M1830 Lazy Tutor

Anson Ho

Simplified statement

- Given a list of binary numbers.

- In each query, find the number of submasks of a given binary mask in the original list.

- 01110_2, 01010_2 and 00000_2 are submasks of 01110_2
- 01111_2, 11010_2 and 11111_2 are not

Prerequisite

- bitwise operation
- https://en.wikipedia.org/wiki/Bitwise_operation

Solution

- dp[i][j] = number of submasks of j such that the bits other than the first i bits are fixed

- counted = , not counted = ✗

j \ submask	0 (first i bits)	1 (first i bits)	0 (last N - i bits)	1 (last N - i bits)
0		✗		✗
1			✗	

Solution

- initialization ($i = 0$):
  - dp[0][j] = number of j in the input list
- transition ($1 \le i \le N$):
  - "unfixing" the i^{th} bit
  - if the i^{th} bit of j is not set
    - dp[i][j] = dp[i – 1][j]
  - if the i^{th} bit of j is set
    - dp[i][j] = dp[i – 1][j] + dp[i – 1][j – 2^i]
- output
  - dp[N][y] if the mask in the query is y
  - i = N means no bits are fixed

The end

Play with Lines

Naive solution

- Brute force to fix two points
- A line can be fixed by fixing 2 points
- For each line, find is it cover at least V points

- Time complexity O(N^3)

Randomize solution

- Note that p is at least 20
- Randomly fixing two point to fix a line
- If such a line exists
- Probability of find such line if exist = Probability(both point line on that line)
- Probability(both point line on that line) >= 0.2 * 0.2 = 0.04
- Repeat the process 200 times
- Probability of find such line if exist = 1 – 0.96^200 = 99.97%
- Able to pass all test data

Randomize solution

- Also able to fix 1 point and brute force all line passing through the it
- Prob of find such line = 0.2 in this method
- Repeat 100 times, probability to find such line = 1 – 0.8^100 = 1

- Time complexity: O(Constant * N) where constant is around 200

M1837 Grid Exploration

Alex Tung
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14 April 2018

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Observation

Let S_1 and S_2 be length-N instructions with one-cycle displacements (dx_1, dy_1) and (dx_2, dy_2), respectively.
If $(dx_1, dy_1) \neq (dx_2, dy_2)$, then the instructions lead the robot to different final positions after $86400 \times M$ seconds.

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- So the first step of the solution is to enumerate all $O(N^2)$ valid one-cycle displacements.

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Given REM, N, dx, and dy.
Consider all length-N instructions with one-cycle displacement (dx, dy).
Count the number of possible final positions after executing just the first REM instructions.

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- So the overall time complexity is $O(N^2)$ per case.

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(x, y) can be reached **if and only if**:

- $-REM \leq x + y \leq REM$,
- $-REM \leq x - y \leq REM$,
- $dx + dy - (N - REM) \leq x + y \leq dx + dy + (N - REM)$,
- $dx - dy - (N - REM) \leq x - y \leq dx - dy - (N - REM)$, and
- $x + y$ and REM have the same parity.

"Simple Arithmetic"

Here is a (rather clumsy, in hindsight) way to count the number of (x, y).
Let $PARITY := REM \bmod 2$.
Let $u := \frac{x+y-PARITY}{2}$ and $v := \frac{x-y-PARITY}{2}$.
The constraints are of the form

- $L_1 \leq 2u \leq R_1$
- $L_2 \leq 2v \leq R_2$

for some L_1, R_1, L_2, R_2.

The Point

Each solution (u, v) corresponds to a solution (x, y).
Plus, the above system is super easy to solve :)

A Much Cleaner Solution (Credits: Charlie Li)

- We can split the instruction into two parts.
- Operations in the first part will be executed $\lfloor \frac{86400 \times M}{N} + 1 \rfloor$ times. (There are $86400 \times M \bmod N$ such operations.)
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- Clearly, given valid (d_1, d_2), one can get a valid final position.
- Moreover, if $(d_1, d_2) \neq (d'_1, d'_2)$, then they give rise to different final positions. (It has to do with $86400 \times M \geq N$.)

The Point

The first and second parts are independent.

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- Let $f(i)$ be the number of possible final positions after i moves from the origin.
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- Then the answer is $f(R) \times f(N - R)$, where $R := 86400 \times M \bmod N$.
- It turns out that $f(i) = (i + 1)^2$. (Prove it!)
- Kudos to Charlie for finding such an elegant $O(1)$ solution!

M1838 Hit the Bubbles!

Alex Tung
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14 April 2018

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- Think of it as adding a score of $v[i]$ to all P in the range.
- Range update $\rightarrow$ difference "array" :)

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- Specifically, we store $(l[i] - dx \times y[i], v[i])$ and $(r[i] - dx \times y[i] + 1, -v[i])$ for each i.
- To force the turret position to be in $[L, R]$, simply add (L, X) and $(R+1, -X)$ to the list.

Does it work?

- NO!!!

Does it work?

- NO!!!
- You must not add $(\mathit{endpoint}, \mathit{delta})$ coorresponding to *negative* $v[i]$s.

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- Number from 0 to $2r - 1$ the states $(0, \mathit{right}), (1, \mathit{right}), \ldots, (r-1, \mathit{right}), (r, \mathit{left}), (r-1, \mathit{left}), \ldots, (1, \mathit{left})$.

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- After one second, bubble state changes from S to $(S + 1) \pmod{2r}$.
- We want the bubble to be at state x at time y. So it should be at $x - y \bmod 2r$ at time 0.
- Beware: if $v[i] < 0$, we actually **don't want** it to be at state x at time y.

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- Next we decide the initial configuration of the bubbles.
- It can be calculated easily by observing the periodic behaviour.
- Overall time complexity: $O(NlogN)$.
- Be careful when $v[i] < 0$.

No sort, no search

solution

Lau Chi Yung

2018/04/14

Problem

- write 13 3
- write 15 5
- write 15 7
- ask 14 2 8
- ask 15 2 8
- write 16 5
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		13						
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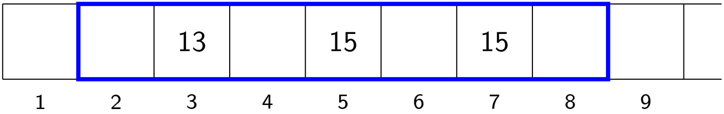

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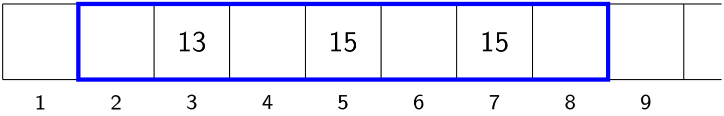

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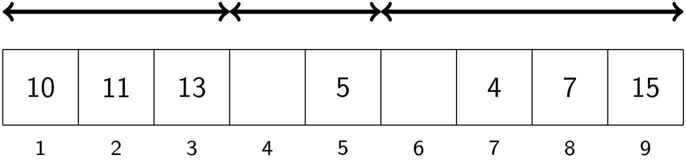
Problem

- Given a range, determine if it is sorted
- If it is sorted, find if a number exists
  - binary search *with empty slots*

Determine if a range is sorted

- At any time, we remember all disjoint sorted sub-ranges
- When we write to a slot, only the sub-ranges near that slot will have to be modified



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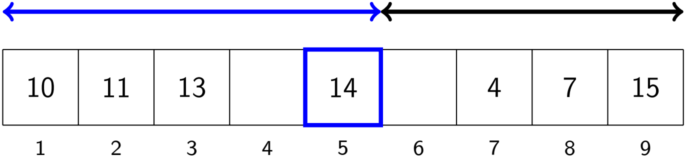

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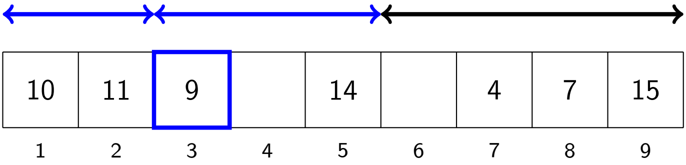

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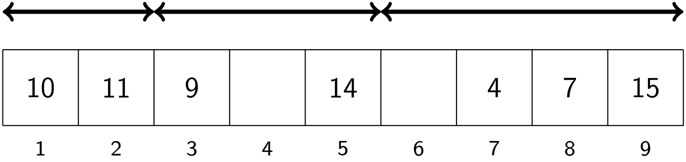

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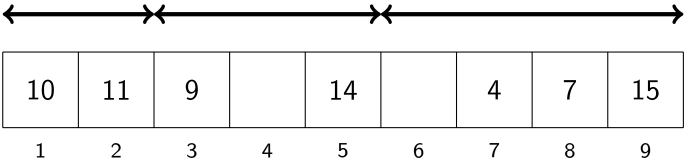
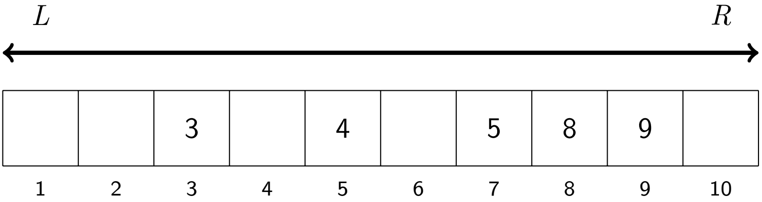
- $[L, R]$ is sorted if and only if L and R lie in the same sub-range
- Implementation: record the *right boundaries* of the ranges in a `set::<int>`, i.e. $\{2, 5, 9\}$
  Time complexity: $O(N \log N)$



Binary search with empty slots

1. Shrink $[L, R]$ to $[L', R']$ such that both boundaries are not empty
2. Calculate $M = \frac{L'+R'}{2}$
3. Calculate M' = the *nearest* slot to M that is not empty
4. Set $L' = M'$ or $R' = M'$, and recurse the process
5. (you should handle many index ± 1 special cases)
6. With `std::map<int, int>`, time complexity $= O(N \log^2 N)$



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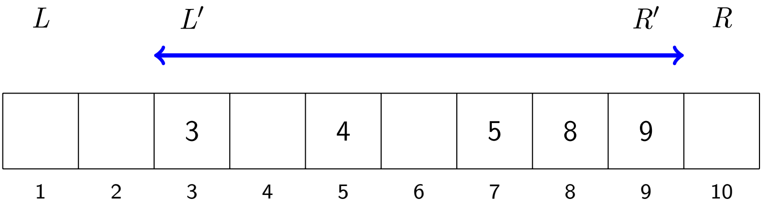

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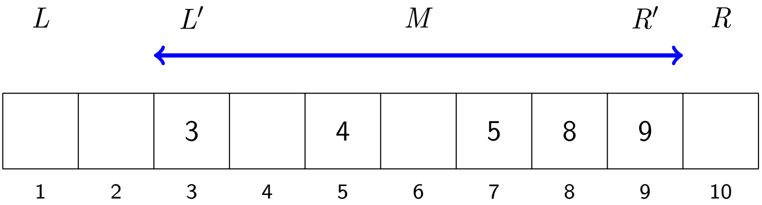

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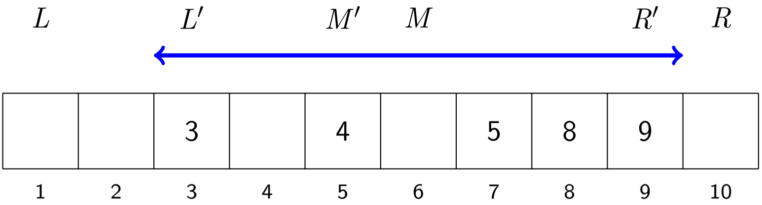

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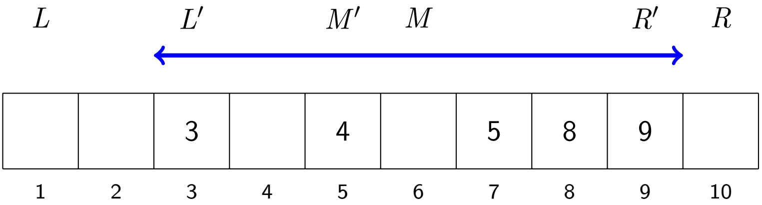
5. (you should handle many index ± 1 special cases)
6. With `std::map<int, int>`, time complexity $= O(N \log^2 N)$



Overall time complexity

$O(N \log^2 N)$