starting time of v and
    - Finishing time of u > finishing time of u

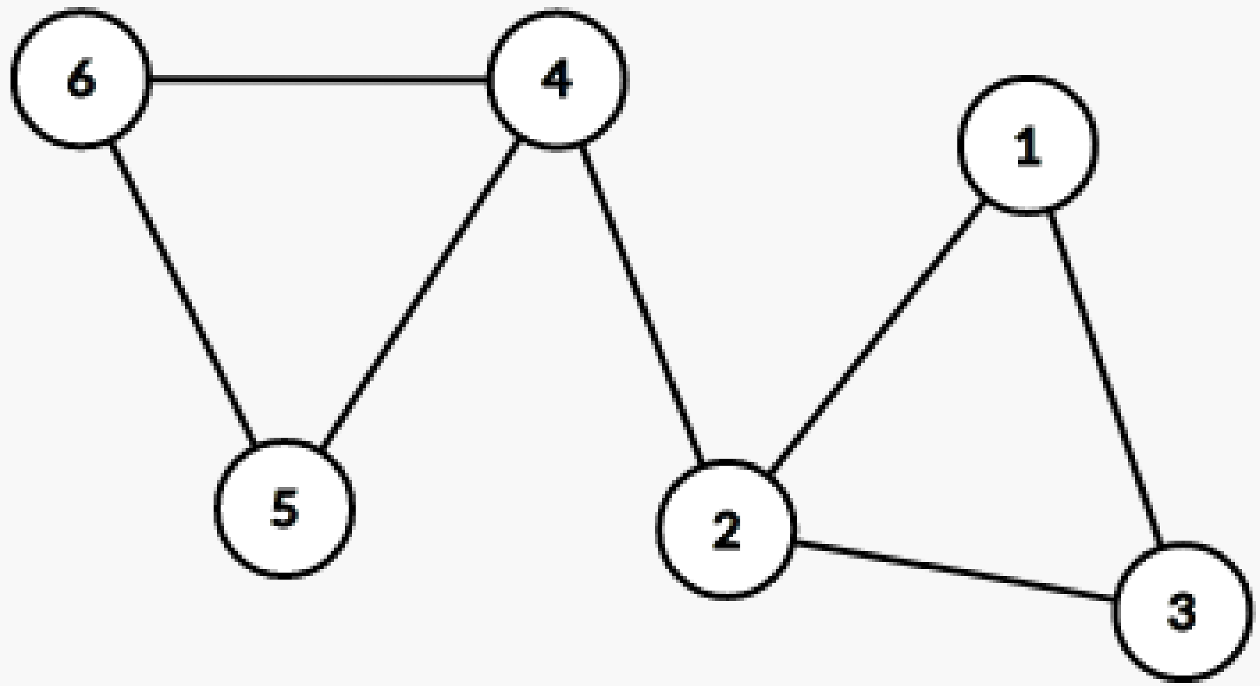
Determination of Edges

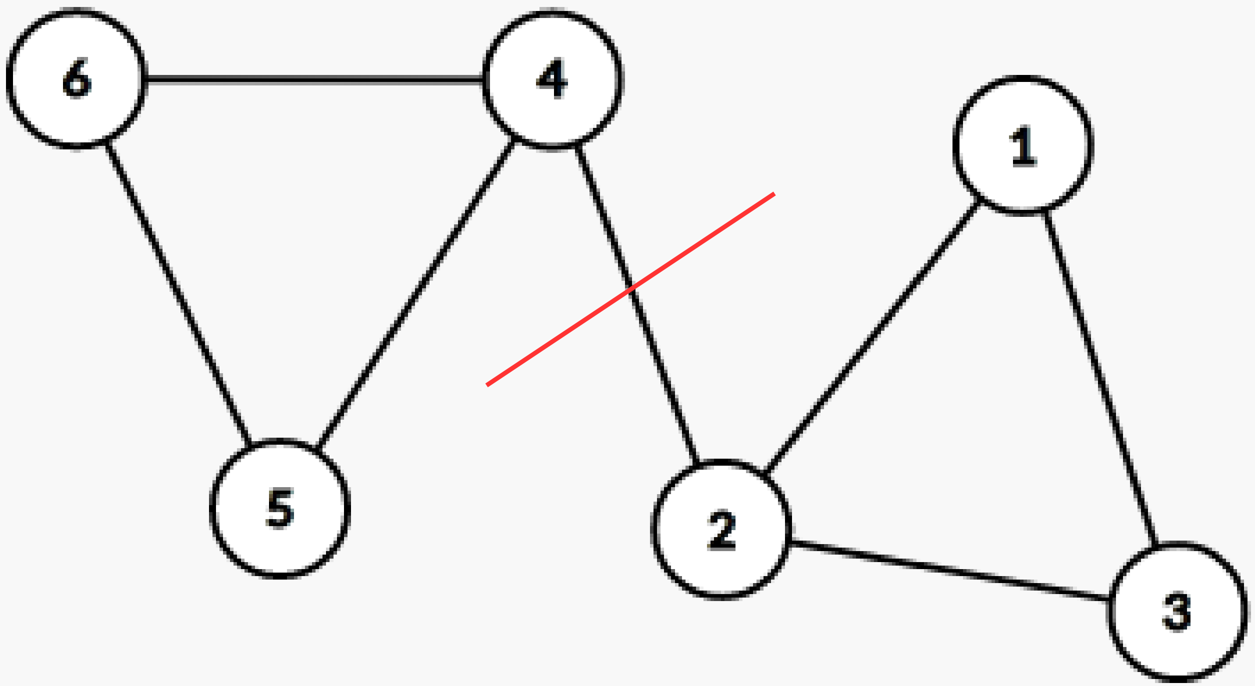
- **It is back edge**
  - **If starting time of u > starting time of v and**
  - **Finishing time of u < finishing time of v**
- **It is cross edge**
  - **If starting time of u > starting time of v and**
  - **Finishing time of u > finishing time of v**

Cut Edges(Bridges)

- **Definition – a bridge is an edge of a graph whose deletion increases its number of connected component**
- **We could easily found all bridges in O(N + M) using DFS**

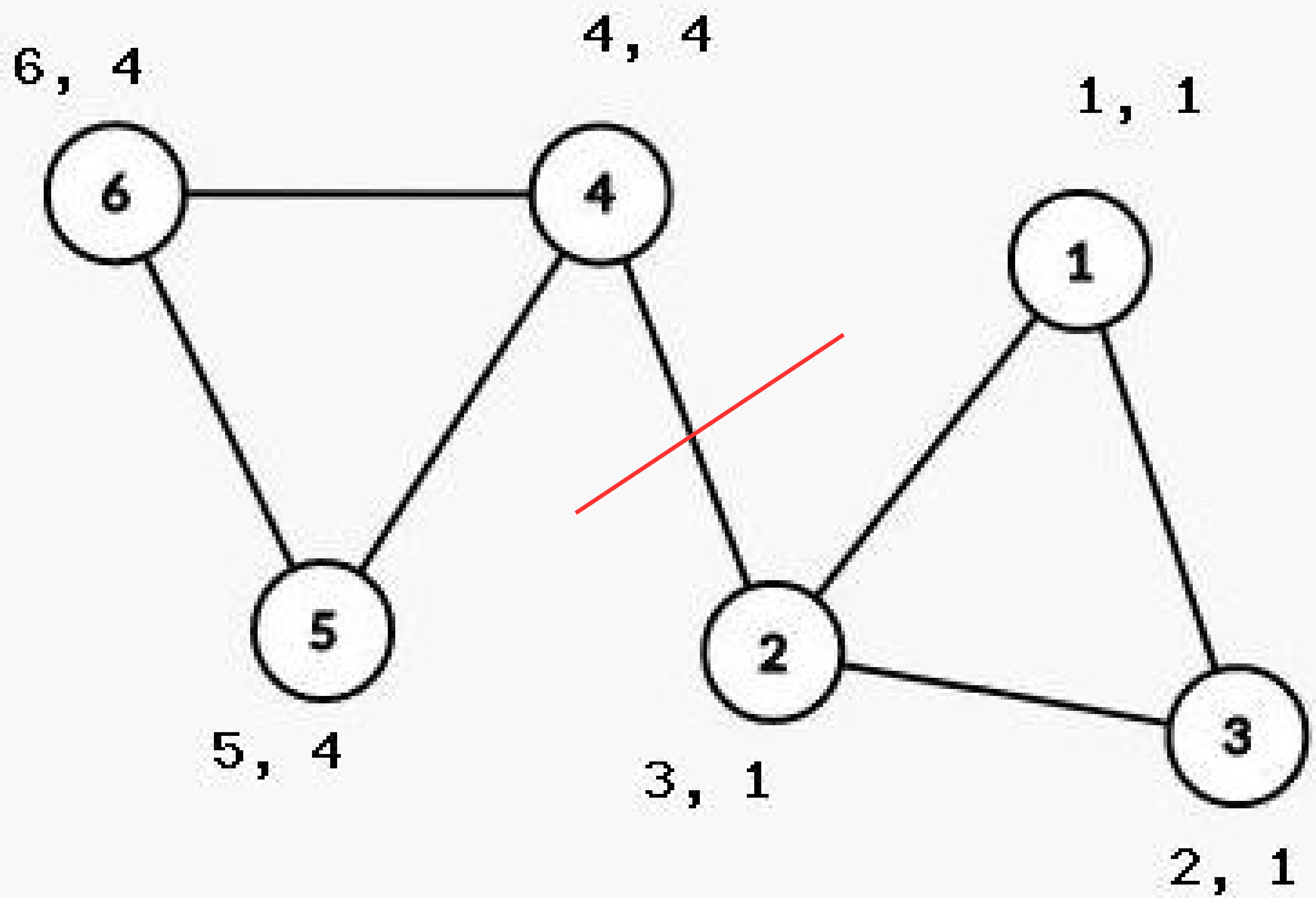
AMAZING


6
4
1
5
2
3


6
4
1
5
2
3

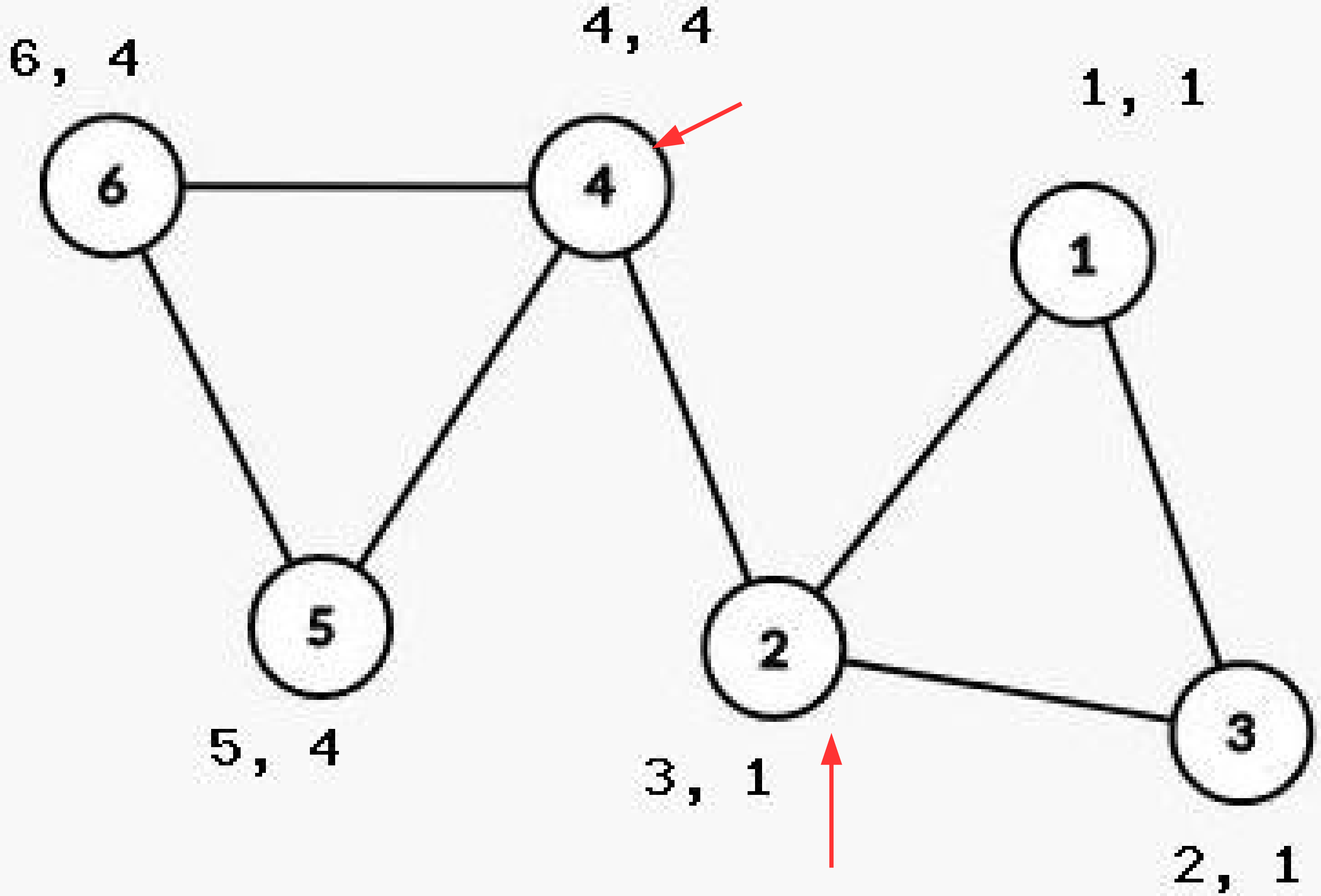
Finding Cut Edges with DFS

- Let’s maintain a array, named low. low[v] = min(start_time[w] where there is at least a vertex u in subtree of v such that there is an edge between w and u).
- Then for every vertex v, if its child u, where start_time[v] < low[u]. Edge (v, u) is a cut edge.


6, 4
4, 4
1, 1
6
4
1
5
2
3
5, 4
3, 1
2, 1

Cut Vertices(Articulation Point)

- Definition – a articulation point is a vertex of a graph whose deletion increases its number of connected component
- Similar techinque could be used to find cut vertices
- For every vertex v, if its child u, where start_time[v] <= low[u] AND (v is not the root or number of child of v > 1).
- Consider the case where n = 2 and there is edge between node 1 and 2. Second condition is necessary as removing 1 or 2 would not increase the number of connected component.



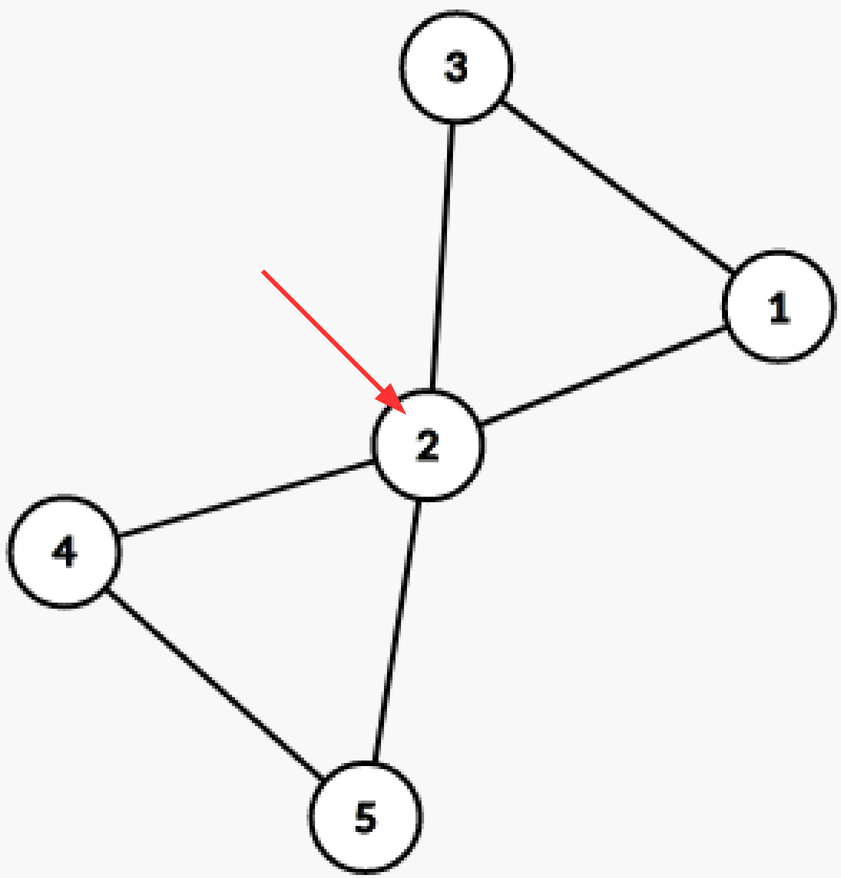
6, 4
4, 4
1, 1
6
4
1
5
2
3
5, 4
3, 1
2, 1

Practice Problems

- **Bridges :**
  - **http://www.spoj.com/problems/EC_P/**
- **Articulation Points :**
  - **http://poj.org/problem?id=1523**
  - **http://www.spoj.com/problems/SUBMERGE/**

Biconnected Component

- **Biconnected component is a maximal biconnected subgraph such that there is no articulation point.**
- **But for various OI problems, we often use the other definition, the bridge-connected component. When we delete all bridges, the vertices that are connected is in the same bridge-connected component.**
- **Be sure what you are looking for, biconnected component or bridge-connected component, they sound the same but actually have great difference.**
- **For example : n = 5.**
  - **(1, 2), (2, 3), (3, 1), (2, 4), (4, 5), (5, 2) ← these are edges.**

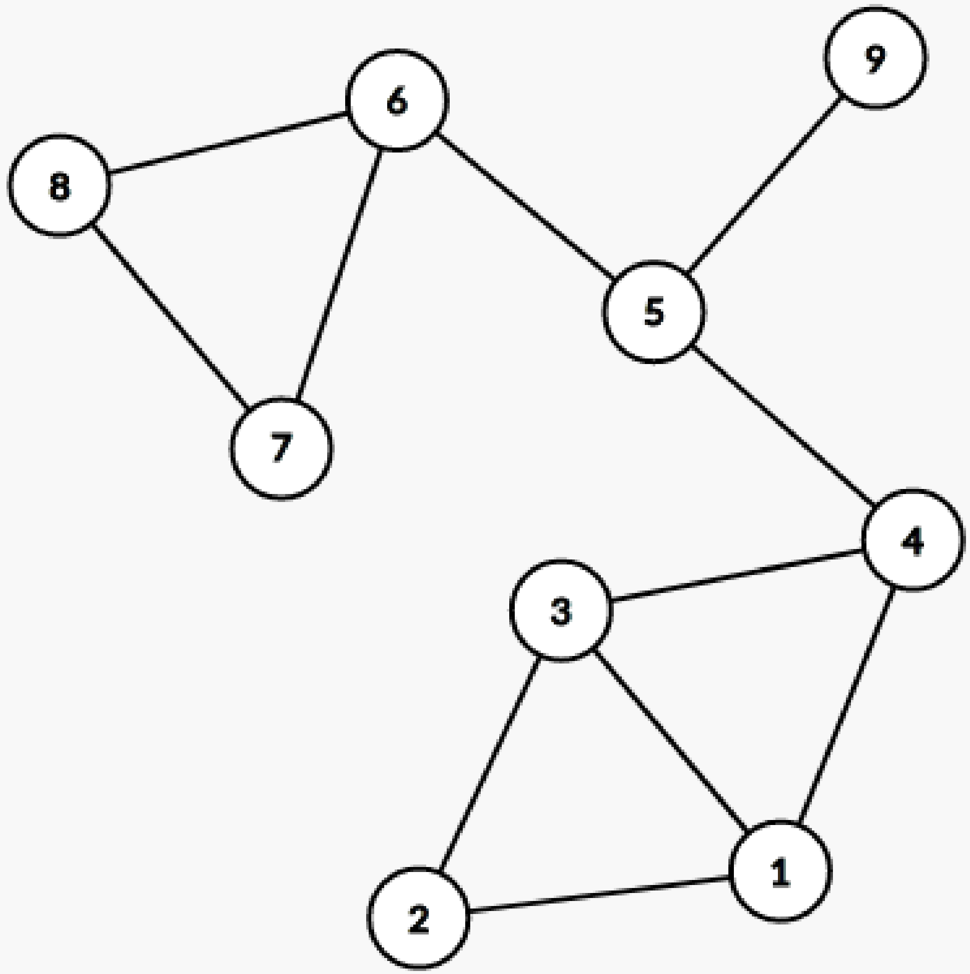

3
1
2
4
5

Bridge-Connected Component

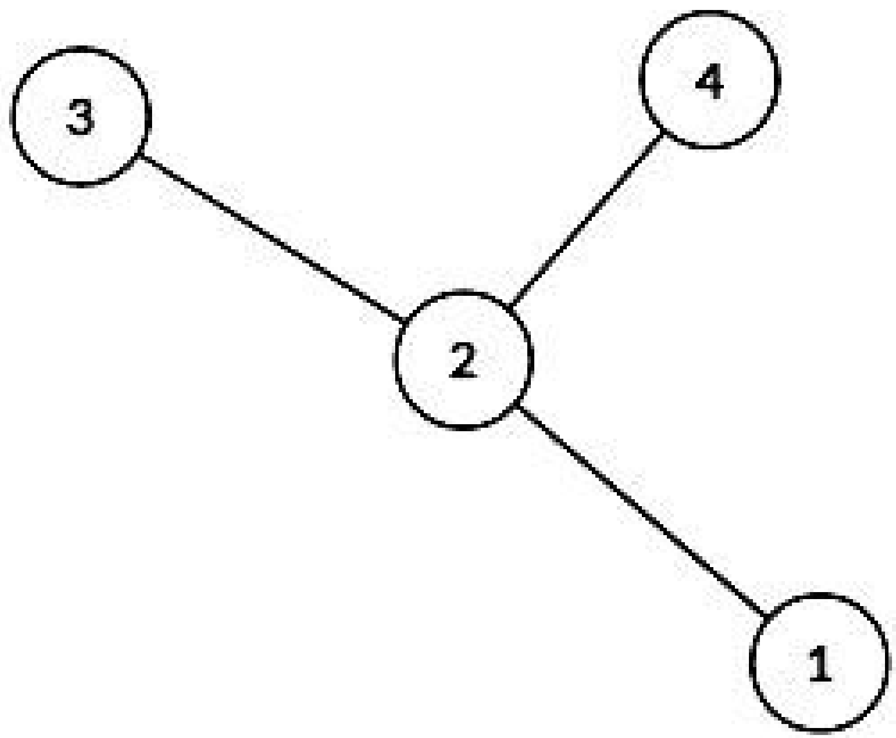
- How to find BCC ?
- SIMPLE !
  - Mark all bridges
  - DFS from 1 to N but do not use bridges to travel
  - Each time produce a BCC

Shrink Trick

- When we shrink all the bridge-connected component in to a node, we would get a tree.
- This help us reduce some hard graph problems into easy tree problems.
- Because in every bridge-connected component, we have got a property. For every vertex v, it could reach vertex u in two ways without duplicate edges
- Problems :
  - http://www.spoj.com/problems/GRAFFDEF/
  - http://codeforces.com/contest/118/problem/E

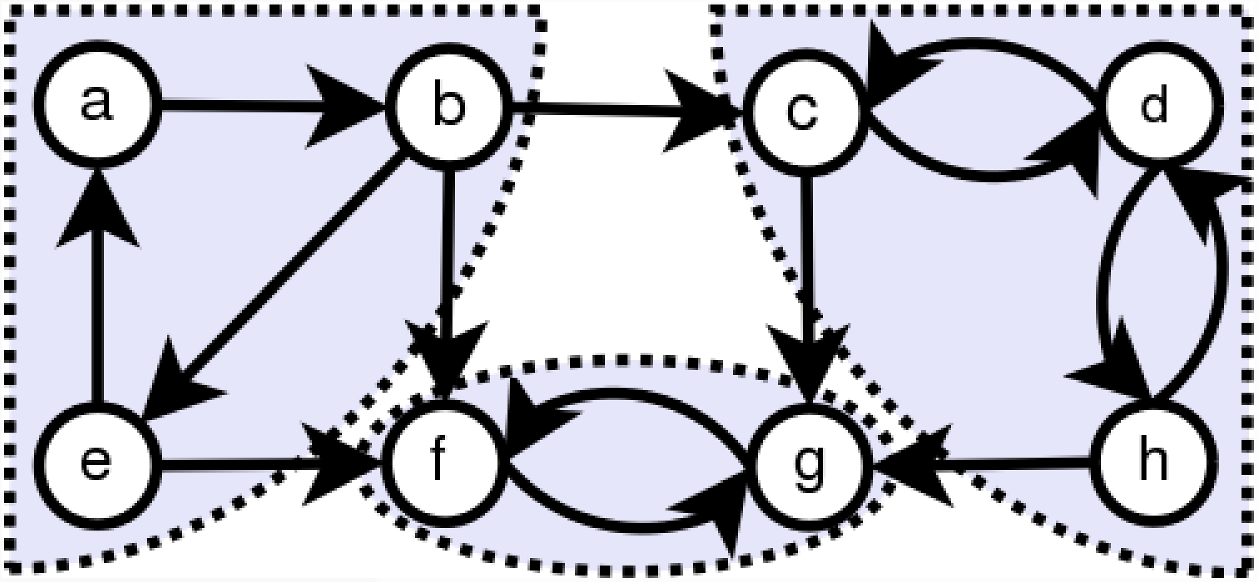


9
6
8
5
7
4
3
1
2


3
4
2
1

Strongly Connected Component

- CAUTION : SCC is discussed based on directed graph, bridges, ap and other stuffs are all based on undireced graph.
- A graph is strongly connected if every vertex v, could reach to every vertex u(including v itself). And it is maximal(you could not add edges or vertices in this subgraph from the original without breaking this property).



(picture from wikipedia)

Strongly Connected Component

- One way to find scc in linear time is to use Kosaraju's algorithm.
- We first do dfs, for unvisited node from 1 to n

  dfs(v) :

  mark v as visited

  for all unvisited node u connected to v, dfs(u)

  append v in vector V
- We then do dfs again(called rdfs(v, k)), from the back of the vector, each time increasing parameter k by 1

  rdfs(v, k) :

  mark v as visited, v is belong to scc group k

  for all unvisited node v connected to u, dfs(u) **** CAREFUL

Strongly Connected Component

- Again, you could apply shrink trick to solve problems easily.
- Here are some practice problems for scc :
  - http://codeforces.com/problemset/problem/427/C
  - http://www.spoj.com/problems/TFRIENDS/
  - http://acm.timus.ru/problem.aspx?space=1&num=1198

Weakly Connected Component

- **Again we are discussing it based only on directed graphs.**
- **A graph is weakly connected if every vertex v and u, at least v could reach u or u could reach v. And the graph is maximal.**
- **How to find ? I'll leave it as a practice**
  - **https://judge.hkoi.org/task/M1321**

Reference

- http://codeforces.com/blog/entry/16221
- https://en.wikipedia.org/wiki/Strongly_connected_component
-

Q&A

- **If you have further question, you are welcomed to facebook me or message me on codeforces(user name : ulna)**