J181 Wings and Nuggets

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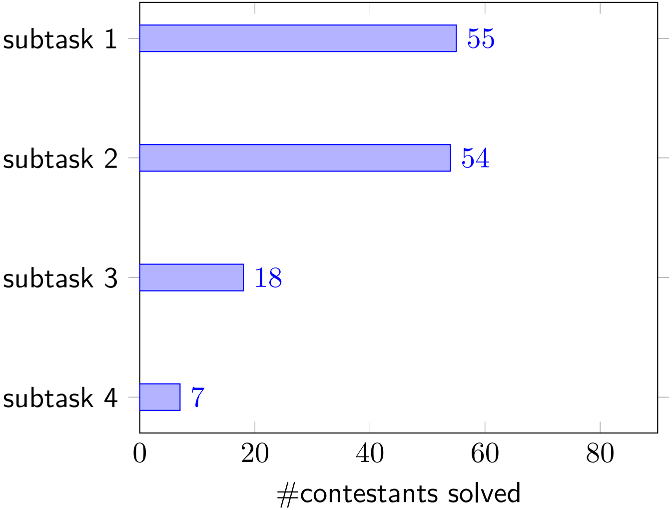
Statistics



subtask 1
subtask 2
subtask 3
subtask 4
55
54
18
7
0
20
40
60
80
#contestants solved

Problem Statement

Food	Price
pack of 2 Wings	W_2
pack of 4 Wings	W_4
pack of 4 Nuggets	N_4
pack of 6 Nuggets	N_6
pack of 9 Nuggets	N_9

- Subtask 1: minimum cost to buy X Wings
- Subtask 2: maximum number of Wings with \$$Y$
- Subtask 3: minimum cost to buy X Nuggets
- Subtask 4: maximum number of Nuggets with \$$Y$

Subtask 1: minimum cost to buy X Wings

Constraints:

- $1 \leq W_2 < W_4 \leq 100$
- $1 \leq X \leq 100$

Suppose we should buy k_2 packs of 2 Wings and k_4 packs of 4 Wings. Then,

- $0 \leq k_2 \leq \lceil \frac{X}{2} \rceil$
- $0 \leq k_4 \leq \lceil \frac{X}{4} \rceil$
- $2k_2 + 4k_4 \geq X$
- Want to minimize $k_2 W_2 + k_4 W_4$

Subtask 1: minimum cost to buy X Wings

Constraints:

- $1 \leq W_2 < W_4 \leq 100$
- $1 \leq X \leq 100$

Suppose we should buy k_2 packs of 2 Wings and k_4 packs of 4 Wings. Then,

- $0 \leq k_2 \leq \lceil \frac{X}{2} \rceil$
- $0 \leq k_4 \leq \lceil \frac{X}{4} \rceil$
- $2k_2 + 4k_4 \geq X$
- Want to minimize $k_2 W_2 + k_4 W_4$

Solution 1: Naive Exhaustion

```
ans ← ∞
for k4 ← 0 to ⌈X/4⌉ do
    for k2 ← 0 to ⌈X/2⌉ do
        cost ← k2W2 + k4W4
        if 2k2 + 4k4 ≥ X then
            ans ← min(ans, cost)
        end if
    end for
end for
output ans
```

Time complexity: $O(X^2)$

Subtask 1: minimum cost to buy X Wings

Constraints:

- $1 \leq W_2 < W_4 \leq 100$
- $1 \leq X \leq 100$

Suppose we should buy k_2 packs of 2 Wings and k_4 packs of 4 Wings. Then,

- $0 \leq k_2 \leq \lceil \frac{X}{2} \rceil$
- $0 \leq k_4 \leq \lceil \frac{X}{4} \rceil$
- $2k_2 + 4k_4 \geq X$
- Want to minimize $k_2 W_2 + k_4 W_4$

Solution 2: Exhaustion

$ans \leftarrow \infty$
for $k_4 \leftarrow 0$ **to** $\lceil \frac{X}{4} \rceil$ **do**
  $k_2 \leftarrow \max(0, \lceil \frac{X - 4k_4}{2} \rceil)$
  $cost \leftarrow k_2 W_2 + k_4 W_4$
  $ans \leftarrow \min(ans, cost)$
end for
output ans

Time complexity: $O(X)$

Subtask 1: minimum cost to buy X Wings

Constraints:

- $1 \leq W_2 < W_4 \leq 100$
- $1 \leq X \leq 100$

Suppose we should buy k_2 packs of 2 Wings and k_4 packs of 4 Wings. Then,

- $0 \leq k_2 \leq \lceil \frac{X}{2} \rceil$
- $0 \leq k_4 \leq \lceil \frac{X}{4} \rceil$
- $2k_2 + 4k_4 \geq X$
- Want to minimize $k_2 W_2 + k_4 W_4$

Solution 3: Math

- Buy W_2 only
- If $W_4 \leq 2W_2$, replace every two W_2 with a W_4

Time complexity: $O(1)$

Subtask 2: maximum number of Wings with \$$Y$

Constraints:

- $1 \leq W_2 < W_4 \leq 100$
- $1 \leq Y \leq 10000$

Suppose we should buy k_2 packs of 2 Wings and k_4 packs of 4 Wings. Then,

- $0 \leq k_2 W_2 + k_4 W_4 \leq Y$
- Want to maximize $2k_2 + 4k_4$

Subtask 2: maximum number of Wings with \$$Y$

Constraints:

- $1 \leq W_2 < W_4 \leq 100$
- $1 \leq Y \leq 10000$

Suppose we should buy k_2 packs of 2 Wings and k_4 packs of 4 Wings. Then,

- $0 \leq k_2 W_2 + k_4 W_4 \leq Y$
- Want to maximize $2k_2 + 4k_4$

Solution 1: Exhaustion

$\mathit{ans} \leftarrow 0$
for $k_4 \leftarrow 0$ **to** $\lfloor \frac{Y}{W_4} \rfloor$ **do**
  $k_2 \leftarrow \max(0, \lfloor \frac{Y - k_4 W_4}{W_2} \rfloor)$
  $\mathit{wings} \leftarrow 2k_2 + 4k_4$
  $\mathit{ans} \leftarrow \max(\mathit{ans}, \mathit{wings})$
end for
output ans

Time complexity: $O(Y)$

Subtask 2: maximum number of Wings with \$$Y$

Constraints:

- $1 \leq W_2 < W_4 \leq 100$
- $1 \leq Y \leq 10000$

Suppose we should buy k_2 packs of 2 Wings and k_4 packs of 4 Wings. Then,

- $0 \leq k_2 W_2 + k_4 W_4 \leq Y$
- Want to maximize $2k_2 + 4k_4$

Solution 2: Math

- If $W_4 \geq 2W_2$, only buy W_2;
- Otherwise, only buy W_4 plus maybe one W_2

Time complexity: $O(1)$

Subtask 3: minimum cost to buy X Nuggets

Constraints:

- $1 \leq N_4 < N_6 < N_9 \leq 100$
- $1 \leq X \leq 100$

Suppose we should buy k_4 packs of 4 Nuggets, k_6 packs of 6 Nuggets and k_9 packs of 9 Nuggets. Then,

- $0 \leq k_4 \leq \lceil \frac{X}{4} \rceil$
- $0 \leq k_6 \leq \lceil \frac{X}{6} \rceil$
- $0 \leq k_9 \leq \lceil \frac{X}{9} \rceil$
- $4k_4 + 6k_6 + 9k_9 \geq X$
- Want to minimize $k_4 N_4 + k_6 N_6 + k_9 N_9$

Subtask 3: minimum cost to buy X Nuggets

Constraints:

- $1 \leq N_4 < N_6 < N_9 \leq 100$
- $1 \leq X \leq 100$

Suppose we should buy k_4 packs of 4 Nuggets, k_6 packs of 6 Nuggets and k_9 packs of 9 Nuggets. Then,

- $0 \leq k_4 \leq \lceil \frac{X}{4} \rceil$
- $0 \leq k_6 \leq \lceil \frac{X}{6} \rceil$
- $0 \leq k_9 \leq \lceil \frac{X}{9} \rceil$
- $4k_4 + 6k_6 + 9k_9 \geq X$
- Want to minimize $k_4 N_4 + k_6 N_6 + k_9 N_9$

Solution 1: Naive Exhaustion

similar to subtask 1, with one more nested for-loop

Time complexity: $O(X^3)$ or $O(X^2)$

Subtask 3: minimum cost to buy X Nuggets

Solution 2: Least Common Multiple

- Unit prices: $\frac{N_4}{4}, \frac{N_6}{6}, \frac{N_9}{9}$
- Assume that N_4 is the cheapest in terms of unit price
- Claim: in an optimal solution,
  - $0 \leq k_6 \leq 5$
  - $0 \leq k_9 \leq 3$
- Why? Because we can replace every 6 N_6 (or 4 N_9) with 9 N_4 without worsening our solution
- Exhaut k_6 and k_9, calculate k_4
- Time complexity: $O(\frac{\text{LCM}(4,6,9)^{3-1}}{4 \times 6 \times 9} \times 4) = O(6 * 4) = O(1)$

Subtask 4: maximum number of Nuggets with \$$Y$

Constraints:

- $1 \leq N_4 < N_6 < N_9 \leq 100$
- $1 \leq Y \leq 10000$

Suppose we should buy k_4 packs of 4 Nuggets, k_6 packs of 6 Nuggets and k_9 packs of 9 Nuggets. Then,

- $0 \leq k_4 N_4 + k_6 N_6 + k_9 N_9 \leq Y$
- Want to maximize $4k_4 + 6k_6 + 9k_9$

Subtask 4: maximum number of Nuggets with \$$Y$

Constraints:

- $1 \leq N_4 < N_6 < N_9 \leq 100$
- $1 \leq Y \leq 10000$

Suppose we should buy k_4 packs of 4 Nuggets, k_6 packs of 6 Nuggets and k_9 packs of 9 Nuggets. Then,

- $0 \leq k_4 N_4 + k_6 N_6 + k_9 N_9 \leq Y$
- Want to maximize $4k_4 + 6k_6 + 9k_9$

Solution 1: Exhaustion

Similar to subtask 2, with one more nested for-loop

Time complexity: $O(Y^2)$

Time limit exceeded

Subtask 4: maximum number of Nuggets with \$$Y$

Constraints:

- $1 \leq N_4 < N_6 < N_9 \leq 100$
- $1 \leq Y \leq 10000$

Suppose we should buy k_4 packs of 4 Nuggets, k_6 packs of 6 Nuggets and k_9 packs of 9 Nuggets. Then,

- $0 \leq k_4 N_4 + k_6 N_6 + k_9 N_9 \leq Y$
- Want to maximize $4k_4 + 6k_6 + 9k_9$

Solution 2: Exhaustion

Exhaust $k4$ and calculate $k6$ and $k9$? Need to use LCM again. Better directly go for the $O(1)$ solution.

Subtask 4: maximum number of Nuggets with \$$Y$

Constraints:

- $1 \leq N_4 < N_6 < N_9 \leq 100$
- $1 \leq Y \leq 10000$

Suppose we should buy k_4 packs of 4 Nuggets, k_6 packs of 6 Nuggets and k_9 packs of 9 Nuggets. Then,

- $0 \leq k_4 N_4 + k_6 N_6 + k_9 N_9 \leq Y$
- Want to maximize $4k_4 + 6k_6 + 9k_9$

Solution 3: Math

- Similar to subtask 3
- If N_4 is the cheapest in terms of unit price, exhaust $0 \leq k_6 \leq 5$ and $0 \leq k_9 \leq 3$, calculate k_4
- Time complexity: $O(1)$