Road Service

JEREMY CHOW 30/6/2018

Problem Statement

- JOI 2018 Spring Camp Day 2 Problem B

- Given a tree with N vertices
- Add K new edges to the graph so that the total distance is minimized

- Total distance = sum of shortest paths between every two points

Problem Statement

- N = 1000
- **Output only task**

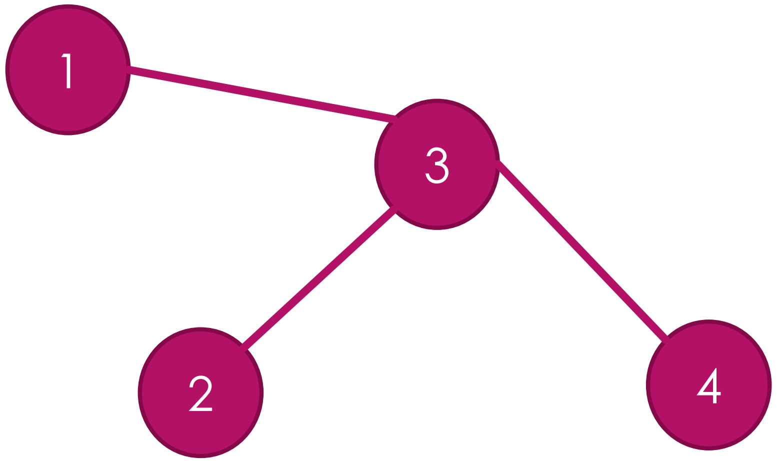
- You are **NOT** required to find out the optimal answer
- Nearly-optimal is good enough

Direction

- Which kind of graph have small pairwise distance?
- Chain
- Total distance = (1+2+3)+(1+2)+1=10

Direction

- Which kind of graph have small pairwise distance?
- Star
- Total distance = (1+2+2)+(1+2)+1=9



Direction

- Which kind of graph have small pairwise distance?
- Star or Chain?

- Actually, Chain graph's total pairwise distance is $O(N^3)$
- But Star graph's total pairwise distance is only $O(N^2)$

- Difference between those two type graph will get bigger when N is large

Direction

- So, if we concentrate all k extra edges to one particular vertex
- Make the graph like a star graph

- We can get graph that gives good result

Solution ~6 points

- Randomly output K edges
- You should at least get this 6 points.

Solution ~30 points

- Output 1-2, 1-3, 1-4, …, 1-k+1
- Turn the graph into star shape
- Easy 30 points ☺

Solution ~30 points

- You may try exhaust the center
- Trying x-1, x-2, x-3, … x-k

- Output the best solution

- But it won’t improve lots

Solution ~60 points

- Exhaust the center of the star

- But which vertices should we choose to connect with the center?
- Random is obviously not good

- We want to make every vertex as close to the center as possible at the end
- So that the total pairwise distance is most likely very small

Solution ~60 points

- One ways is to connect to the vertices having large degree
- Larger degree -> most likely more vertices will closer to center
- Good ways to connect the vertices
- You may use another “good ways” to connect the vertices

Solution ~60 points

- Try making every vertex as center
- Connect center with k vertices having large degree
- Compare the total pairwise distance with your current best solution
- Output the best solution

- You will get 60 points if you implement it correctly

Solution ~60 points

- How to calculate the total pairwise distance?
- Do BFS on every vertex
- $O(N^2)$

- Notice that you should avoid adding edge that is already exist in the original graph
- Also avoid adding self loop

Solution ~100 points

- Let the set of vertices connecting to the center be X
- Actually connecting center to the vertices having large degree will give us a pretty good initial state

- We will try to improve our solution starting from that
  - Hill climbing

Solution ~100 points

- Let each step be the following:
  - Remove one node from X and add one node to X
  - i.e. Change one of the edge connecting to the center

- There are O(NK) possible steps you can choose at the same time
  - O(K) vertices you want to remove, O(N) vertices you want to add

Solution ~100 points

- You may try all possible steps and choose the best step
- Repeat this process until you can’t make any improvement

Solution ~100 points

- However, we find that the evaluation function (total pairwise distance) takes too much time to calculate
- $O(N^2)$

- We can replace the evaluation function by following
  - Sum of distance from the center to other vertices
- $O(N)$

Solution ~100 points

- Also, you may not try to improve all the initial solutions with different center
- You can just improve solutions having top 10 initial score (evaluated by the function motioned before)
- Help the program to run faster

Solution ~100 points

1. Calculate the initial solution with center i $(1 \le i \le n)$
2. For the initial solution having top 10 initial score :
   - Try every possible step (remove and add 1 node)
   - Choose the best step and update the solution
   - Repeat this process until you can't make any improvement more
   - Update the answer

- With this algorithm, you will get 90~100 points

Solution ~100 points

- If you replace “connect the center to the vertex with large degree” with “connect the center to the vertex such that the total distance reduce the most” and repeat it for K times

- You will get ~90 points even without the improvement (Hill climbing)

Solution ~100 points

- If you greedy select (k+1) "good" vertex first, then exhaust the center

- You may also get ~90 points

Solution ~100 points

- If you think the full solution algorithm run too slow

- You may limit the number of times to make improvement
- Also, you may not trying all possible step and using some randomization
  - Randomly remove one node and add the best node to X
  - Each improvement takes O(N)

- You will get ~80 points if you do that

Solution ~100 points

- Also notice that test case 1 constraint is small
- You may find the optimal solution by exhaustion
  - Or with hand (if you are smart enough)

Techniques

- Output only task
- Can start working after looking the input data
- Visualizing the input may often helps
- You can even solve some input by hand (Test data 1)

- Do not require a rigorous solution
- Well designed greedy or DP algorithm can get you high score
- You can also try to improve your solution by hill climbing algorithm after determining the initial state

Techniques

- Try to make good use of your 5 hours contest time
- Do not spend all the times on it
- Try to find easy and short solutions first
- Get some nice points as fast as possible

- Try to design an algorithm that can be rerunning after a small change of parameter

Techniques

- There is no running time limit for your program
- You may just let it run a long time locally
- The official solution also takes a long time to output 1 test case

- When the program is running, you can move on to the next tasks
- Just make sure your program will search for some “good” solution
- It feels bad when you only get few points after running the program for 1 hour

Little patterns, big canvas

solution

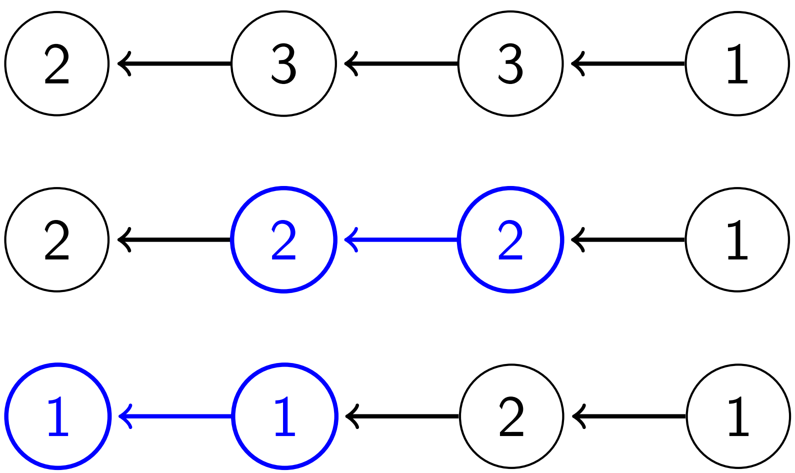
Lau Chi Yung

2018/07/2

Subtask 1

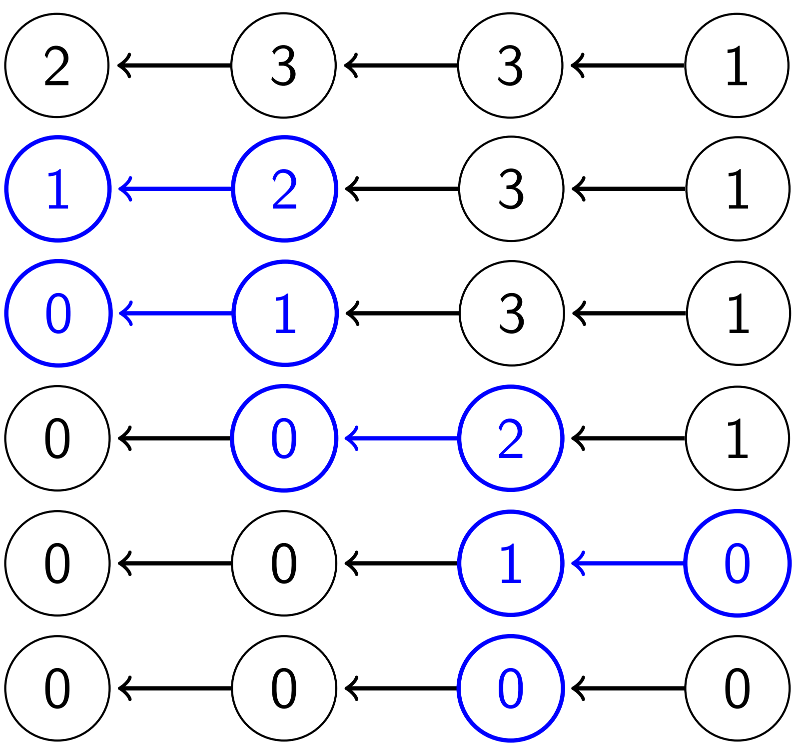
$p_i = i - 1$

- One linear chain from 1 to N
- The direction of the arrows does not matter
- Amber can draw simultaneously on any adjacent nodes



Subtask 1

- No matter what, when pattern 1 is being drawn, pattern 2 should be drawn simultaneously
- Better draw pattern 1 before drawing anything else, so that pattern 2 will have fewer strokes left
- Strategy: greedily draw from left to right
- Time complexity: $O(N)$

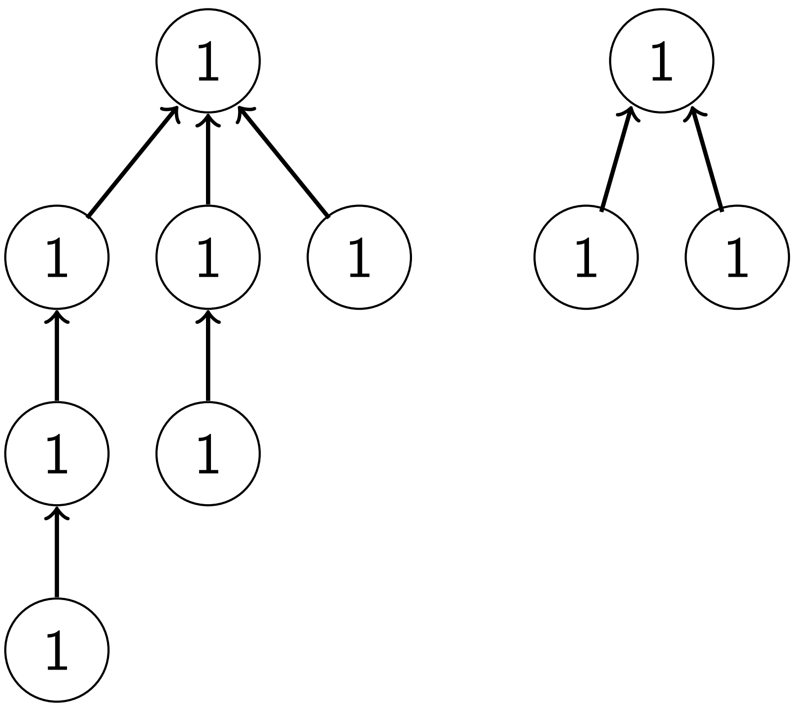


$\leftarrow$ right hand has nothing to do

Subtask 2

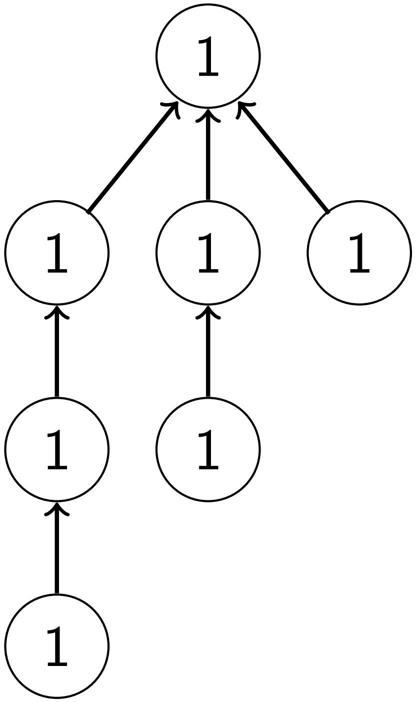
$N \leq 1000, s_j = 1$

- Model patterns as vertices, and partner patterns as parent
- We get a forest (not only one single tree)
- Each pattern only consists of one stroke
- Again, direction of arrows does not matter



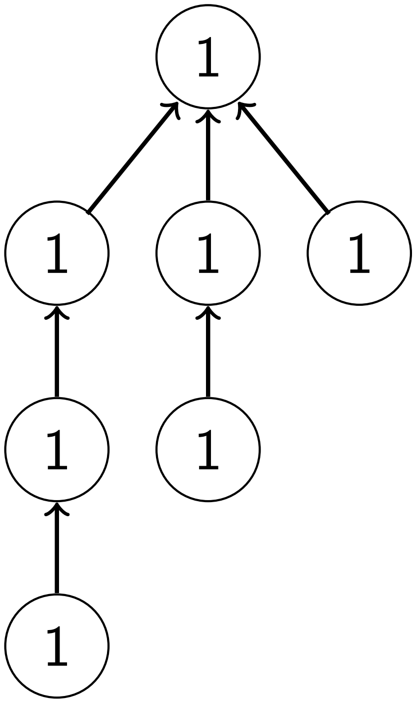
Subtask 2

- We can never draw patterns on different trees simultaneously $\Rightarrow$ we can deal with each tree one by one
- To ease our implementation, let tree roots be all nodes r where $p_r = 0$



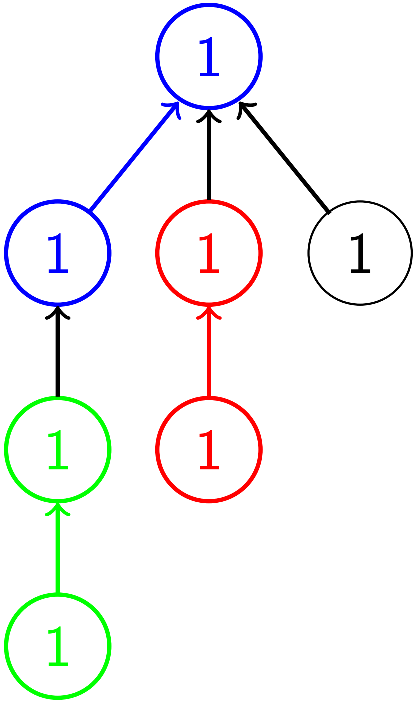
Subtask 2

- Always prefer drawing two patterns simultaneously
  $\Rightarrow$ want to find the largest number of edges such that all vertices in these edges are distinct
- i.e. want to find the maximum matching



Subtask 2

- Always prefer drawing two patterns simultaneously
  $\Rightarrow$ want to find the largest number of edges such that all vertices in these edges are distinct
- i.e. want to find the maximum matching
- answer $= N - |\text{maximum matching}|$
- answer $= 7 - 3 = 4$



Subtask 2

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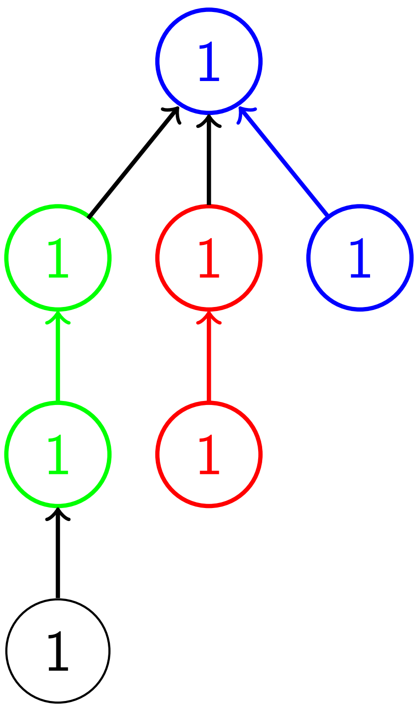


maximum matching may not be unique

Subtask 2

- How to find maximum matching?
- Tree is a bipartite graph
  - simply divide the tree into odd level nodes and even level nodes
- Bipartite matching - *Hopcroft-Karp algorithm*
- Underlying mechanism is same as *Dinic's algorithm*
- Time complexity: $O(E\sqrt{V}) = O(V\sqrt{V})$
  - *for trees, $E = V - 1$

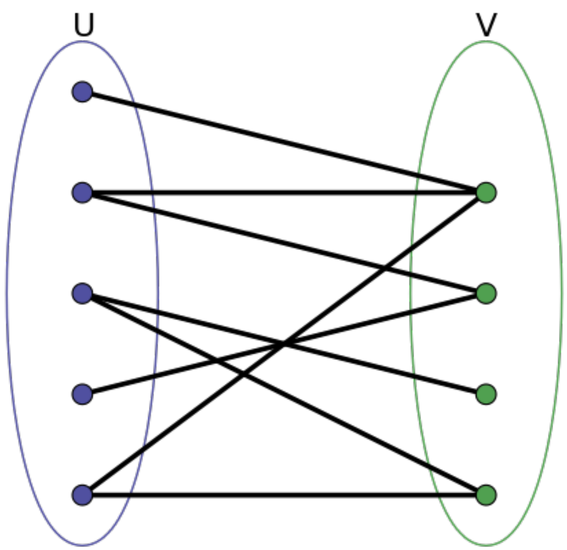


Figure: Bipartite graph

Subtask 3

$s_j = 1$

- Same as subtask 2, want to find maximum matching
- Maximum matching on trees can be solved with Dynamic Programming
- $DP_0(u) = |\text{maximum matching}|$ in the subtree rooted at u without matching u
- $DP_1(u) = |\text{maximum matching}|$ in the subtree rooted at u with or without matching u

$$DP_0(u) = \sum_{v \in \text{children of } u} DP_1(v)$$

$$DP_1(u) = \begin{cases} DP_0(u) + 1 & \text{if some } v \text{ is not matched} \\ DP_0(u) & \text{otherwise} \end{cases}$$

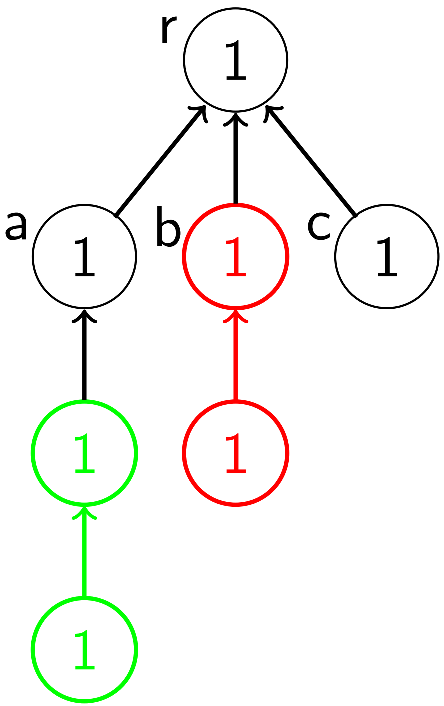
$$\text{answer} = \sum_{p_r = 0} DP_1(r)$$

Time complexity: $O(N)$

Subtask 3

For example

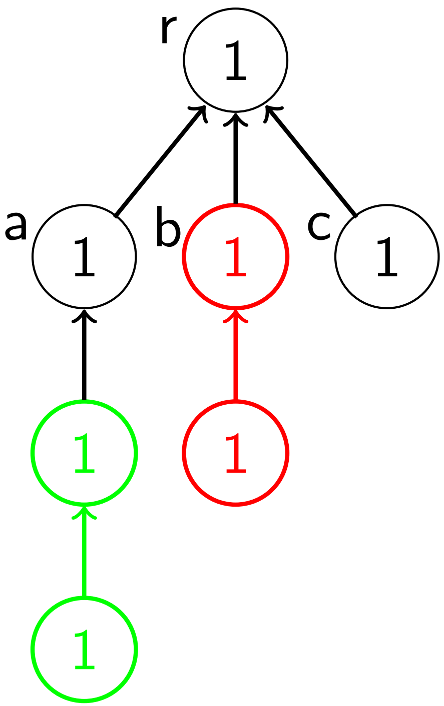
- $DP_0(a) = 1$, $DP_1(a) = 1$



Subtask 3

For example

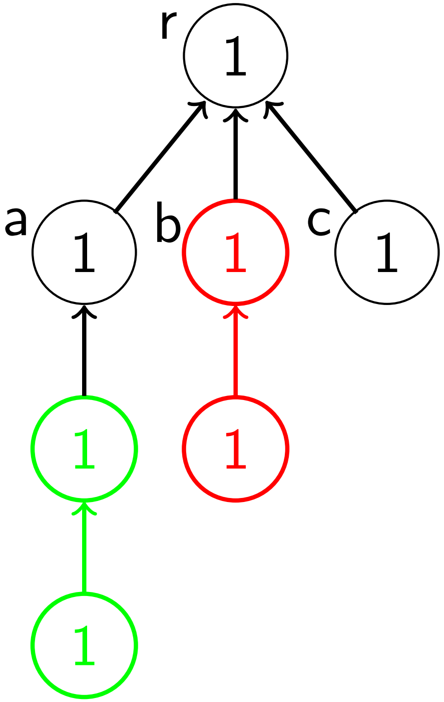
- $DP_0(a) = 1$, $DP_1(a) = 1$
- $DP_0(b) = 0$, $DP_1(b) = 1$



Subtask 3

For example

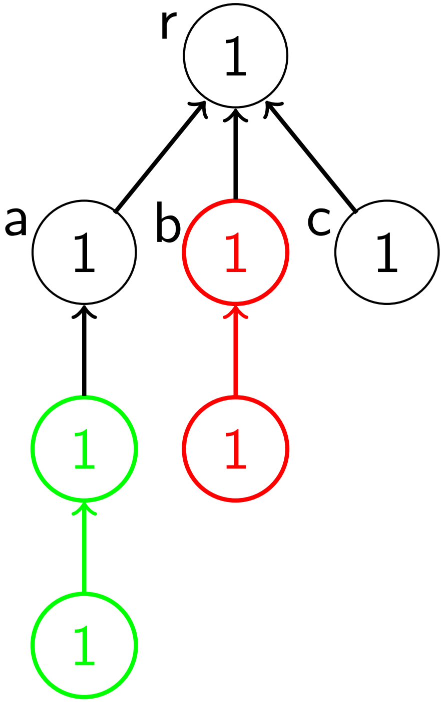
- $DP_0(a) = 1,\ DP_1(a) = 1$
- $DP_0(b) = 0,\ DP_1(b) = 1$
- $DP_0(c) = 0,\ DP_1(c) = 0$



Subtask 3

For example

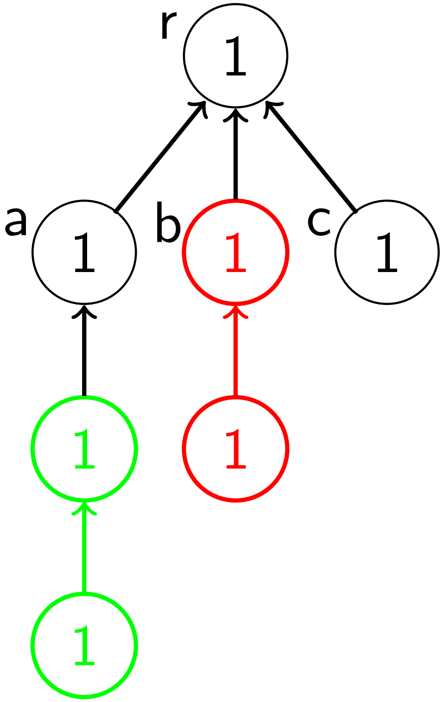
- $DP_0(a) = 1,\ DP_1(a) = 1$
- $DP_0(b) = 0,\ DP_1(b) = 1$
- $DP_0(c) = 0,\ DP_1(c) = 0$
- $DP_0(r) = 1 + 1 + 0 = 2$



Subtask 3

For example

- $DP_0(a) = 1,\ DP_1(a) = 1$
- $DP_0(b) = 0,\ DP_1(b) = 1$
- $DP_0(c) = 0,\ DP_1(c) = 0$
- $DP_0(r) = 1 + 1 + 0 = 2$
- $DP_1(r) = DP_0(r) + 1 = 3$



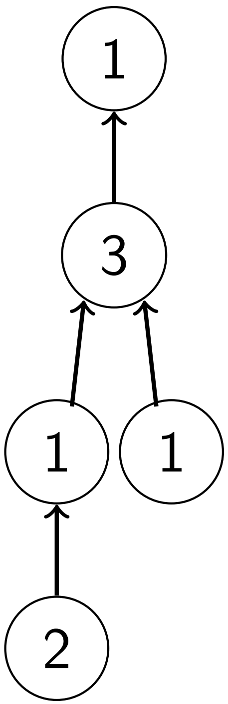
Subtask 4

$N \leq 1000$

- Designed for solutions aiming to solve subtask 5 that are not efficient enough

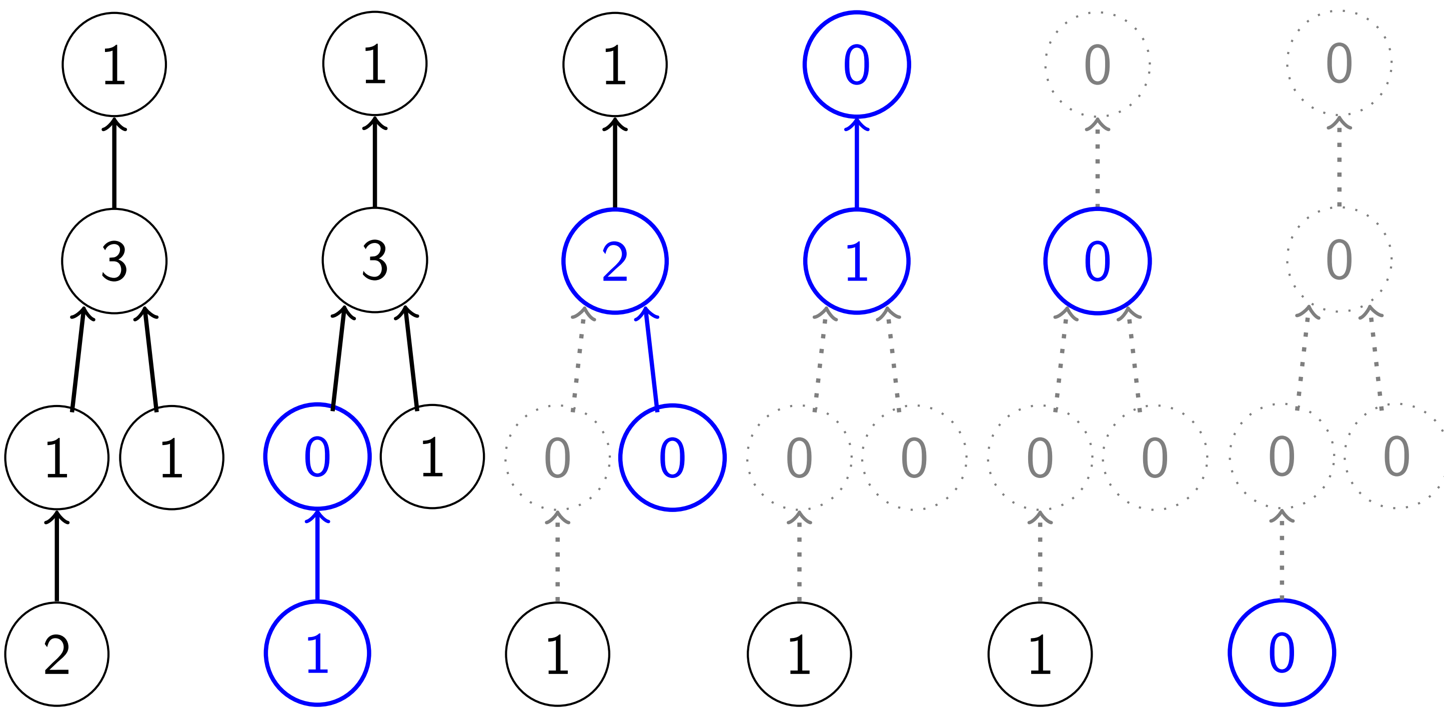
Subtask 5

- No matter what, when a leaf is being drawn, its parent should be drawn simultaneously
- Better draw the leaves before drawing anything else, so that their parents will have fewer strokes left



Subtask 5

- Repeatedly draw on an existing leaf and its parent
- When a node is completed, remove that node



Subtask 5

- Implement with DFS; or
- perform a topological sort, and then linear scan
  - *topological sort* sorts vertices of a directed acyclic graph (DAG) in a way that the parent of a vertice comes before itself
  - a rooted tree with edges directed from children to parent forms a DAG
  - *topological sort* is implemented as DFS
- Time complexity: $O(N)$

Osu!

Charlie Li 2018/07/02

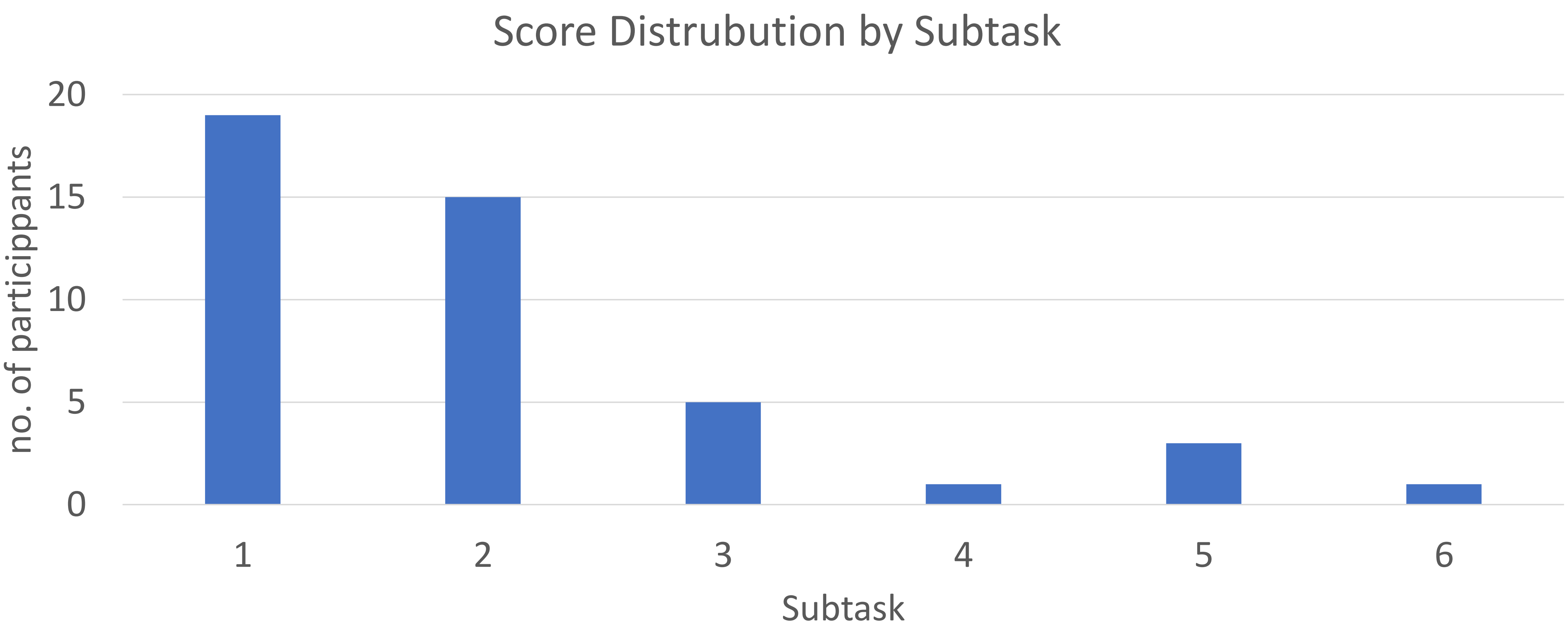
Problem statement

- Given a sequence of N queries of the following type
  - Add a point (S, T) to set A or set B
  - Query $\min(S_A \times S_B + T_A + T_B)$ where $(S_A, T_A) \in A$ and $(S_B, T_B) \in B$
- Initially, there is a plan (A, 0) in A and a plan (B, 0) in B.

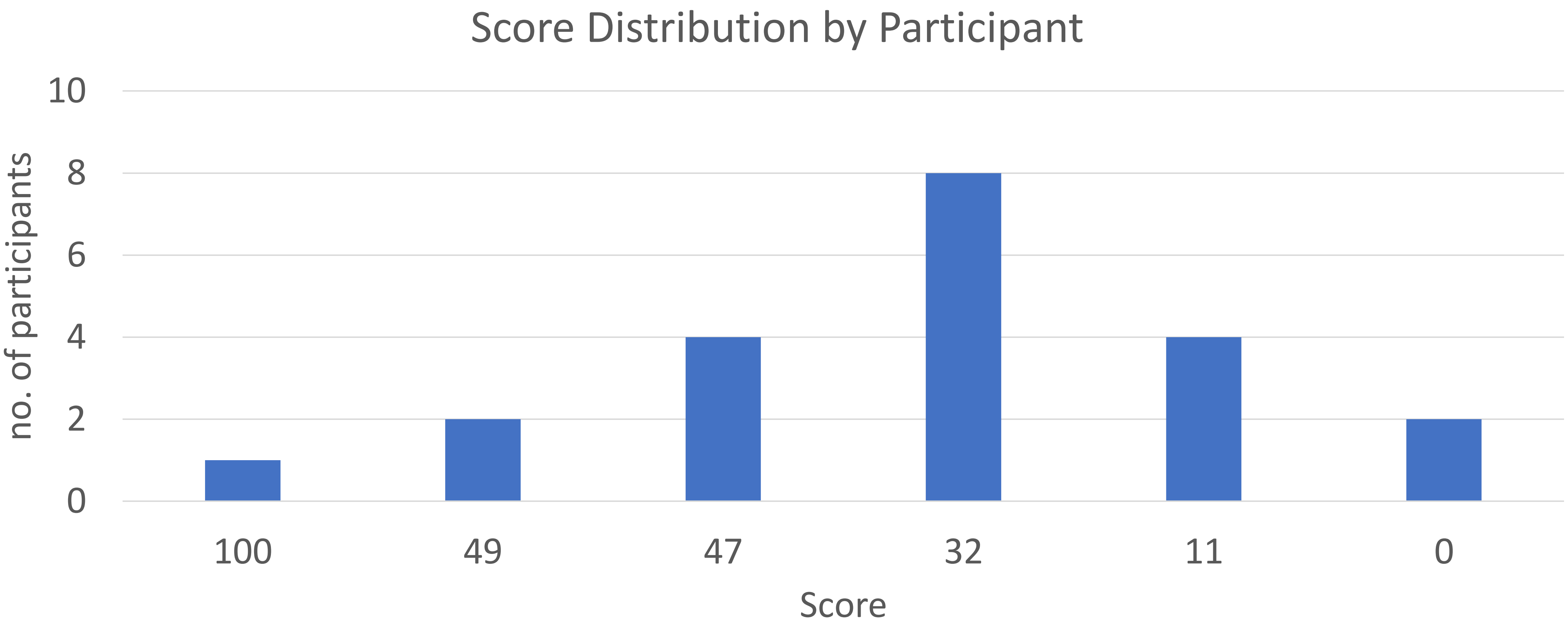
- Constraints:
  - $1 <= N <= 500000$, $1 <= S_i, T_i <= 10^9$

Subtasks

- Subtask 1 (11 points): N <= 500
- Subtask 2 (21 points): N <= 5000
- Subtask 3 (15 points): N <= 50000
- Subtask 4 (12 points): N <= 100000
- Subtask 5 (17 points): S_i, T_i <= 10000
- Subtask 6 (24 points): No additional constraints



Score Distrubution by Subtask
no. of particippants
20
15
10
5
0
1
2
3
4
5
6
Subtask


Score Distribution by Participant
no. of participants
10
8
6
4
2
0
100
49
47
32
11
0
Score

Subtask 1

- Use array to store the elements of A and B

For every query

Set ans to INF

For every element (S_A, T_A) of A

For every element (S_B, T_B) of B

Update ans if $S_A \times S_B + T_A + T_B < ans$

- Time complexity: $O(N^3)$
- Expected score: 11

Subtask 2

- When we look at the solution for subtask 1, we can see that many of the pairs are calculated repletely which is not necessary.
- Instead, we can maintain the current answer.
- When a new plan is added, we try to see if this new plan gives a smaller answer and update current answer.
- This can reduce the overall time complexity to $O(N^2)$

Subtask 2

```
For every query
    if it is type 1
        if the plan is for part A
            insert the plan into A
            for every plan in B
                try to update answer
        if the plan is for part B
            insert the plan into B
            for every plan in A
                try to update answer
    if it is type 2
        output answer
```

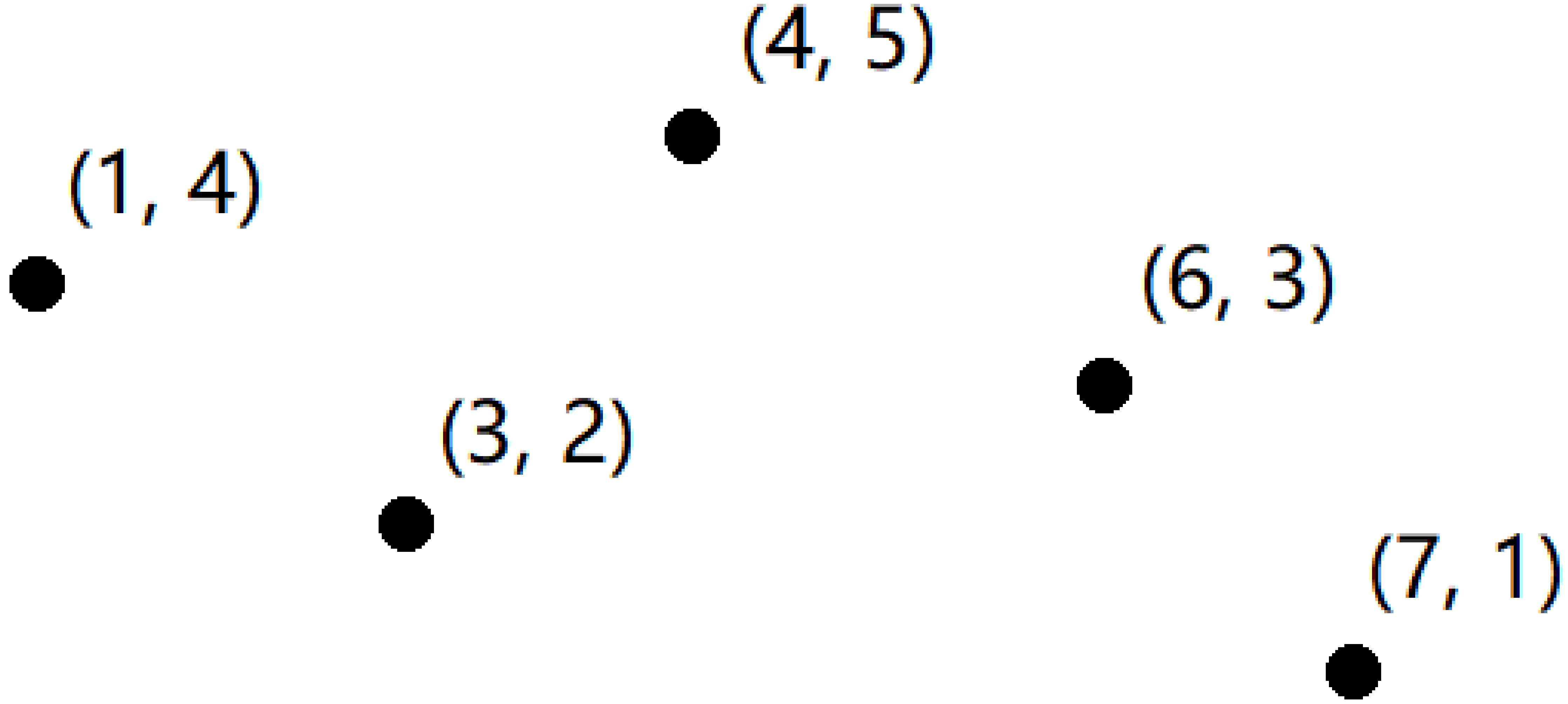
Subtask 2

- Some contestant even manage to pass subtask 3 using this algorithm.

Observations

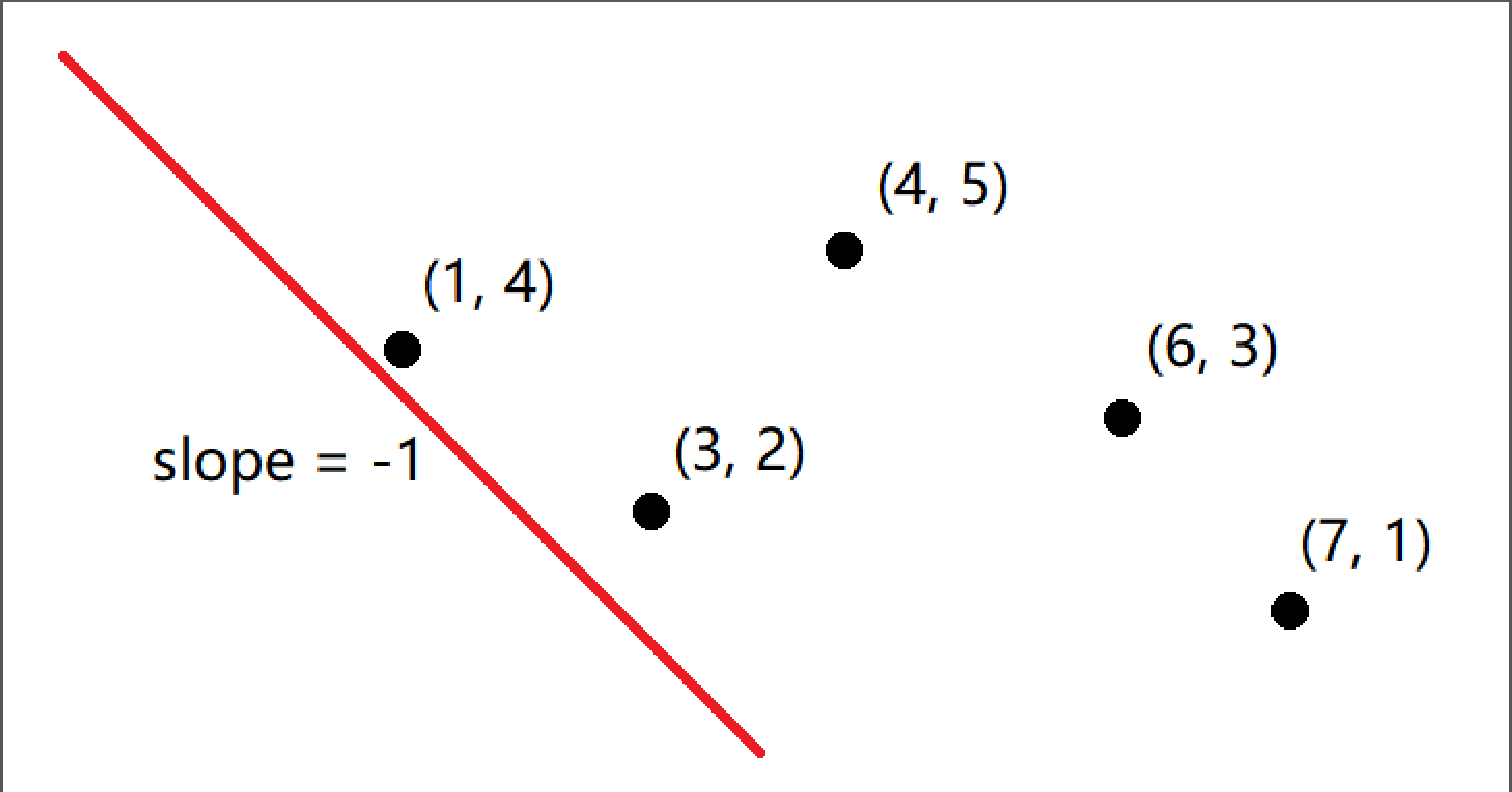
- Consider the following simplified version.
- Given a set A.
- Given a point (S_B, T_B) in B.
- You are to find $\min(S_A \times S_B + T_A + T_B)$ where $(S_A, T_A) \in A$

- It is obvious that if $S_A = S_A'$ and $T_A < T_A'$ then choosing (S_A, T_A) must be better than choosing (S_A', T_A') , so we can remove (S_A', T_A') , let's assume S_A are distinct from now on.


(1, 4)
(4, 5)
(3, 2)
(6, 3)
(7, 1)

Observations

- Recall what we have learnt in senior secondary school.
- To find $\min(x \times S_B + y + T_B)$, we can use linear programming.
- Let’s draw a line with slop $-S_B$


(4, 5)
(1, 4)
(6, 3)
slope = -1
(3, 2)
(7, 1)

Observations

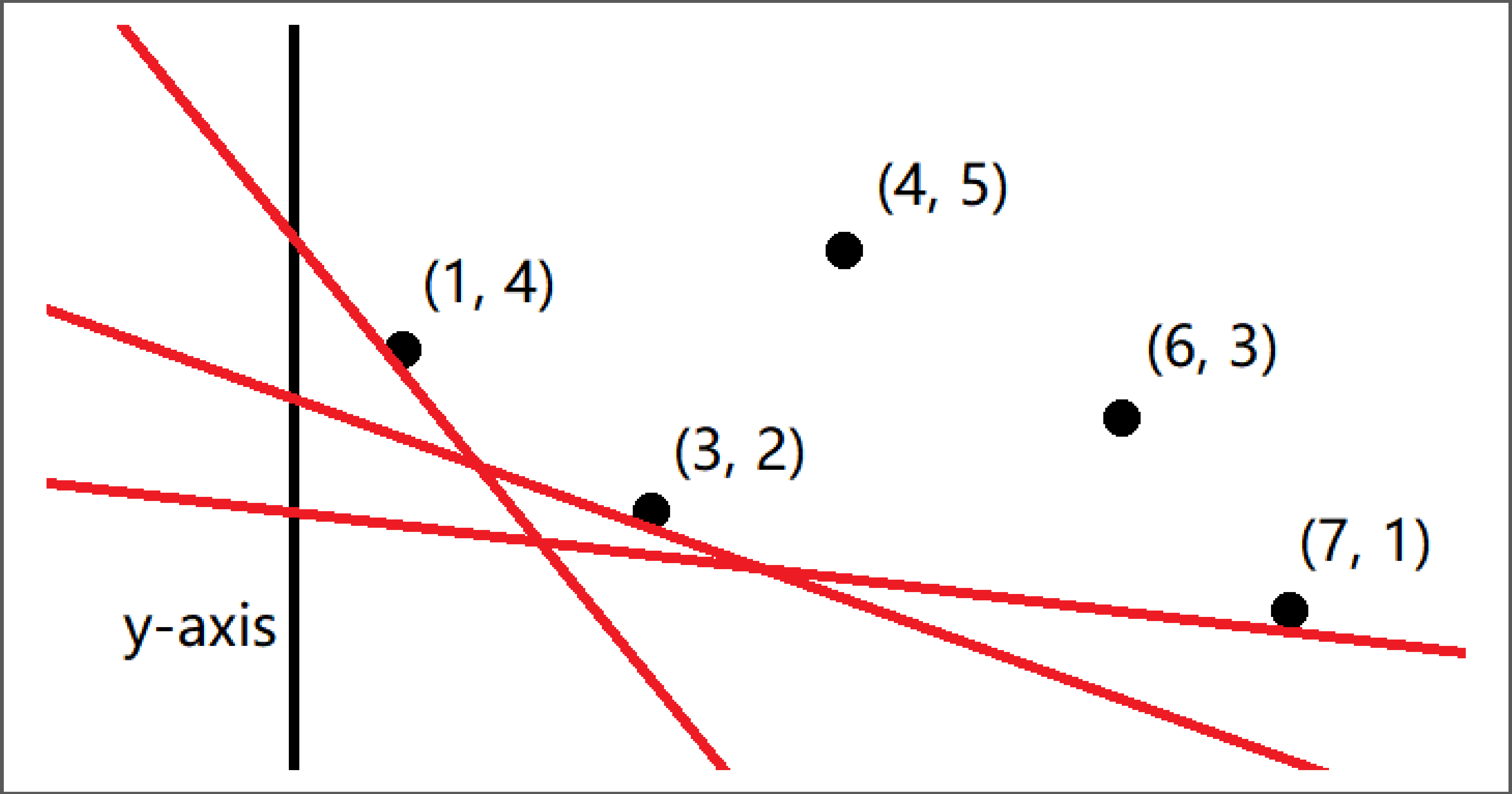
- We know that the point which touches the line with smallest y-intercept is corresponding to the minimum solution.

- So we know that when $S_B = 1$
  - $S_B \times 1 + T_B + 4$ is the minimum

- But, can this really speed up our solution?

Observations

- By drawing lines with different slope, we found that some points will never optimal.


(4, 5)
(1, 4)
(6, 3)
(3, 2)
(7, 1)
y-axis

Observations

- Let H be the set useful points
- (For those who knows convex hull: the set H is actually a lower hull)

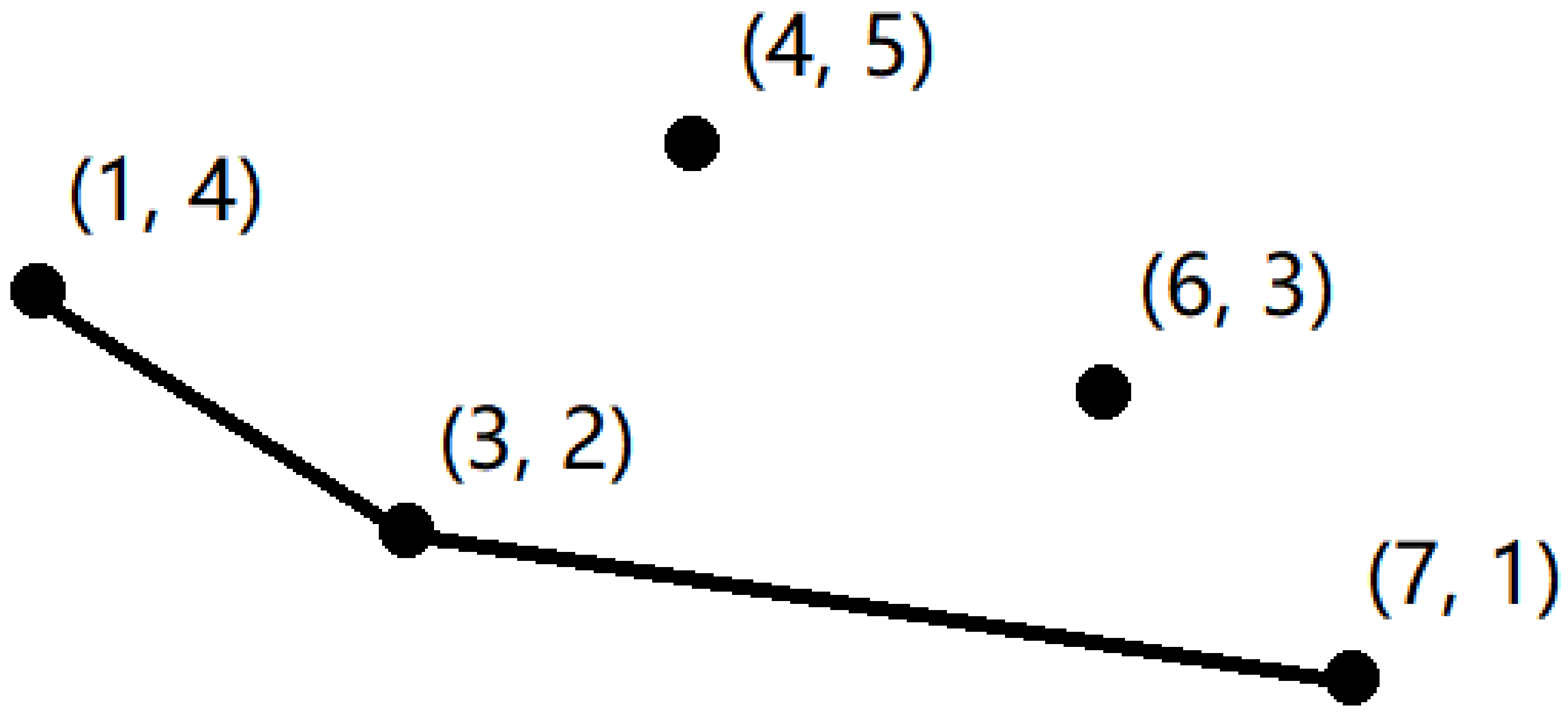
- We can see that if S_B is very big -> the line is very steep -> we will always choose the leftmost point.
- So the leftmost point must be useful.

Observations

- Let $r = (r_x, r_y)$ be the right most point in H.

- We can see that a point on the right side of r which makes the smallest slope with r will also be useful.

- So how can we use this fact to build the set H?


(4, 5)
(1, 4)
(6, 3)
(3, 2)
(7, 1)

Observations

- Consider joining the useful points one by one, we can see that the slope is increasing.
- So we may use a stack to build H.
- Let p1 be the leftmost point.
- Initially, we will p1 and the point p2 which makes a smallest slope with p1(we can find this point just using a linear search)
- Then we know that any point with x-coordinates between p1 and p2 must not be a useful point, so we can also remove them.

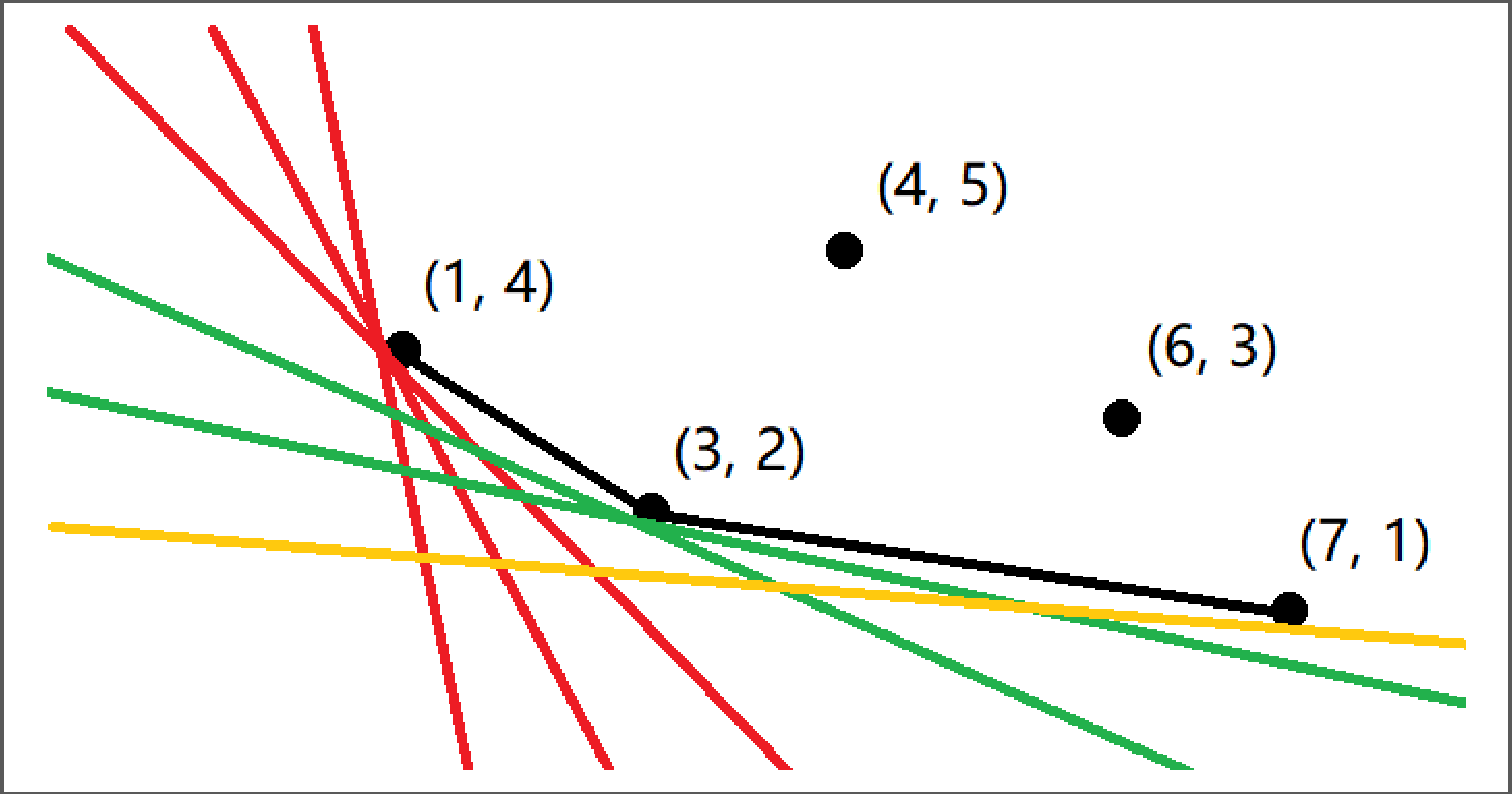
Observations

- Then we can use a stack to build H.

```
Push p1 and p2 into H
r = p2, r2 = p1
For any remaining point p in A from left to right up to (A, 0)
        while slope(r2, r) > slope(r, p)
                pop H
                r = last of H
                r2 = second last of H
        push p into H
```

Observations

- After building up H, we can find an optimal useful point for any given slope very fast.
- How?


(4, 5)
(1, 4)
(6, 3)
(3, 2)
(7, 1)

Observations

- We can see that if the slope is very small (steeper), then we will choose the left most point.
- If the slope is bigger than the slope between two consecutive points in H, we can see that choosing the left point must not be an optimal solution.
- So we can perform a binary search on H to find the optimal point.

- Since we can reuse H for every point in B, so we can find the optimal solution between all pairs of plans in O(N log N)

Subtask 2*

For every query,

use the above method the build H_A and H_B.

For every point p_A in A

do a binary search on H_B and update answer

For every point p_B in B

do a binary search on H_A and update answer

- Time complexity: $O(N^2 \log N)$
- Expected score: 32

Subtask 2*

- Actually, you can replace the binary search by linear search if you have sorted the points.

- A small trick here,
- You can include the plan (INF, 0) to A and B
- This can make sure that there are always at least 2 points in A and B so that we can avoid the case that our H will have only one point

Subtask 3

- Does the constraint looks strange?

- As you may guess by looking at the constraint, we can do square root decomposition on this question.

Subtask 3

- We will maintain the current answer and a partially online version of H.

- We will rebuild H_A and H_B in every $\sqrt{N}$ queries

- For every type 1 query of inserting points to A (or B),
  - We can insert the point to new_A (or new_B)
  - Do a binary search on H_B and update answer
  - Do a linear search on new_B and update answer
- For every type 2 query, we can simply output current answer.

Subtask 3

- Rebuild part
  - Single time : O($N \log N$)
  - We will perform O($\sqrt{N}$) times.
  - Overall : O($N\sqrt{N} \log N$)
- Query & update part
  - Single time : O($\log N + \sqrt{N}$) = O($\sqrt{N}$)
  - We will perform O(N) times.
  - Overall : O($N\sqrt{N}$)
- Time complexity: O($N\sqrt{N} \log N$)
- Expected score: 47

Subtask 4

- The constraint is just a bit larger than that of subtask 3, maybe getting rid of a log N in the solution is useful.

- We can see that the log N only appears in the rebuild part and that is because of sorting.

- Can we maintain the order of data using less time?

Subtask 4

- Can we maintain the order of data using less time?
- Yes, we can!

- When we insert points into new, we can insert according to order, which is $O(N^{0.5})$

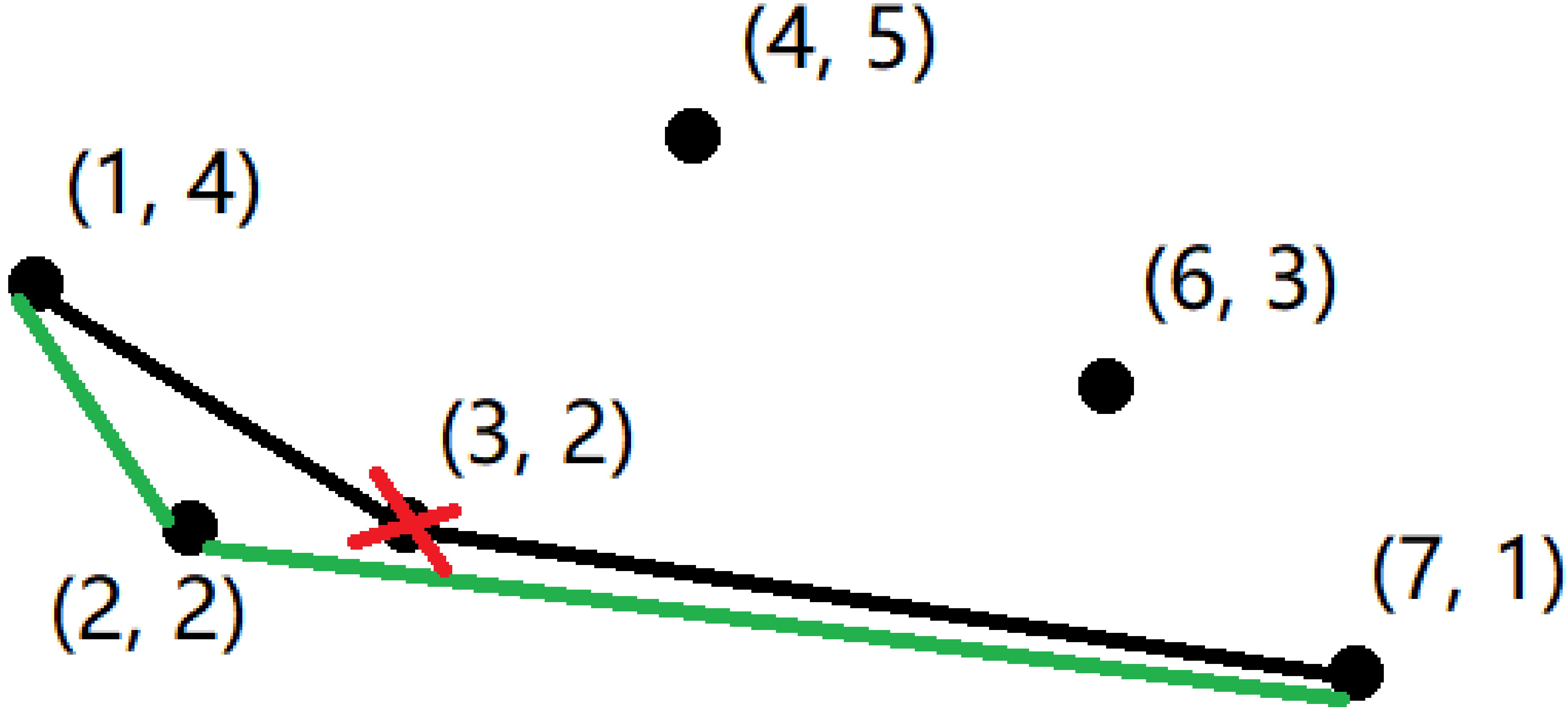
- Then in the rebuild part, we do not need to sort the points but merge two arrays of sorted points into one.

Subtask 4

- Rebuild part
  - Single time : O(N)
  - We will perform O($\sqrt{N}$) times.
  - Overall : O($N\sqrt{N}$)
- Query & update part
  - Single time : O($\log N + \sqrt{N}$) = O($\sqrt{N}$)
  - We will perform O(N) times.
  - Overall : O($N\sqrt{N}$)
- Time complexity: O($N\sqrt{N}$)
- Expected score: 59

Observations

- We can see that adding new points will only make some useful point useless but not useless point useful



(4, 5)
(1, 4)
(6, 3)
(3, 2)
(2, 2)
(7, 1)

Observations

- Suppose R is the maximum coordinate of the points.

- There are at most $O(\sqrt{R})$ points in H.

- Consider the points (1, R), (2, R-1), (3, R-1-2)…
- Slope -1 to -$\frac{\sqrt{8R+1}-1}{2}$ is included in H
  - so there are at most $O(\sqrt{R})$ points in H.

Subtask 5

- Base on a solution of subtask 2, 3 or 4.
- When a point is removed in building H, we remove it totally so that we will never consider this point when building H later.

- The time complexity will be the same but the runtime will be around 10 times faster (for R <= 10000)

Subtask 6

- There are 2 solutions for subtask 6.
- The first one is to maintain the set H online.
- We can achieve this by storing H in a set.
- When we try to insert a new point (s, t) into the set, we may need to remove a consecutive part in H.
- This involves usage of iterator for set and edge cases handling, seems difficult to discuss here…
- So try to implement yourself, this can train your coding skills.

Subtask 6

- Suppose we can maintain H online with O($\log N$) time.
- We should also maintain the current best answer like in subtask 3.
- We just replace the query & update part by using the set H.

- Ps. Don't forget that we have to maintain plan from two parts, ie. 2Hs

- Time complexity: O($N \log N$)
- Expected score: 100 if no bugs
- Solution: https://judge.hkoi.org/submission/284650/details?sharing=wCgqOGiNL6Hth1e3UhX1o4ZWE

Subtask 6

- The other one is easier
- Recall what you have learnt yesterday again….

Subtask 6

- CDQ D&C seems useful here.
- But how?

Subtask 6

- Actually, type 2 is NOT meaningful in this algorithm.
- We should treat every insertion of point as both update and query.
- So we will store the answer on type 1 query.
- To find the answer of a type 2 query, we just need to find the running min.

- Time complexity: O($N \log^2 N$)
- Expected score: 100
- Solution: https://judge.hkoi.org/submission/284647/details?sharing=iDXniTG2axPNrIburijL08ZitU

Subtask 6

- We can even get rid of one log by using linear search and merge sort.

- Time complexity: O($N \log N$)
- Expected score: 100
- Solution: https://judge.hkoi.org/submission/284632/details?sharing=bKqdUhFUWKvbsCB6pI1Z0qw5G0
- However, the runtime is not improved.

Civilizations

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Source: KOI 2017 (High School)

Task Description

- Initially there are K painted cells in a N x N grid
- Each year the painted cells expand to their adjacent cells (4 directions)
- Find number of years until the painted cells become connected

Subtask 1: $O(N^3)$

- Repeat (at most N times)
  - Longest path is corner to the opposite corner
- Check if the painted cells are connected using DFS or BFS | $O(N^2)$
- For each painted cell, expand to adjacent cells | $O(N^2)$

Subtask 2: $O(N^2 \lg N)$

- Note that in Subtask 1 we try ans = 1, 2, 3, ...
- Instead, we can use binary search on answer
- After fixing the time t, we can use BFS from each civilization's origin, and mark all nodes with distance <= t
- Finally, check if the civilizations are connected

Subtask 3: $O(N^2)$

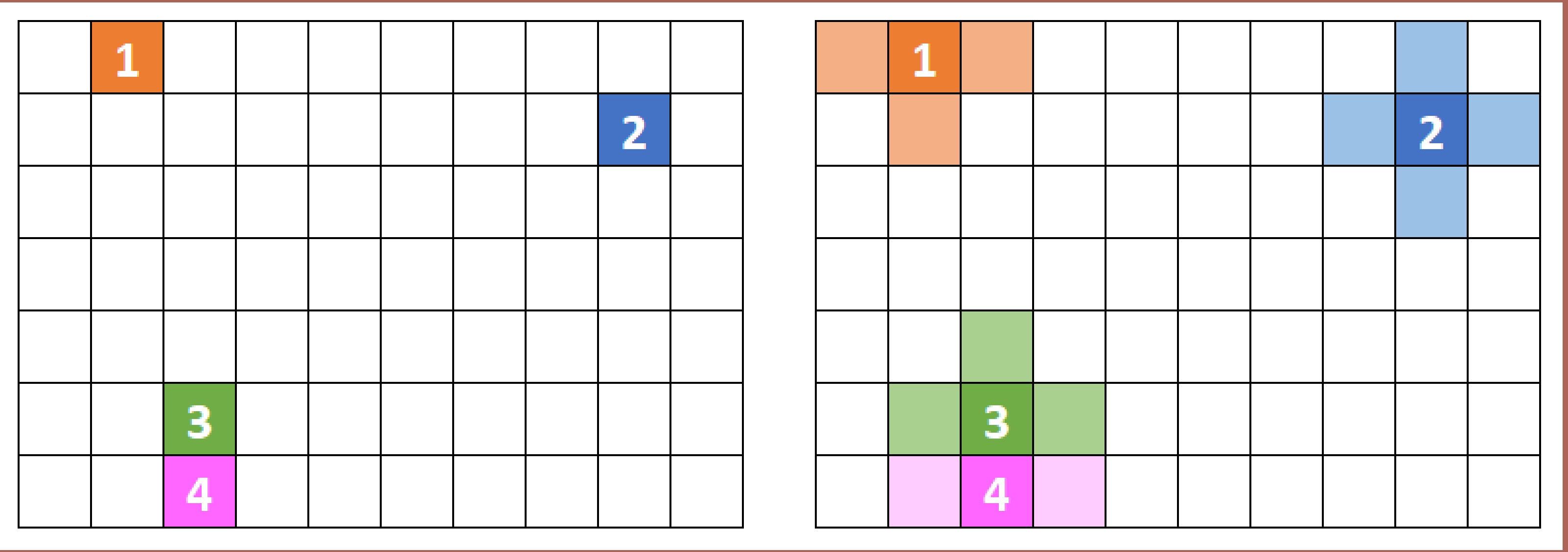
- To solve the problem, we optimize subtask 1's solution
- Repeat (at most N times)
- Check if the newly painted cells are connected using ~~DFS or BFS~~ disjoint-set union-find | $O(N)$
- For each newly painted cell, expand to adjacent cells | $O(N)$

Subtask 3: O(N^2)

- Use a vector to store "newly painted cells"
- For each input civilization:
  - Put the origin into the vector
  - Create a disjoint set with group_size[i] = 1
- For year = 0, 1, 2 ....
  - For each last year's newly painted cells, union them with the adjacent cells
    - When merging disjoint set a into b: parent[b] = a; "transfer" the civilization count: group_size[a] += group_size[b]
  - If a disjoint set contains K civilizations, output day
  - For every last day's newly painted cells, add their unvisited adjacent cells into another vector

Year 0

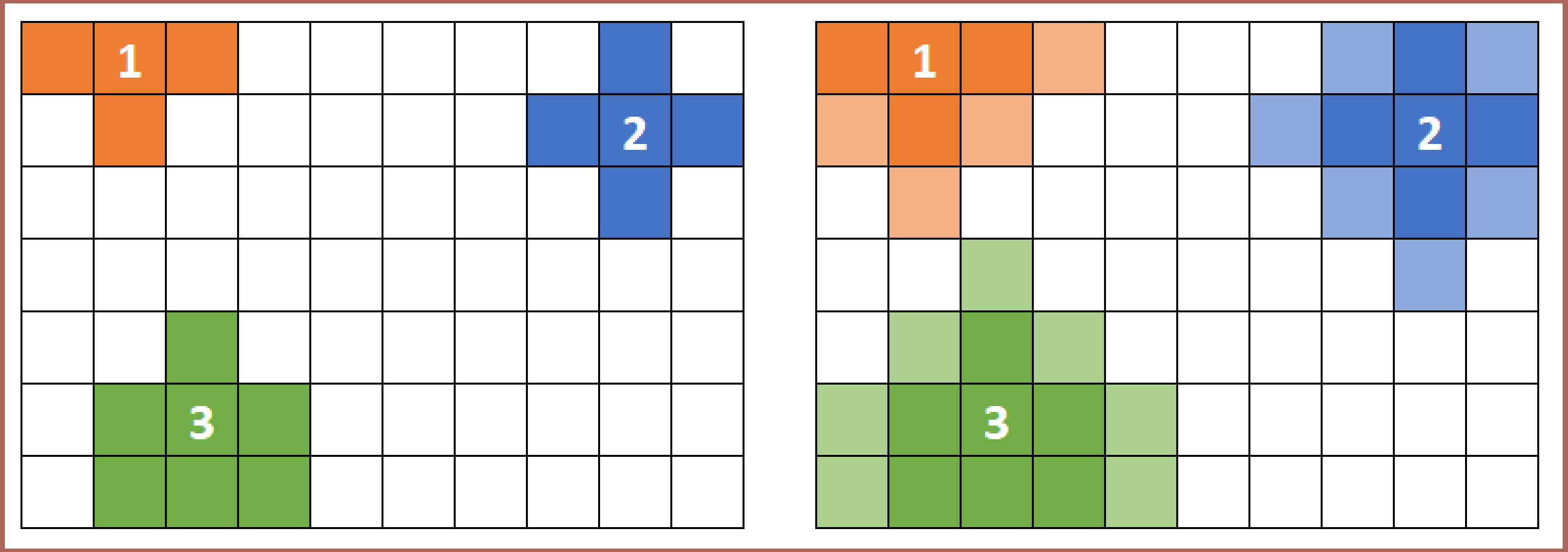
Merge | Expand



Disjoint Set	Size	Parent
1	1	1
2	1	2
3	1	3
4	1	4

Year 1

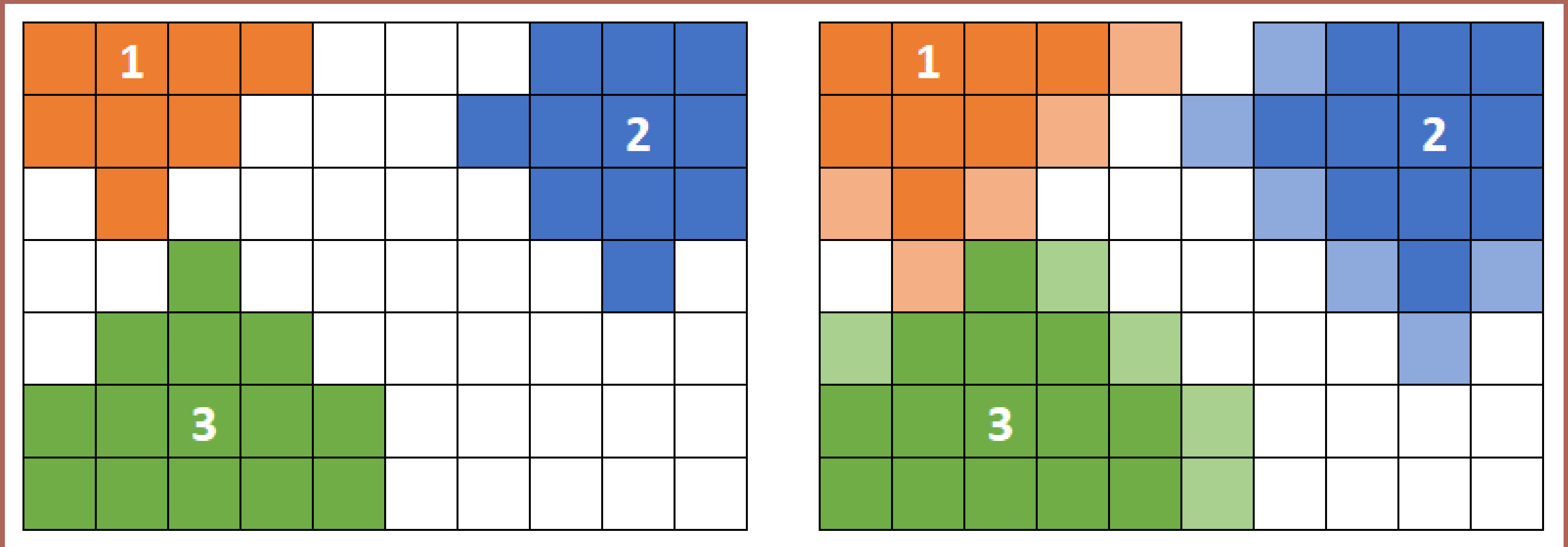
Merge | Expand



Disjoint Set	Size	Parent
1	1	1
2	1	2
3	2	3
4	--	3

Year 2

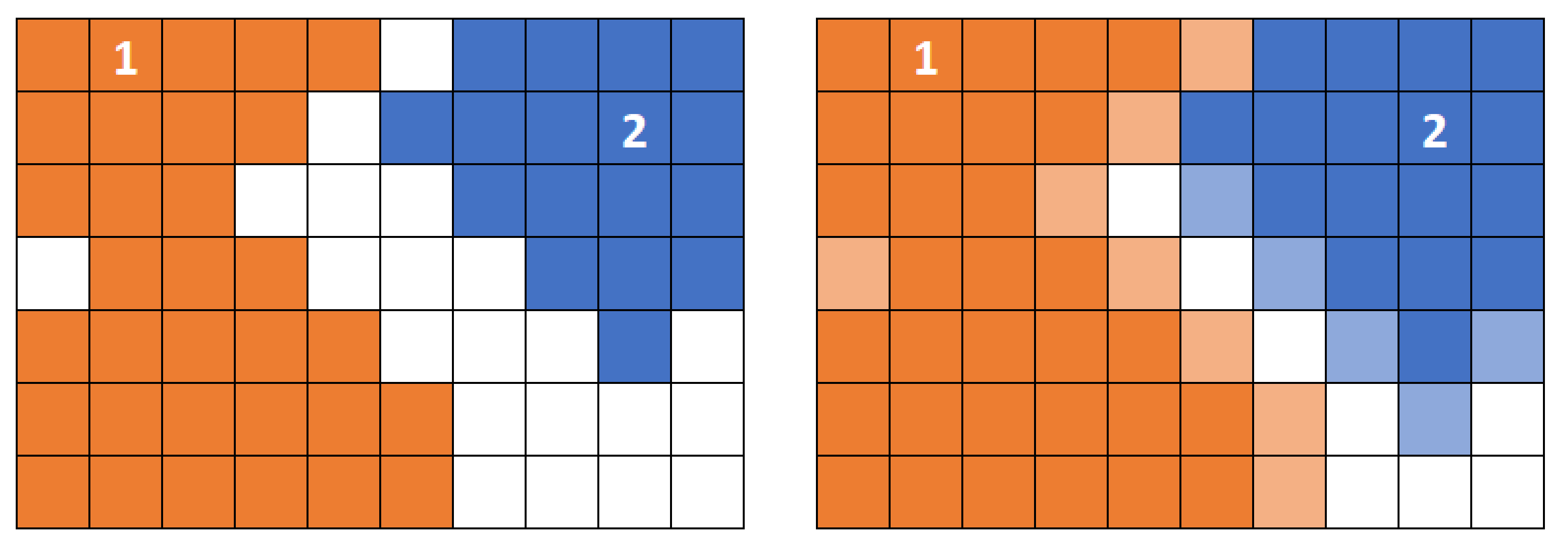
Merge　　　　Expand



Disjoint Set	Size	Parent
1	1	1
2	1	2
3	2	3
4	--	3

Year 3

Merge Expand

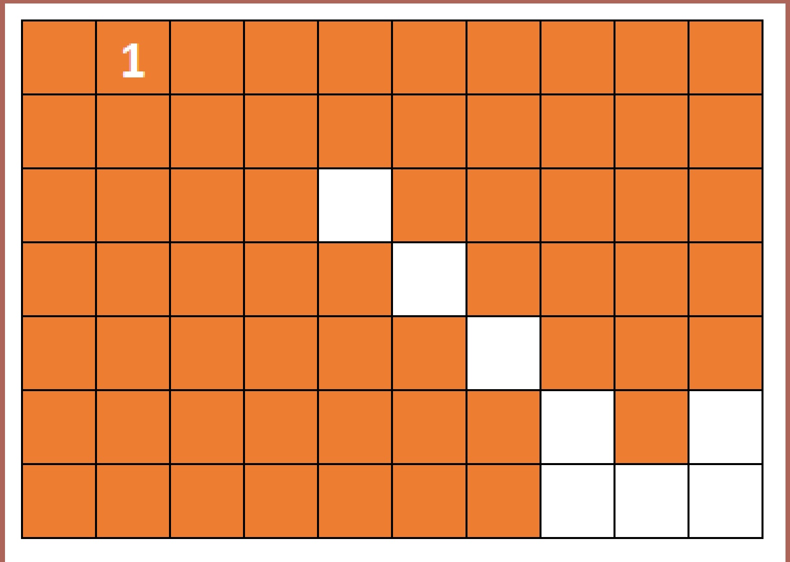


Disjoint Set	Size	Parent
1	3	1
2	1	2
3	--	3
4	--	3

Year 4

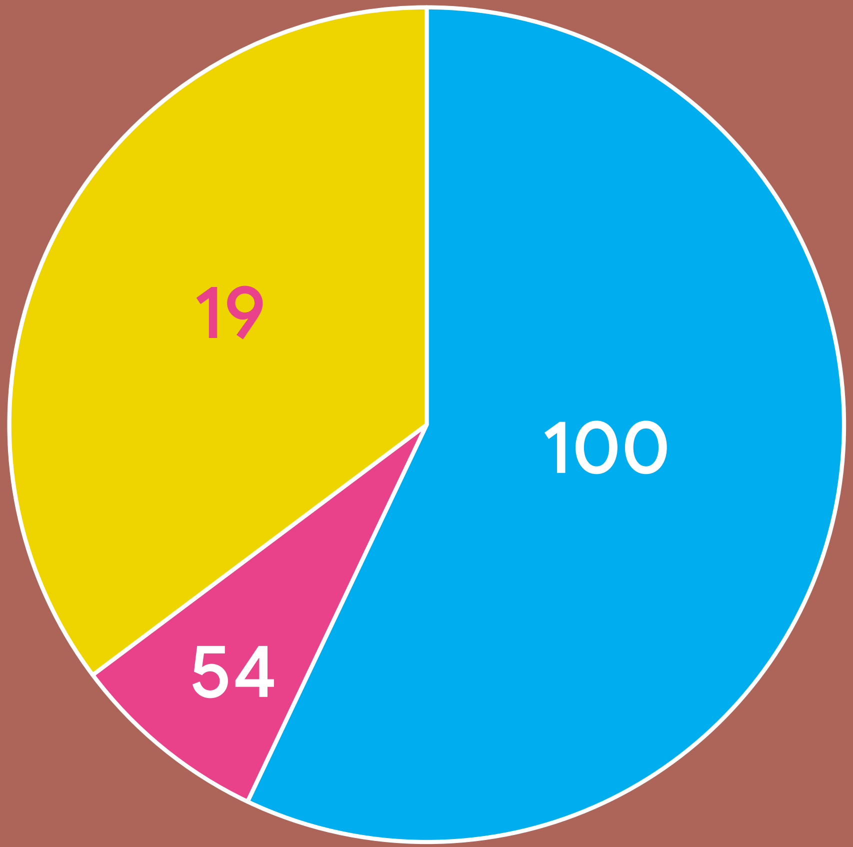
Merge

Expand



Disjoint Set	Size	Parent
1	4	1
2	--	1
3	--	3
4	--	3

Score distribution



Note: Area weighted by score