TFT Q4 (Hackerland's Got Talent) Editorial

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Assumption

For simplicity, assume that $q[1..n]$ is sorted in ascending order.

Easy Partials

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  - Consider $dp[mask][last]$.
  - $mask$: a bitmask representing the performed acts.
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  - Easy $O(n)$ transition.
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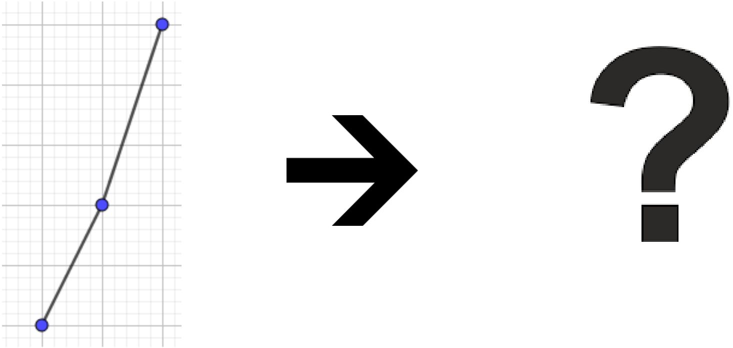
- Subtask 4: Output pattern $1, n, 2, (n-1), \ldots$.
  (**Exercise**: Prove that it is one of the optimal solutions.)

Target: reduce the number of candidate solutions (permutations $p[1..n]$) we need to consider.

Observation 1

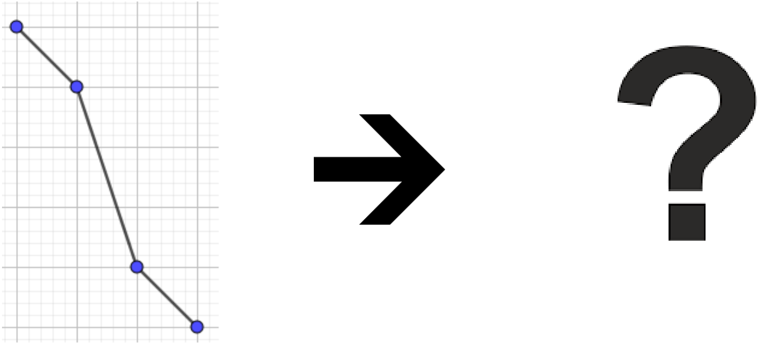
If $p[i] < p[i+1] < p[i+2]$ for some i, we can ignore the permutation.



“Local” Optimality Criteria

Observation 2

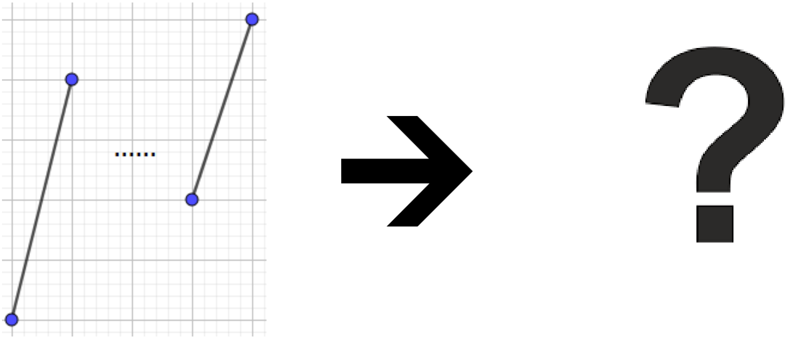
If $p[i] > p[i+1] > p[i+2] > p[i+3]$ for some i, we can ignore the permutation.



"Local" Optimality Criteria

Observation 3

If there exists $i + 1 < j$ such that $p[i] < p[j] < p[i + 1]$ or $p[j] < p[i] < p[j + 1]$, we can ignore the permutation.



A Better Solution

- We can assume that, if $p[i] < p[i+1]$, then it is one of the pairs $(1, n), (2, n-1), \ldots$ (thanks to Observation 3).
- This already allows for a better solution.
- $dp[mask][pos]$: *mask* is on "used" **pairs**, not individual acts.
- This passes subtask 5. Time complexity: $O(2^{\frac{n}{2}} \times poly(n))$

"Exhaustion"

Observation 4

If we fix where to put a pair, it is easy to choose the best way to put pairs. For example, $n = 6$, $w[] = \{1, \mathbf{5}, 2, 3, \mathbf{1}\}$ (**bold** = want to insert a pair). Then we shall put $(1, 6)$ to **5** and $(2, 5)$ to **1**.

- Exhaust all possible ways of putting pairs.
- Criteria:
  - Consecutive indices cannot be both chosen.
  - Distance between two adjacent chosen indices cannot exceed 3.
- Turns out there are not many ways of putting pairs (fewer than 10^6 for $n = 50$. **Exercise**: derive a recurrence formula.).
- This passes subtask 6. Time complexity: $O((1.3247...)^n \times n)$.

"Exhaustion" again!?

- We exhaust whether $4, 7, 10, \ldots, 3k+1, \ldots$ should be given a pair.
- For the rest, it is easy to determine whether a pair should be given.
- Take $5, 6$ as example.
  - If 4 and 7 are chosen, choose neither.
  - If 4 is chosen, choose 6.
  - If 7 is chosen, choose 5.
  - If neither is chosen, compare $w[5]$ and $w[6]$, and choose the better one.
- This gets 100 points. Time complexity: $O(2^{\frac{n}{3}} \times n)$.

The End

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- If no, then I have one for you.

Challenge

Can this task be solved in polynomial time?
If yes, how?
If no, why not?