String Algorithms

HKOI 2017 training

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Content

- introduction
- trie
- KMP
- hashing
- suffix array
- sth else…

What is String?

a string is traditionally a sequence of characters (Wiki)

or more familiarly

- char s[SIZE];
- “Hello World!”
- {‘a’,’b’,’c’}

ASCII code

- American Standard Code for Information Interchange
- 8 bits = 1 byte = sizeof(char)
- 256 different states (0~255)
- ‘0’ = 48 ‘9’ = 48+9 = 57
- ‘A’ = 65 ‘Z’ = 65+26-1 = 90
- ‘a’ = 97 ‘z’ = 97+26-1 = 122
- ‘ ‘ = 32 ‘+’ = 43 ‘-’ = 45
- null= 0 ‘\n’= 10 ‘\r’= 13

Glossary

concatenation

- ”addition” in string

- 1+10 = 11
- “1”+”10” = “110”

- C++: “1””10”
- Haskell: “1”++”10”

Glossary

lexicographical order

- comparison in string
- dictionary order
- alphabetical order
- numerical order of ASCII code

- “1” < “2” < “A” < “B” < “a” < “b”

Glossary

lexicographical order

- same lengths -> numerical order
- “123” < “132”
- “ab” < “ac”

- different lengths -> add zeros
- “a” < “ab” {97,0} < {97,98}
- “ab” < “z” {97,98} < {122,0}

Glossary

prefix

- bicycle
- triangle
- hyperlink

suffix

- girls
- beautiful

Glossary

substring

- prefix of suffix, suffix of prefix
- contiguous subarray

subsequence

- obtained by deleting characters
- order is kept

Glossary

“Alice and Bob”

substring

- “”, “ce a”, “Alice and Bob”

subsequence

- all of the above
- “A B”, “AaBb”

Glossary

palindrome

- same as the reversed string

- “a”, “”
- “radar”, “abccba”

- longest palindromic subsequence
- O(N^2) with DP

I/O

- Input and Output
- What will you use to read a string from stdin?
- (in C++)

I/O

scanf printf

- read until space, line break or EOF
- ‘\0’ is automatically added

- int n;
- char x[5]; // max 4 char.
- scanf(“%d%s”, &n, x);
- printf(“%s”, x);

I/O

gets puts

- read until line break or EOF
- ‘\0’ is automatically added
- input will not include ‘\n’
- output with ‘\n’ at the end

- char x[5];
- gets(x);
- puts(x);

I/O

cin cout

- SLOW
- sync_with_stdio(false)

getchar putchar

- single character
- = scanf printf with %c
- unlocked

I/O

sscanf sprintf

- read from a string
- write to a string

- int n, m;
- char x[12];
- sprintf(x, “%d%d”, n, m);
- sscanf(x, “%d”, &n);

Container

char[]

- static size
- easy for I/O
- null terminator (‘\0’)
  - reserve one space for it

std::vector<char>

- dynamic size
- compatible with other STL tools
  - sort vector< vector<char> >
  - map, set

Container

<string>

- C++ object
- supports +, ==, <, <=, …

- char x[5];
- string str(x);
- str[0]=str[1];
- printf(“%s\n”,str.c_str());

<cstring>

- memcpy strcpy
- memcmp strcmp
- memset
- strlen

- don’t use if you forget how to use
- same time complexity as brute force
- maybe faster in terms of constant

Trie

- not a formal word
- (not found in Oxford Dictionary)

- a tree
- storing multiple string

- searching or counting strings or prefixes

Trie

```
struct node{
    int count,child[26];
    node *child[26];
};
```

- each node represents a character
- stores the positions of its children which are the next character
- stores the count of strings ending with the node or in its subtree

Trie

N: total length

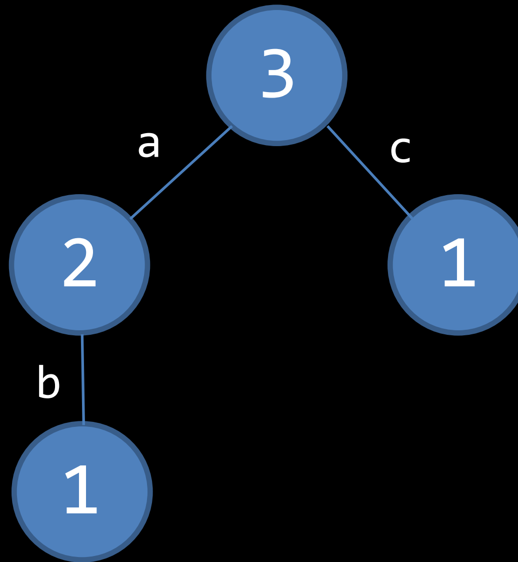
M: length of the target string

- time complexity:
  - insertion: O(M)
  - deletion: O(M)
  - searching/ counting: O(M)
  - sorting: O(26N)
- space complexity: O(26N)

actually all of them are DFS

Trie

insert “abc” to {“a”,”ab”,”c”}



Trie

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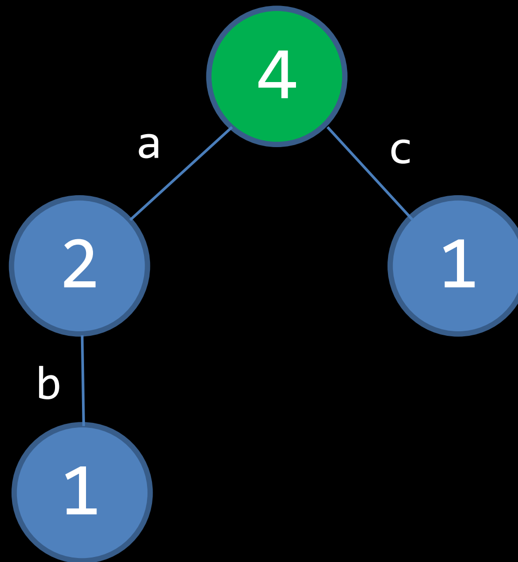

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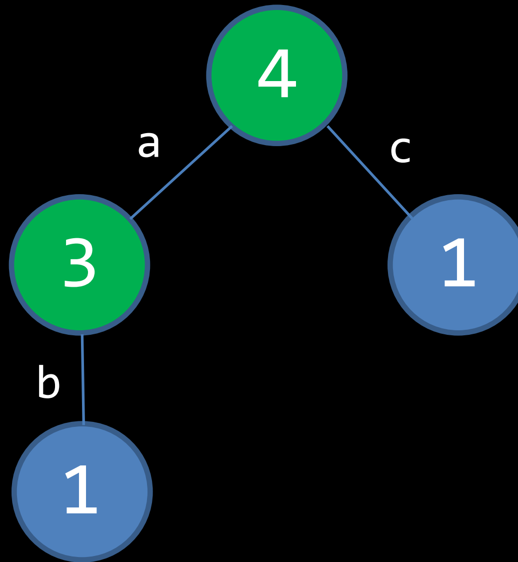

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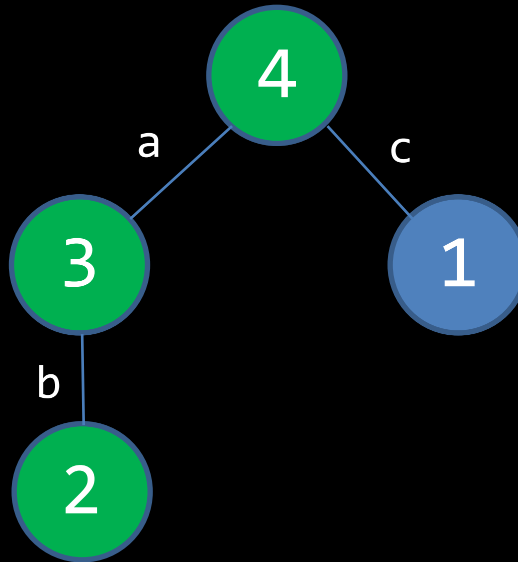

Trie

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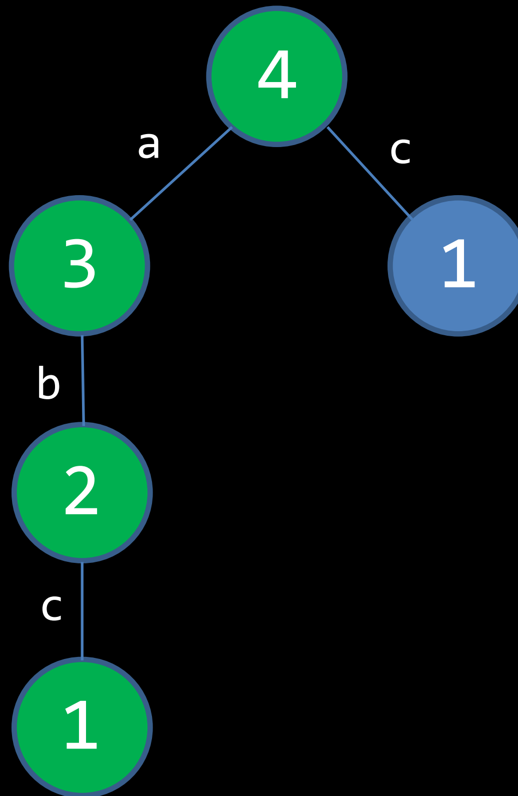
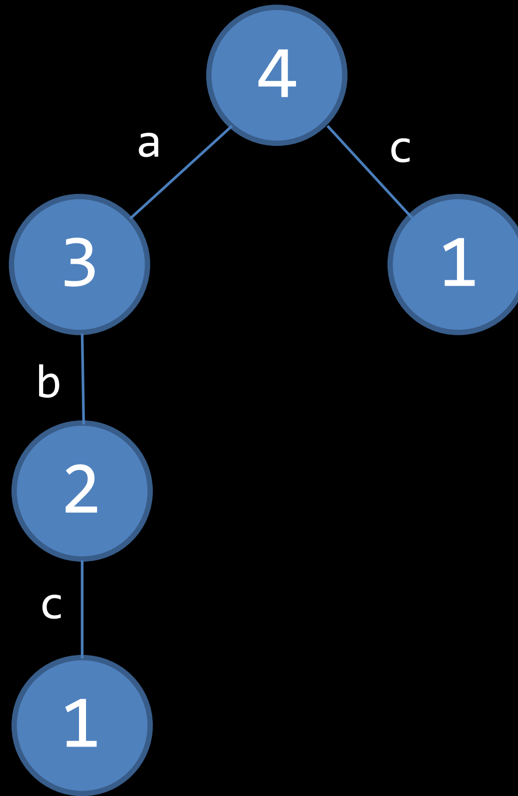


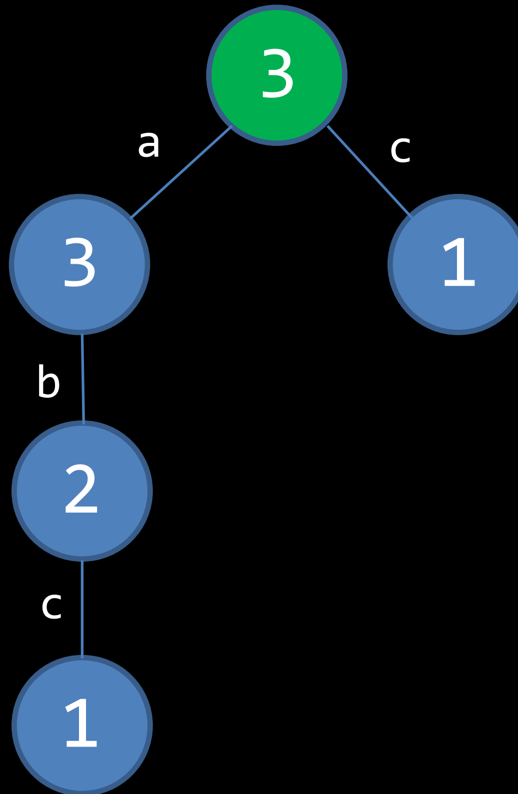
Trie

delete “ab” from {“a”,”ab”,”abc”,”c”}



Trie

delete “ab” from {“a”,”ab”,”abc”,”c”}



Trie

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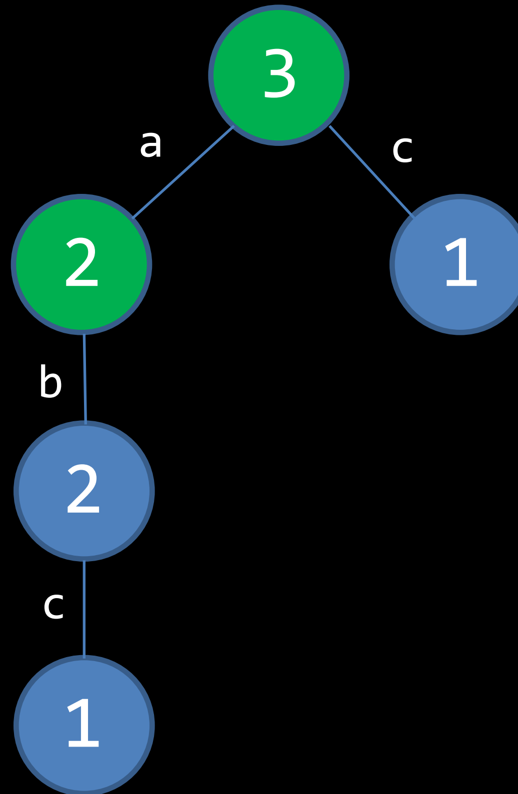

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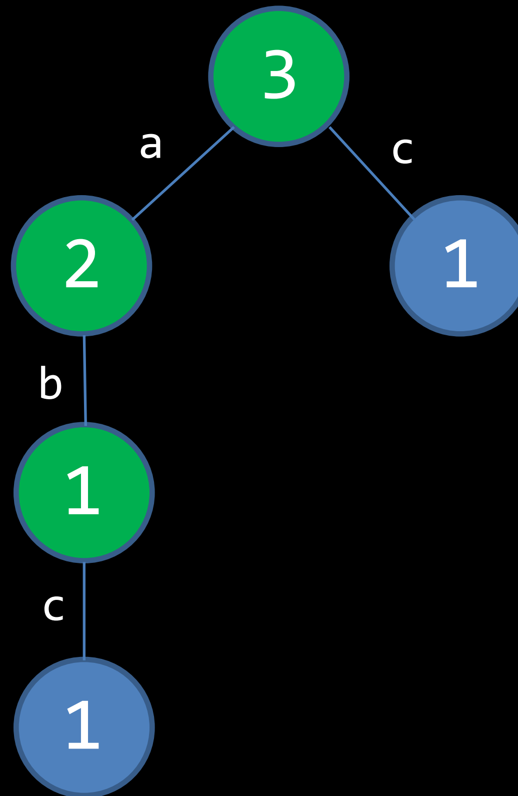
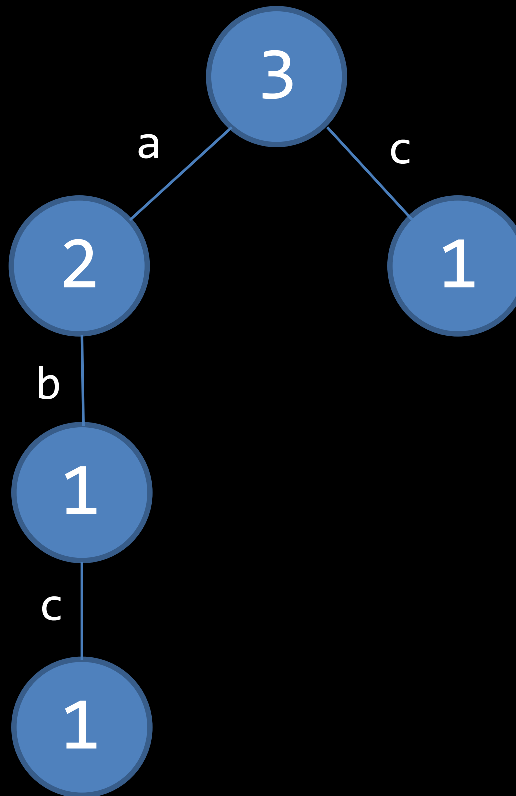


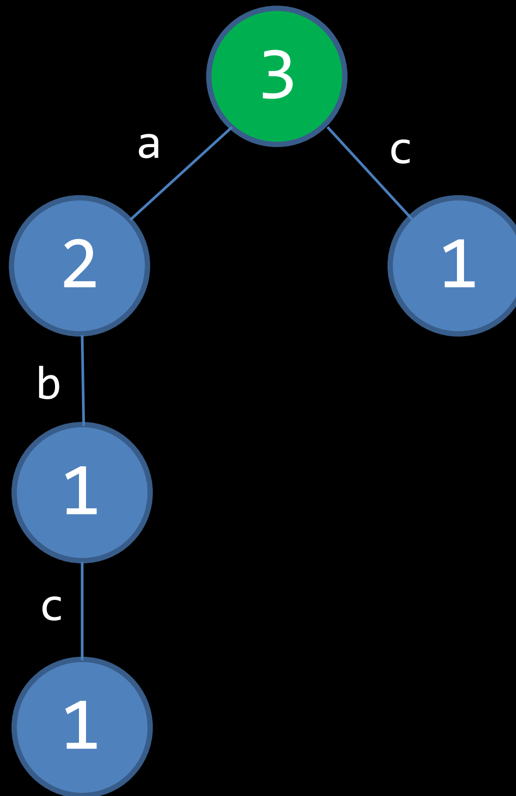
Trie

count string with the prefix “a”



Trie

count string with the prefix “a”



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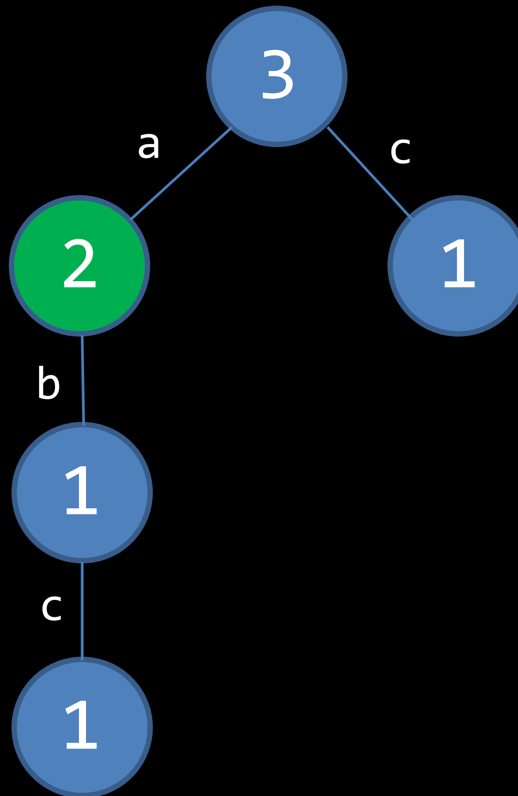
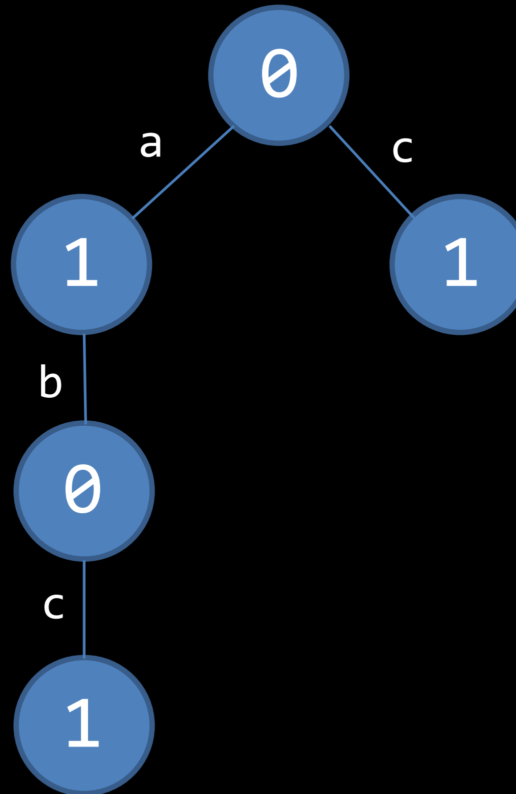


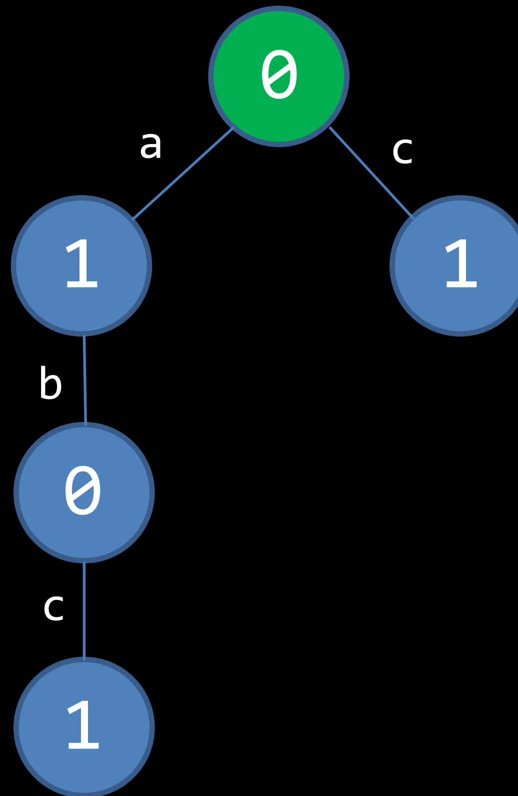
Trie

count string “a”



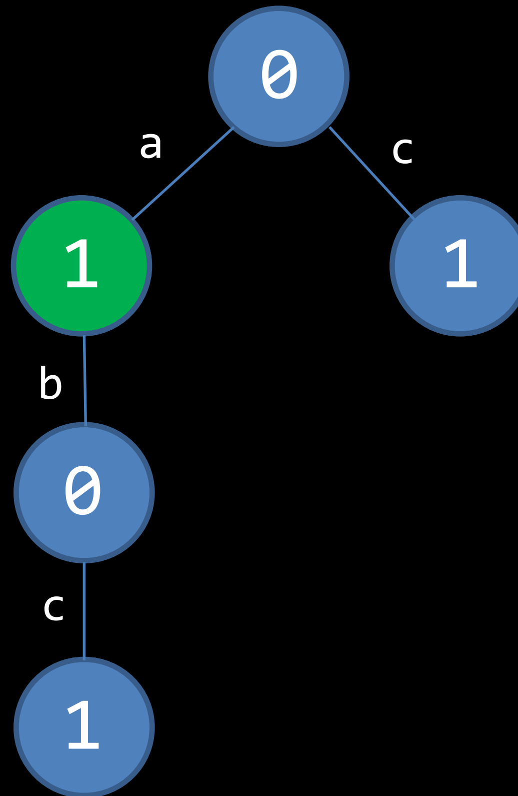
Trie

count string “a”



Trie

count string “a”



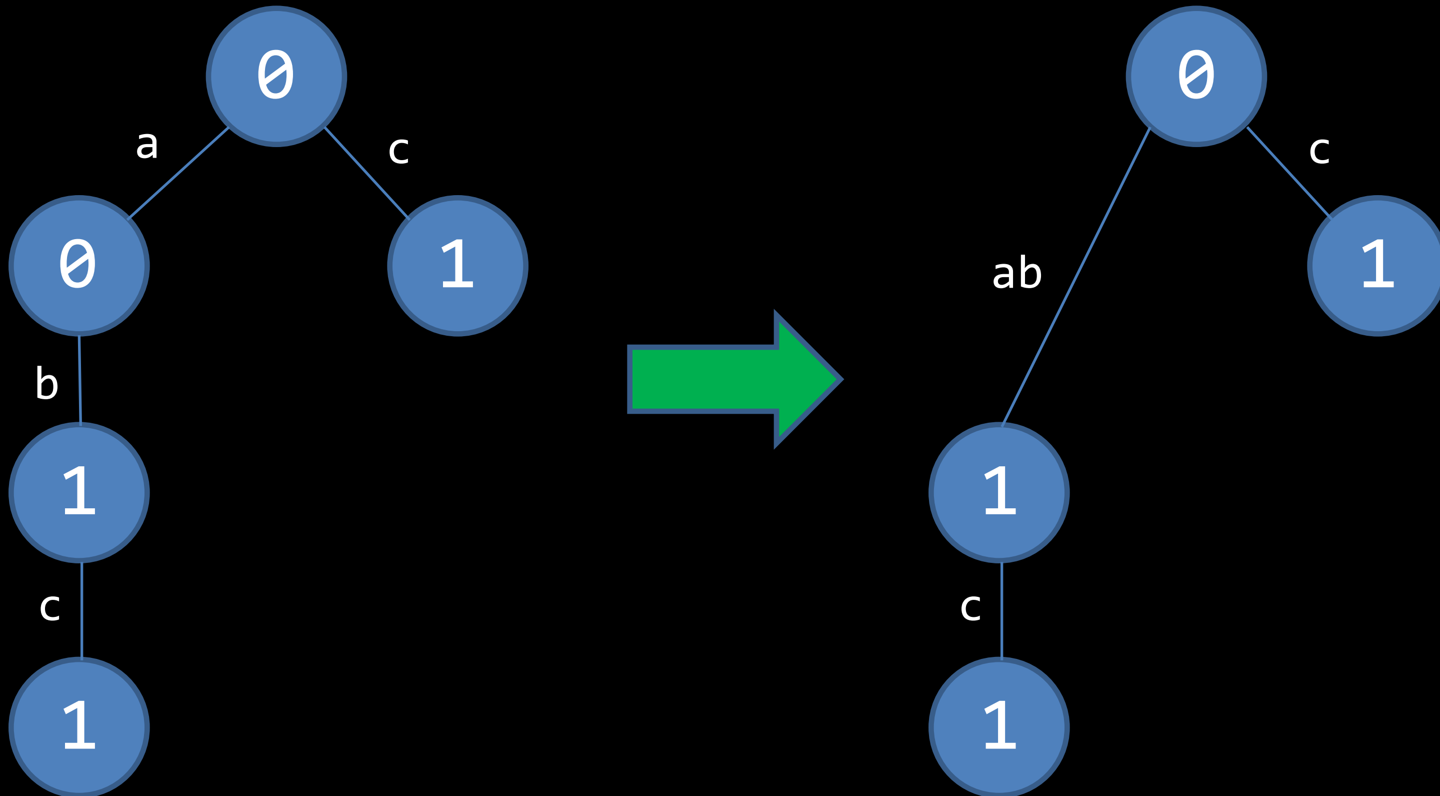
Trie

radix tree/ compressed trie

- a variant (or an optimization)
- some useless nodes in trie are merged with its parent
- harder implementation

Trie

radix tree/ compressed trie



KMP

HKOJ 01002 A Counting Problem

- given 2 string x, y
- length <= 1000
- count the occurrence of y in x

KMP

HKOJ 01002 A Counting Problem

```
for each starting position in x
    if the |y| characters follow
    exactly match with y
        ans++
```

- O(NM)

KMP

HKOJ 01002 A Counting Problem

- given 2 string x, y
- length <= ~~1000~~ 1e5
- count the occurrences of y in x

- a faster algorithm is needed

KMP

- KMP is a solution
  - Knuth-Morris-Pratt Algorithm
- O(N+M)
- How it works?

KMP

brute force

AHKOIHHKOIHKABC
HKOIHK

KMP

brute force

AHKOIHHKOIHKABC
HKOIHK

KMP

brute force

```
AHKOIHHKOIHKABC
 HKOIHK
```

KMP

brute force

```
AHKOIHHKOIHKABC
 HKOIHK
```

KMP

brute force

```
AHKOIHHKOIHKABC
 HKOIHK
```

KMP

brute force

```
AHKOIHHKOIHKABC
 HKOIHK
```

KMP

brute force

```
AHKOIHHKOIHKABC
 HKOIHK
```

KMP

brute force

```
AHKOIHHKOIHKABC
 HKOIHK
```

KMP

brute force

```
AHKOIHHKOIHKABC
 HKOIHK
```

KMP

brute force

AHKOIHHKOIHKABC
HKOIHK

KMP

brute force

```
AHKOIHHKOIHKABC
      HKOIHK
```

KMP

optimization 1

```
AHKOIHHKOIHKABC
 HKOIHK
```

KMP

optimization 1

```
-HKOIHH---------
 HKOIHK
```

KMP

optimization 1

```
-HKOIHH--------
  HKOIHK
```

KMP

optimization 1

```
-HKOIHH--------
   HKOIHK
```

KMP

optimization 1

```
-HKOIHH---------
    HKOIHK
```

KMP

optimization 1

```
-HKOIHH--------
     HKOIHK
```

KMP

optimization 1

HKOIHK -> move 4 units

HKOIHK -> move 4 units

HKOIHK -> move 1 unit

HKOIHK -> move 2 units

HKOIHK -> move 1 unit

KMP

optimization 1

HKOIHK -> move 4 units
HKOIHK -> move 4 units

HKOIHK -> move 1 unit
HKOIHK -> move 2 units

HKOIHK -> move 1 unit

KMP

optimization 1

HKOIHK -> move 4 units

length of move = length of green string – length of underline / 2

KMP

optimization 1

- precompute an array of length of underline in O(M)
- simple dynamic programming

KMP

optimization 1

O(number of operations in the searching stage) =

O(number of increments of the green part + number of decrements of the green part) = $O(N)$

overall time complexity = $O(N+M)$

KMP

optimization 2

HKOIHK -> move 4 units

length of move = length of green string – length of underline / 2

KMP

optimization 2

```
---HKOI#--------
   HKOIHK
```

KMP

optimization 2

```
---HKOI#-------
   HKOIHK
```

KMP

optimization 2

```
---HKOI#--------
   1HKOIHK
```

KMP

optimization 2

```
---HKOI#--------
   12HKOIHK
```

KMP

optimization 2

```
---HKOI#-------
   123HKOIHK
```

KMP

optimization 2

```
---HKOI#-------
   1234HKOIHK
```

KMP

optimization 2

*H*KOI*H*K -> move 4+1 units

HKOIHKOI -> move 4+4 units

HKOIHHKI -> move 5-1 units

H*K*OIH*K*KI -> move 5-1+1 units

KMP

optimization 2

<u>H</u>*K*OI<u>H</u>*K*KI -> move 5-1+1 units

length of move = length of green string – length of underline / 2 + length of italic / 2

KMP

optimization 2

- by Knuth, the K in KMP
- no improvement on time complexity
- thus sometimes being ignored

KMP

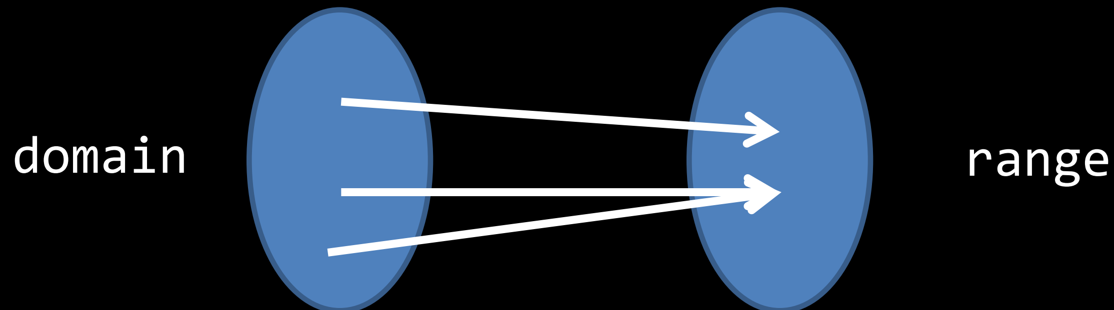
Remarks

- KMP works with one target and multiple input
- e.g.: finding w from x, y, z

Take a break

Hashing

- hash table
  - taught in Data Structures (II)
- string hashing: String->Number
  - a function
  - not necessarily injective



Hashing

- advantages
  - can be stored in array ~hash table
  - can be compared/searched in O(1)

- disadvantages
  - inverse may not be easily found
  - not injective -> may crash
    - e.g.: output extra results in string matching

Hashing

- rolling hash
  - “sliding window” fast (in O(1))

- Rabin-Karp hash
  - common rolling hash
  - addition
    - can be replaced by xor
  - multiplication
  - mod

Hashing

Rabin-Karp hash

$$H = (c_0 a^{n-1} + c_1 a^{n-2} + \ldots + c_{n-2} a + c_{n-1}) mod\ p$$

- H = output number
- c = ASCII code / A=0, B=1, …, Z=25
- a = 256 / 26
- P = a large prime, e.g. 1e9+7

Hashing

"HKOI"

c = 7, 10, 14, 8

a = 26

p = 64997

H = (7*26^3+10*26^2+14*26+8) mod 64997

H = 130164 mod 64997 = 170

Hashing

“GO”

c = 6, 14

a = 26

p = 64997

H = (6*26+14) mod 64997

H = 170 mod 64997 = 170

Hashing

H("HKOI") = 170 = H("GO")

"HKOI" == "GO" ???

Hashing

- assume range is any string
- surjective function
- all output with equal probability

If the input has 2 strings,

P(crash) = 1/64997 = 0.0000154

- negligible?

Hashing

If the input has 700 strings,

P(crash) = 1-(64996/64997)^700

P(crash) = 0.01

- NOT negligible

Hashing

solution 1

- give up hashing
- use exact algorithm
- e.g.: KMP for string matching

Hashing

solution 2a

- use a larger prime

p = 1e9+7

If the input has 700 strings,

P(crash) = 1-[(1e9+6)/(1e9+7)]^700

P(crash) = 0.0000007

Hashing

solution 2b

- use primeS
- compute H more than once with different primes
- P(crash) decreases exponentially

Hashing

solution 2

- cannot be stored in array
- use map/set instead
- increase time complexity by logN

- not always need to store the result

Hashing

“slide”

$$H = (c_0 a^{n-1} + c_1 a^{n-2} + \ldots + c_{n-2} a + c_{n-1})\, mod\ p$$

$$H' = (c_1 a^{n-1} + c_2 a^{n-2} + \ldots + c_{n-1} a + c_n)\, mod\ p$$

$$H' = ((H - c_0 a^{n-1}) \times a + c_n)\, mod\ p$$

$$H = ((H' - c_n) \times a^{-1} + c_0 a^{n-1})\, mod\ p$$

Hashing

"slide"

- use long long if there is possibility of overflow

- subtraction
  - avoid negative numbers

  $(a-b)$ mod p -> $(a+p-b\%p)\%p$

Hashing

"slide"

- inverse
  - taught in Mathematics in OI
  - Fermat's little theorem
    - p needs to be a prime first
  - power mod

  a^-1 mod p -> a^(p-2) mod p

Hashing

segment tree
- taught in Data Structures (III)
- find the hash of substring in $O(\log N)$
- with precomputation in $O(N \log N)$

Arithmetic Sequence

Problem setter: Alex Poon

Challenge

If the constraint is 1 <= N <= 100,000 and R = N for all subtasks

How can we write a program to verify if a sequence is valid in O(N lg N)?

tag: Data structure (III), (String algorithm/data structure (II))

Suffix array

- Indices of all non-empty suffixes
- sorted

- “IOIHKGHKOI”

- “HKGHKOI”
- “HKOI”
- ...

Suffix array

- IOIHKGHKOI
- OIHKGHKOI
- IHKGHKOI
- HKGHKOI
- KGHKOI
- GHKOI
- HKOI
- KOI
- OI
- I

- 5
- 3
- 6
- 9
- 2
- 0
- 4
- 7
- 8
- 1

Suffix array

$O(N^2 \log N)$

- brute force

- user-defined compare function
- or sort `vector<pair<vector<char>,int> >`

Suffix array

O($N\log N\log N$)

- compare suffixes by the length of the power of two

- no. of sort: O($\log N$)
- no. of comparisons in each sort: O($N\log N$)

Suffix array

$O(N\log N\log N)$

len=1,2,4,8...

rank[i]=the rank of the first len characters of the ith suffix

Suffix array

- IOIHKGHKOI
- OIHKGHKOI
- IHKGHKOI
- HKGHKOI
- KGHKOI
- GHKOI
- HKOI
- KOI
- OI
- I

Suffix array

- IO
- OI
- IH
- HK
- KG
- GH
- HK
- KO
- OI
- I_

- len=2

Suffix array

- IO
- OI
- IH
- HK
- KG
- GH
- HK
- KO
- OI
- I_

- 4
- 7
- 3
- 1
- 5
- 0
- 1
- 6
- 7
- 2

Suffix array

O($N \log N \log N$)

len = 1

- trivial

Suffix array

$O(N \log N \log N)$

len > 1

- rank[] is describing the status of len/2 till the end of the sorting of len

Suffix array

O(Nlog$N$log$N$)

len > 1

```
compare(a,b) //overriding '<'
    if r[a]!=r[b] return r[a]<r[b]
    return r[a+len/2]<r[b+len/2]
```

- r[i]=-1 if i>=n

Suffix array

O(*N*log*N*log*N*)

len > 1

- update rank[] at the end of sorting
- handle the case of equality

    r[a]==r[b] and r[a+len/2]==r[b+len/2]

Suffix array

$O(N\log N\log N)$

len >= N

- the whole process is done after this sorting

Suffix array

faster than $O(N\log N\log N)$

- it exists
- harder to understand/implement
- not so important unless you aim for NOI

Suffix array

longest common prefix lcp[]

- useful in many application of suffix array
- brute force needs O(N^2)
- actually O(N) is enough

Suffix array

longest common prefix lcp[]

```
i   : ABC...
i+1 : BCD...
lcp[i] = 0

j   : ABC...
j+1 : ABD...
lcp[j] = 2
```

Suffix array

```
longest common prefix lcp[]

j   : ABC...
j+1 : ABD...
lcp[j] = 2

k   : BC...
k+1 : BD...
lcp[k] >= lcp[j]-1
```

Suffix array

longest common prefix lcp[]

- the 2 new suffixes obtained by deleting the first character may not be neighbors
- the property lcp[k] >= lcp[j]-1 still holds
- can be computed in O(*N*) by the original order of the suffixes

Suffix array

string matching

- binary search
- find the suffix beginning with the target
- i.e. target is the prefix of it
- O($T \log N$)
- can be improved by lcp[]

Suffix array

other application

- longest repeated substring
- longest common substring
- longest palindromic substring

Other string algorithms

- Z algorithm
  – string matching in O(N)

- Manacher algorithm
  – longest palindromic substring in O(N)

Other string algorithms

- suffix tree
  - a tree storing all the suffixes of a string
  - compute suffix array in O(*N*)
- Aho-Corasick Algorithm
  - multi-target version of KMP
  - linear time complexity

Practice

judge.hkoi.org/tag/strings

codeforces.com/problemset/tags/strings

- HKOJ 01002 A Counting Problem
- HKOJ M0932 String Rotation

- UOJ 35 後綴排序
- NOI 2015 品酒大會

Reference

- 演算法筆記
- www.csie.ntnu.edu.tw/~u91029/

The end