Data Structures (II)

- Binary Heap
- Binary Search Tree
- Hash Table

Lau Chi Yung (Steven)

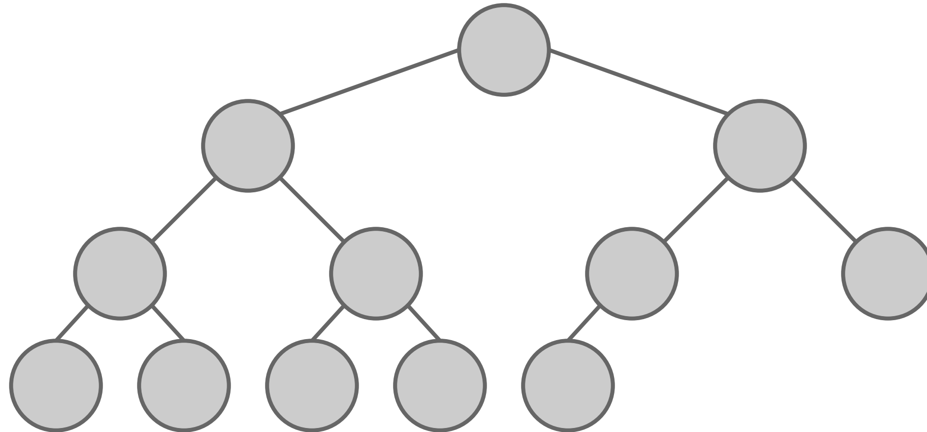
Binary Heap?

- Want a data structure which supports
  - insert element
  - remove minimum

	insert element	remove minimum
plain array	O(1)	O(N)
sorted array	O(N)	O(1)
binary heap	O(log(N))	O(log(N))

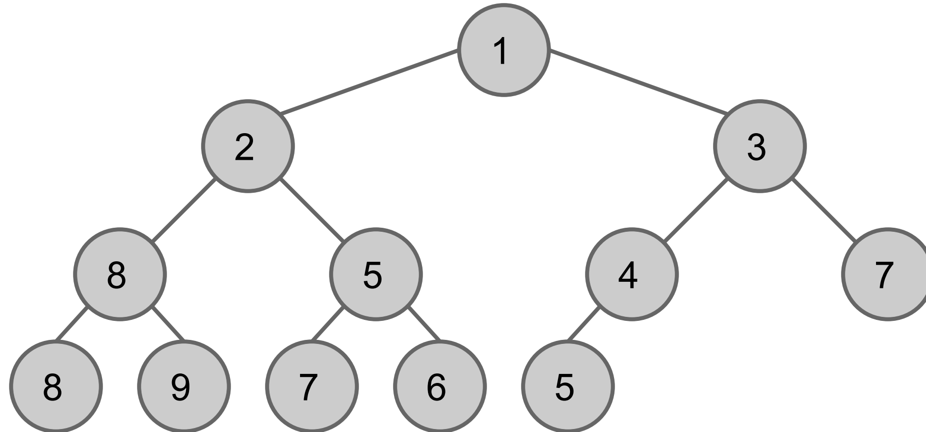
Binary Heap - shape

- Complete binary tree
  - all levels (except possibly the bottom) are fully filled
  - the bottom is filled from left to right



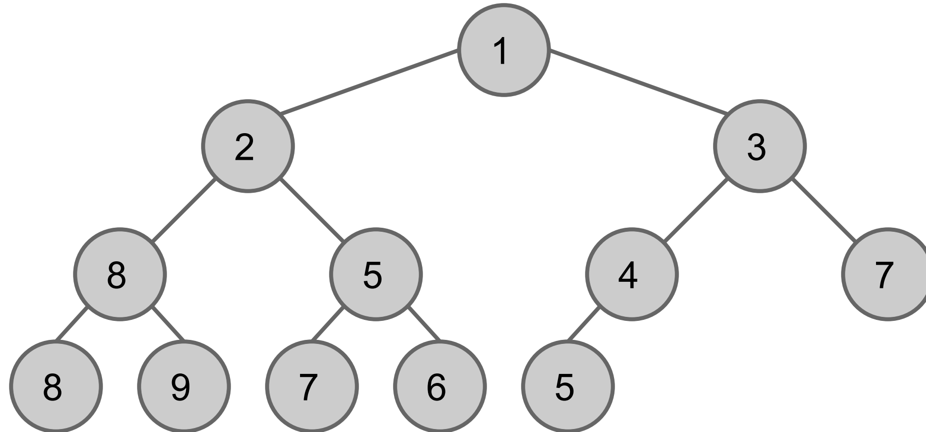
Binary Heap - heap property

- All nodes are greater than or equal to parent



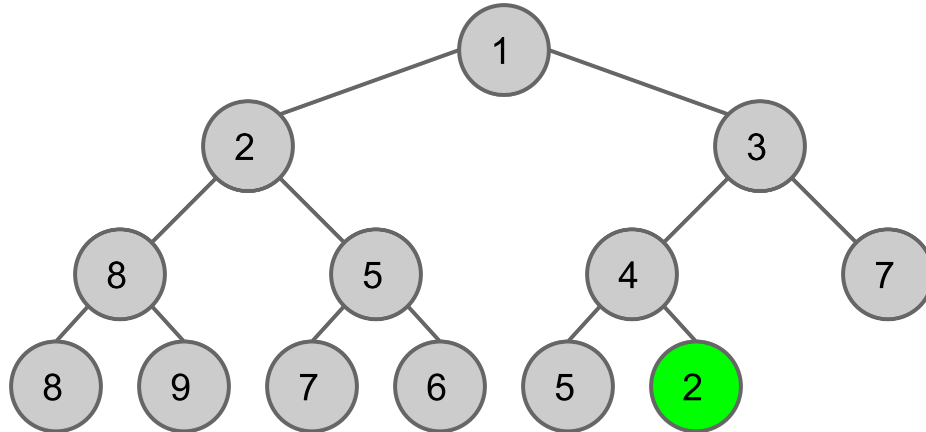
Binary Heap

- Insert element
  - insert at the bottom



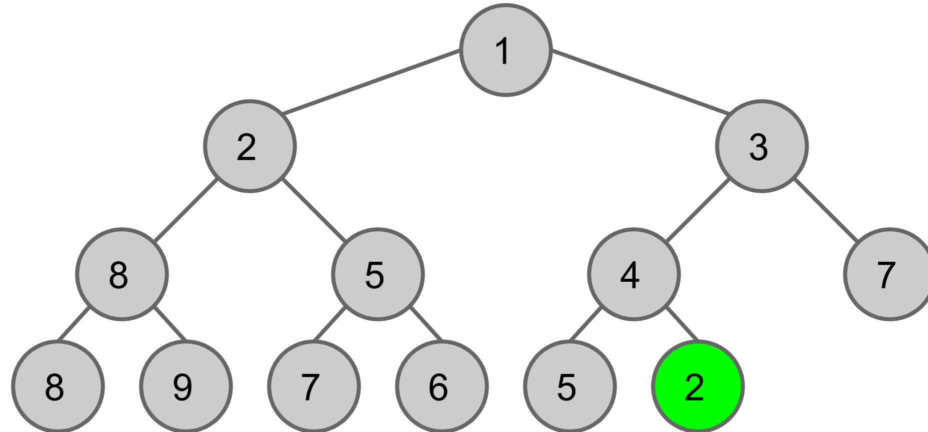
Binary Heap

- Insert element
  - insert at the bottom



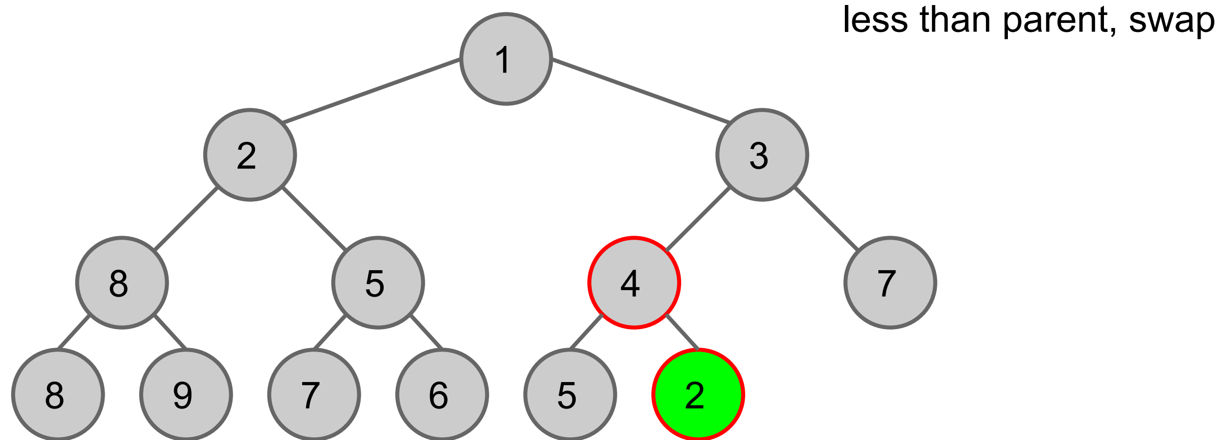
Binary Heap

- Insert element
  - insert at the bottom
  - heap property violated; do *sift-up* to recover



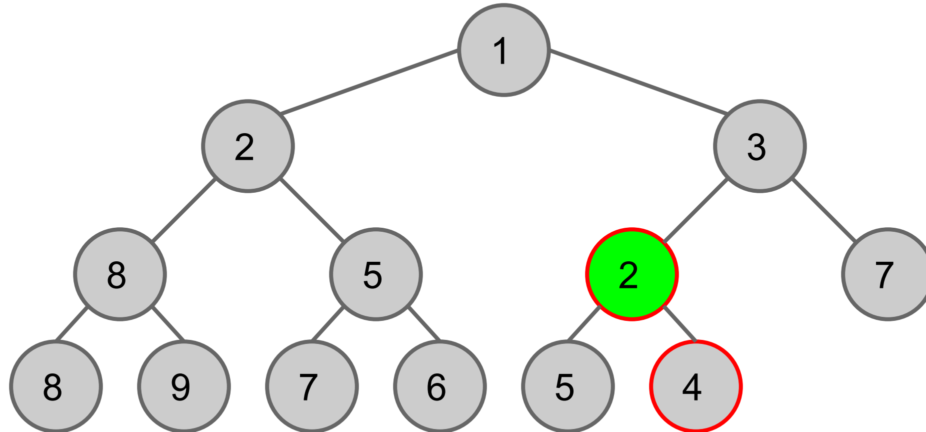
Binary Heap

- Insert element
  - insert at the bottom
  - heap property violated; do *sift-up* to recover



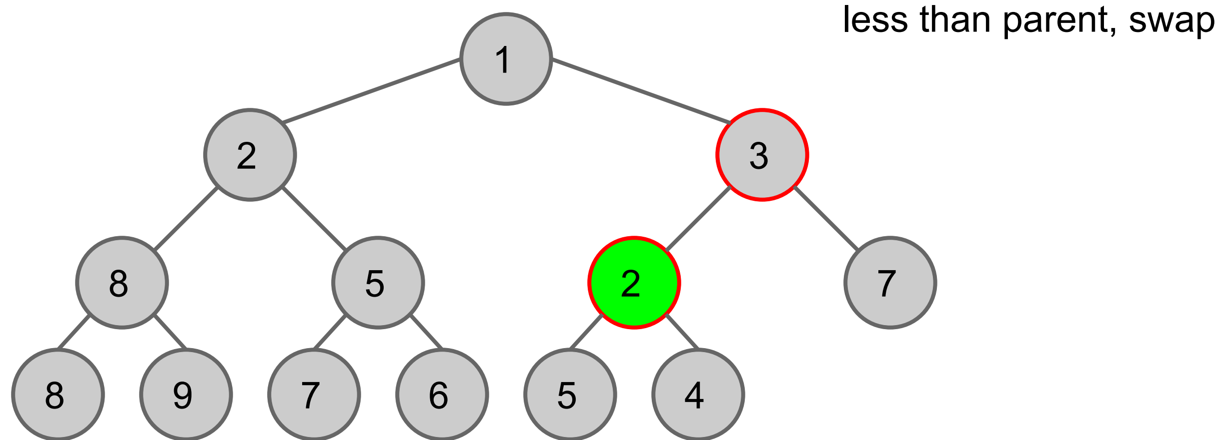
Binary Heap

- Insert element
  - insert at the bottom
  - heap property violated; do *sift-up* to recover



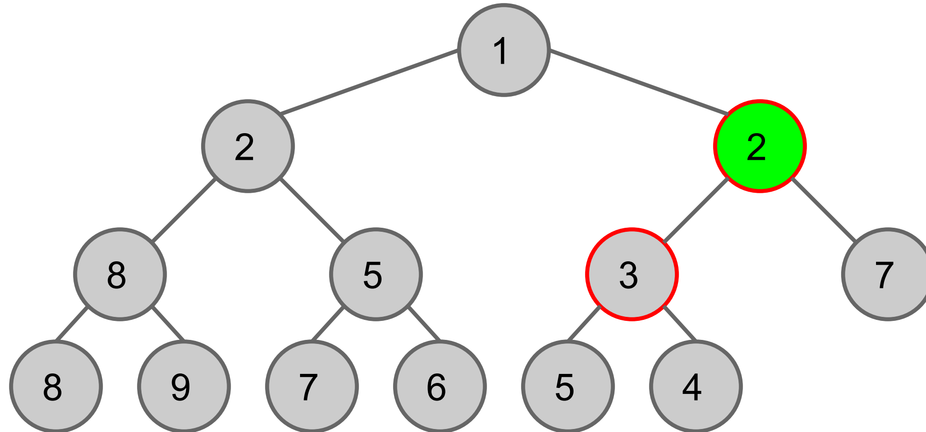
Binary Heap

- Insert element
  - insert at the bottom
  - heap property violated; do *sift-up* to recover



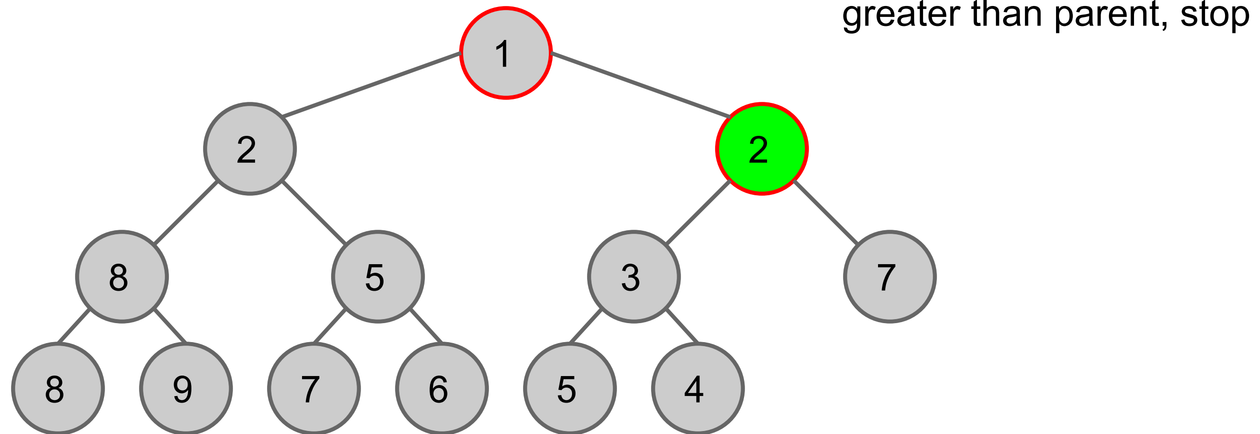
Binary Heap

- Insert element
  - insert at the bottom
  - heap property violated; do *sift-up* to recover



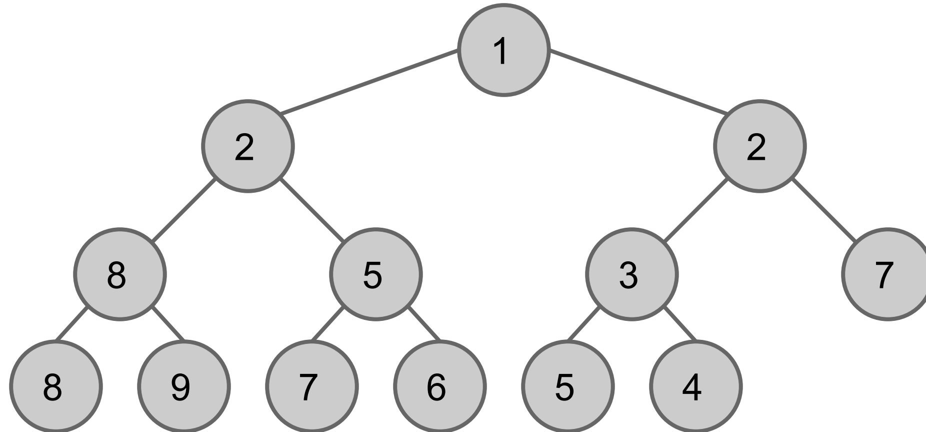
Binary Heap

- Insert element
  - insert at the bottom
  - heap property violated; do *sift-up* to recover



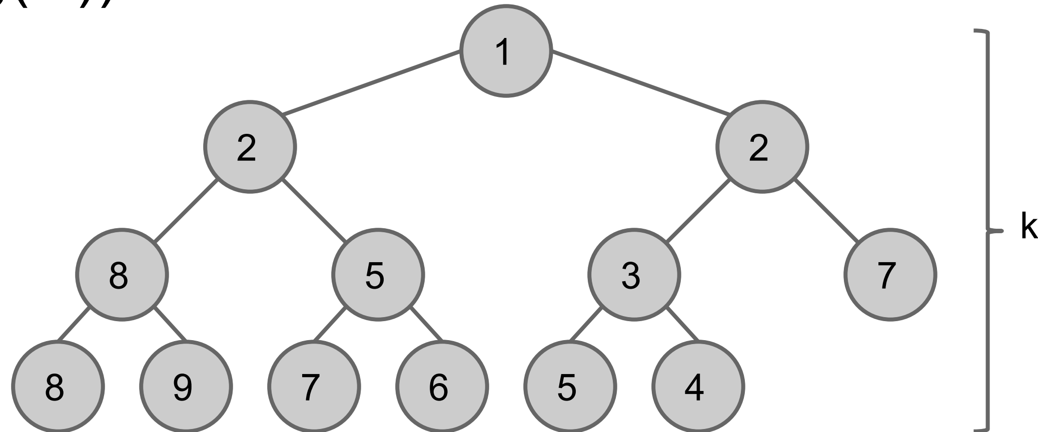
Binary Heap

- Insert element
  - insert at the bottom
  - heap property violated; do *sift-up* to recover



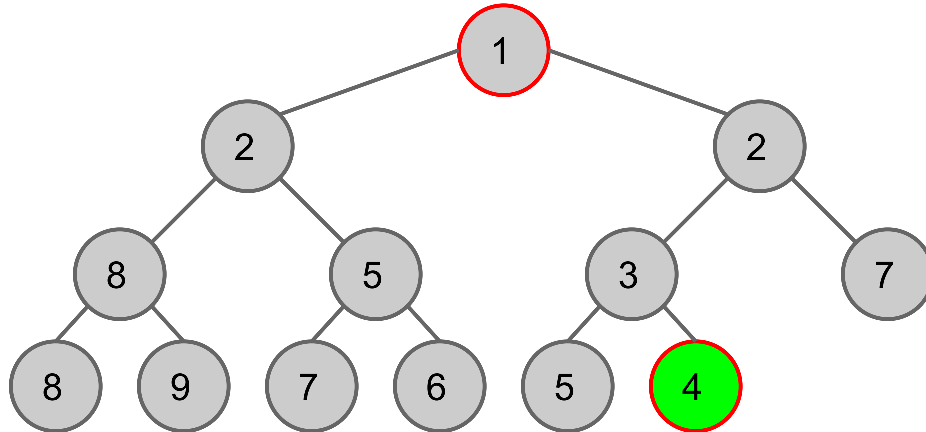
Binary Heap

- Insert element - time complexity
  - maximum number of swaps = k-1
  - $2^{k-1} \le N \Rightarrow k-1 \le \log_2 N$
  - $O(\log(N))$



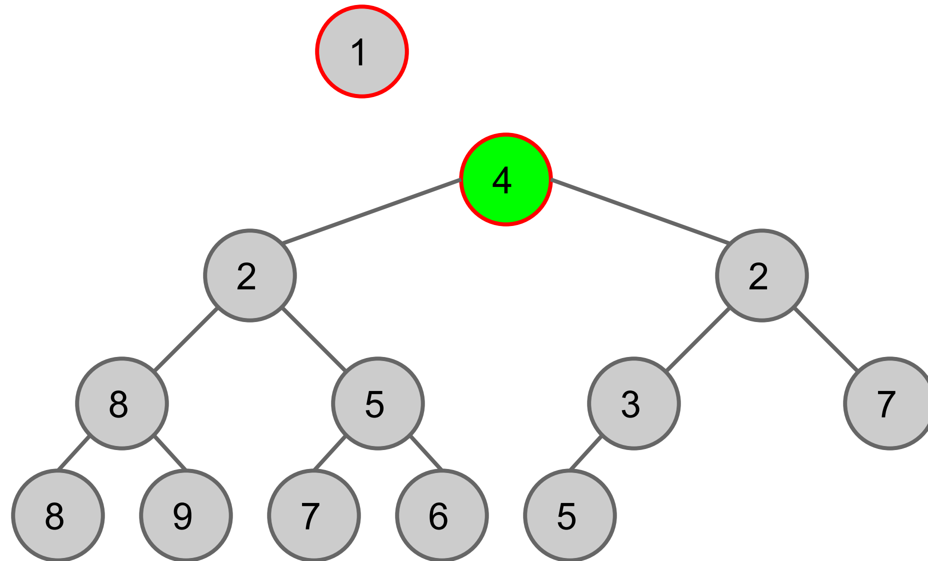
Binary Heap

- Remove minimum
  - replace the root with the last node at the bottom



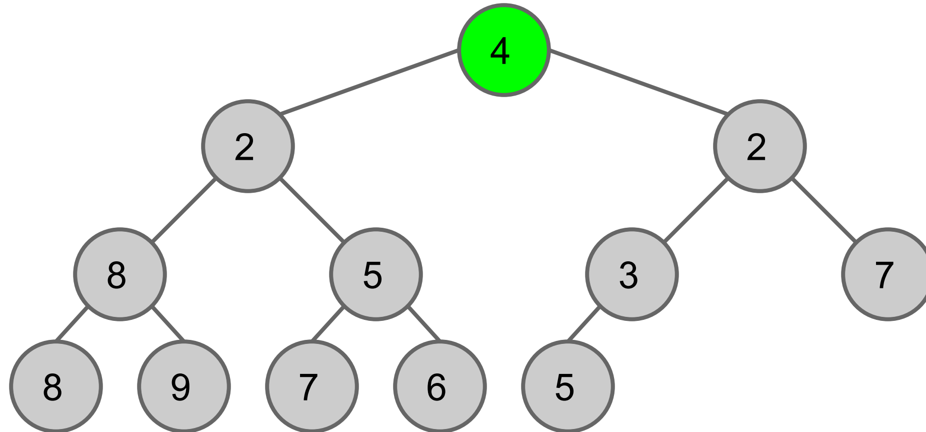
Binary Heap

- Remove minimum
  - replace the root with the last node at the bottom



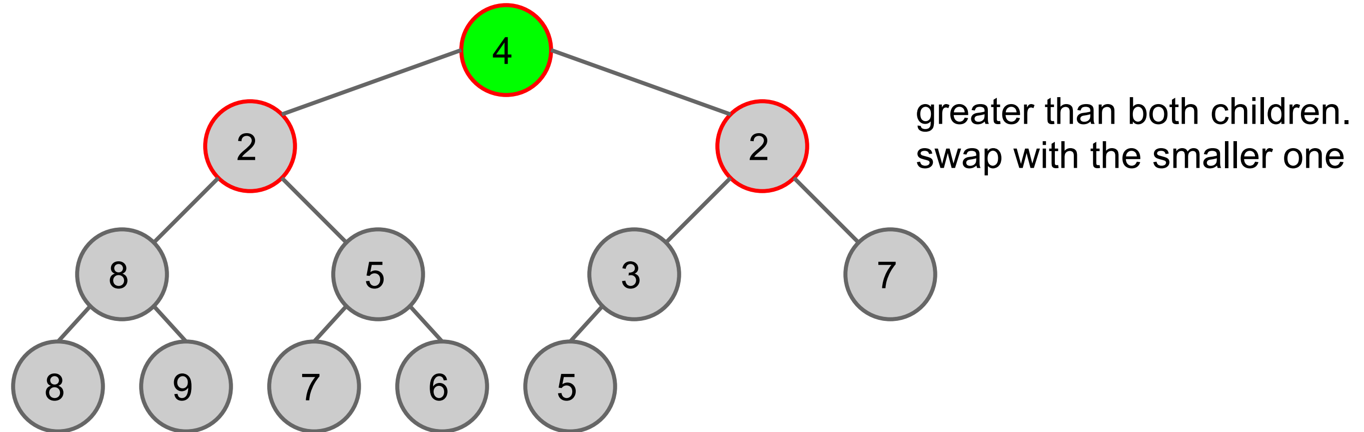
Binary Heap

- Remove minimum
  - replace the root with the last node at the bottom
  - heap property violated; do *sift-down* to recover



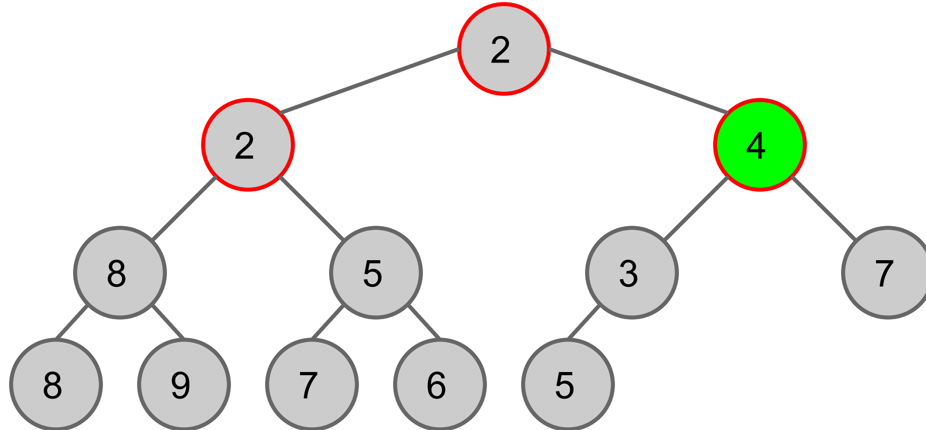
Binary Heap

- Remove minimum
  - replace the root with the last node at the bottom
  - heap property violated; do *sift-down* to recover



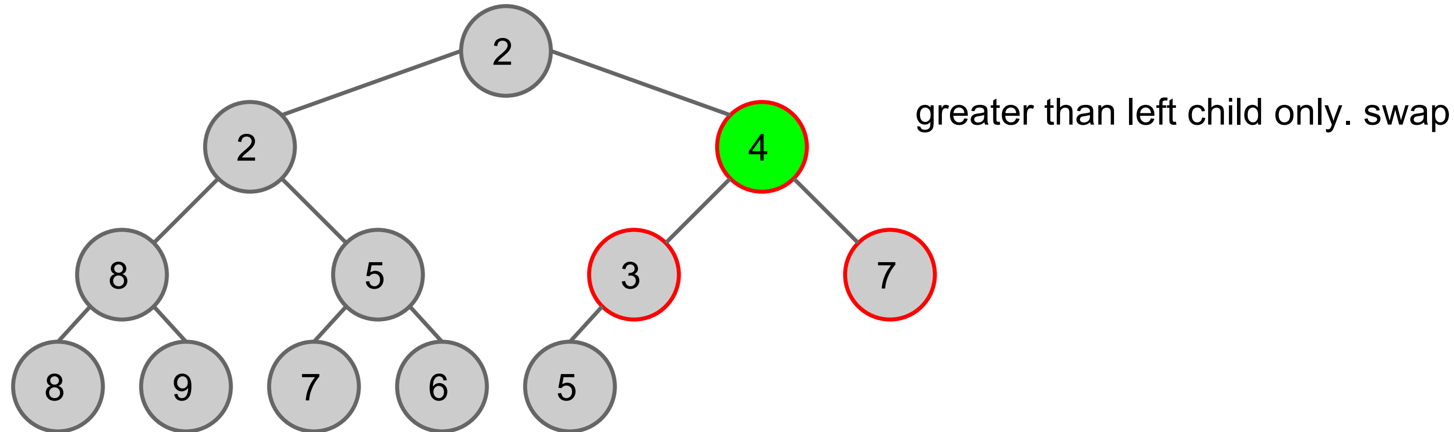
Binary Heap

- Remove minimum
  - replace the root with the last node at the bottom
  - heap property violated; do *sift-down* to recover



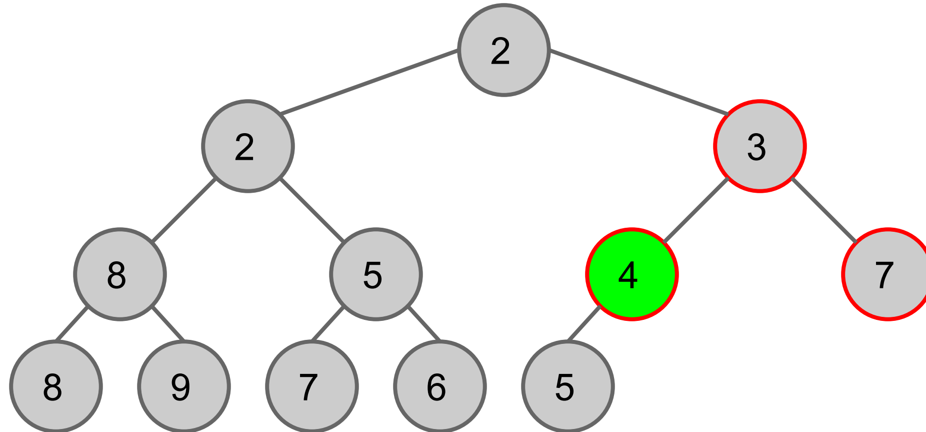
Binary Heap

- Remove minimum
  - replace the root with the last node at the bottom
  - heap property violated; do *sift-down* to recover



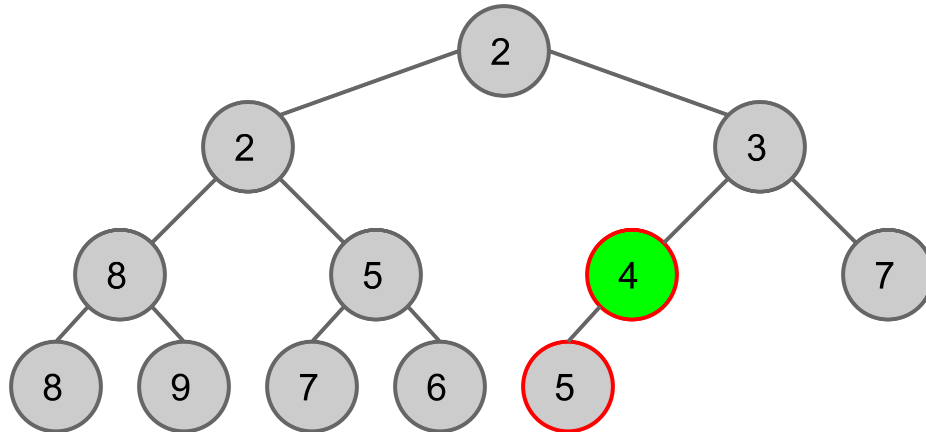
Binary Heap

- Remove minimum
  - replace the root with the last node at the bottom
  - heap property violated; do *sift-down* to recover



Binary Heap

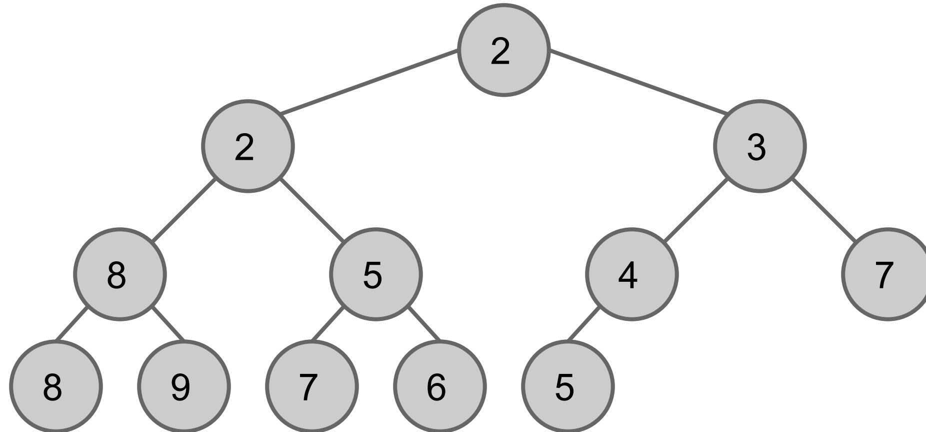
- Remove minimum
  - replace the root with the last node at the bottom
  - heap property violated; do *sift-down* to recover



no smaller children. stop

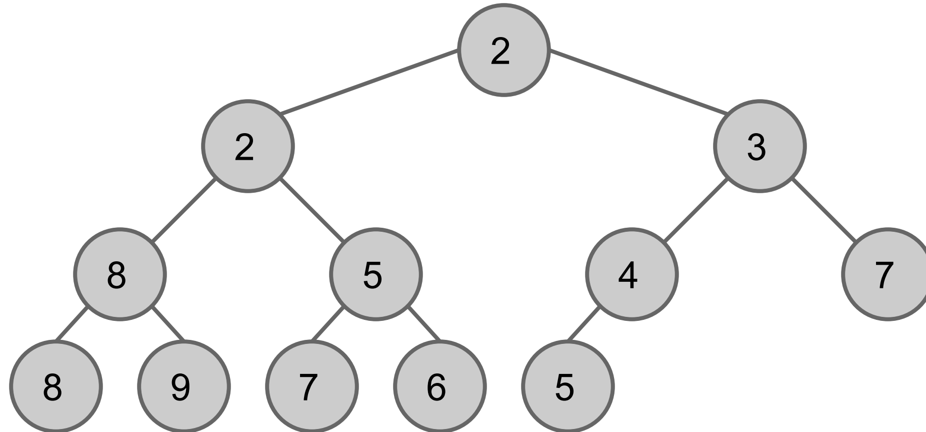
Binary Heap

- Remove minimum
  - replace the root with the last node at the bottom
  - heap property violated; do *sift-down* to recover
  - recovered



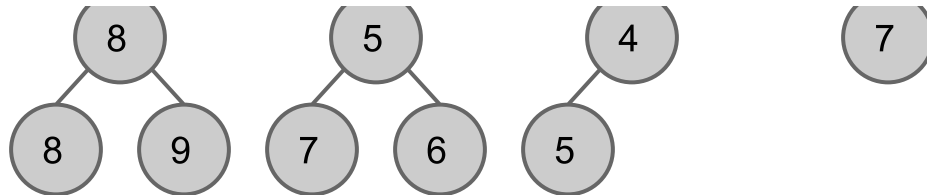
Binary Heap

- Remove minimum
    - Time complexity: O(log(N))

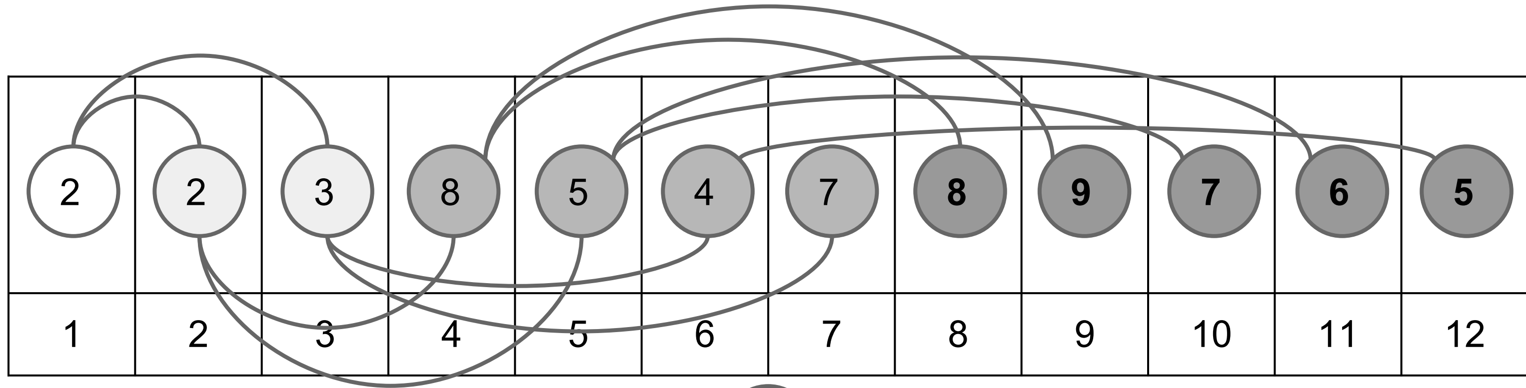


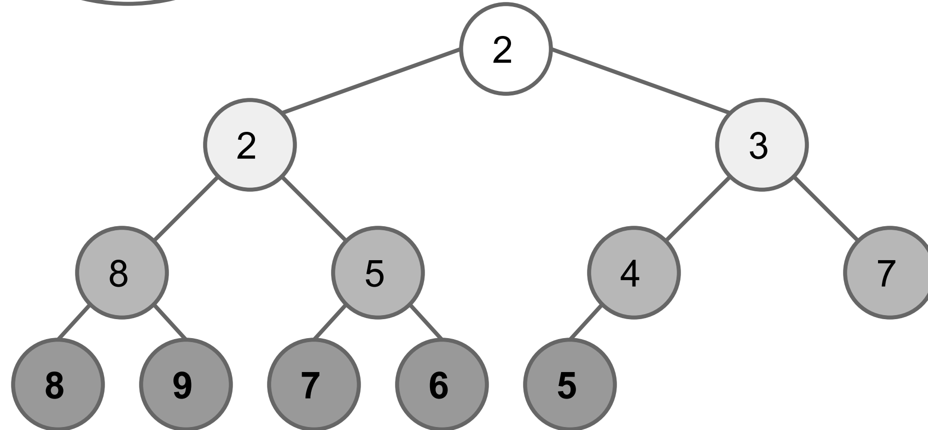
Binary Heap

- Implement in array
  - array[1..N]
  - root: array[1]
  - for array[i],
    - left child: array[i * 2]
    - right child: array[i * 2 + 1]



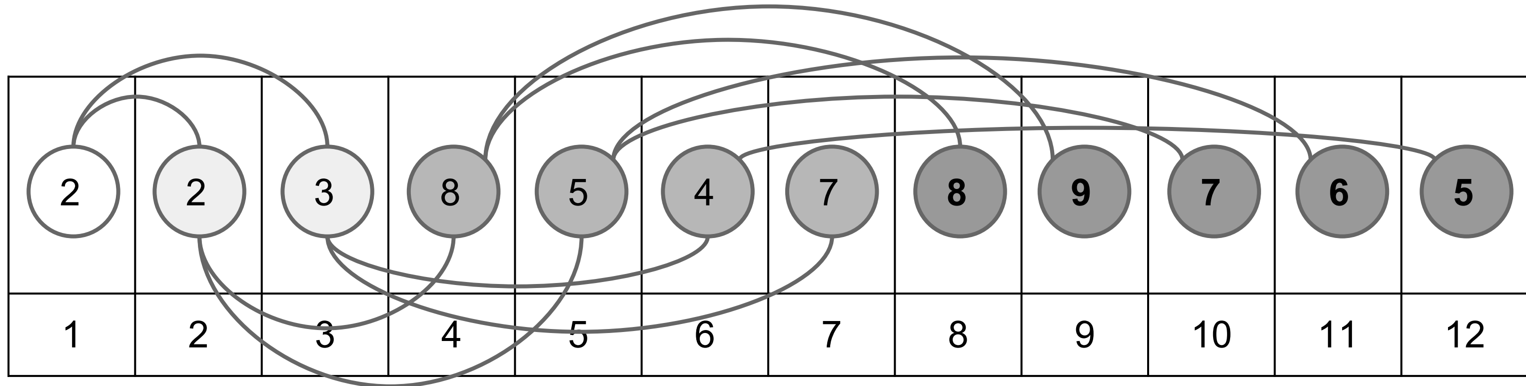
Binary Heap





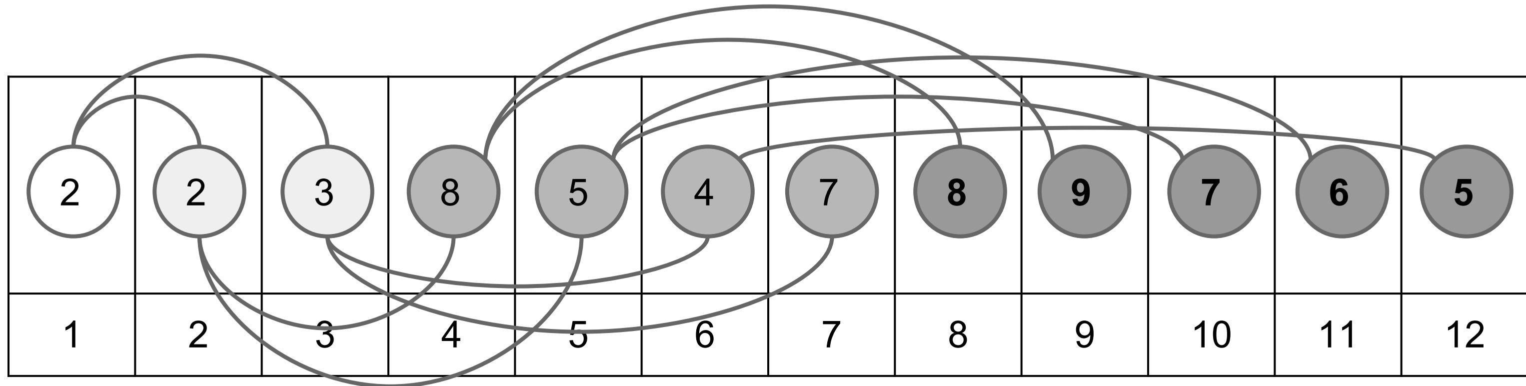
Binary Heap

- for array[i],
  - left child: array[i * 2]
  - right child: array[i * 2 + 1]
  - parent: ?



Binary Heap

- for array[i],
  - left child: array[i * 2]
  - right child: array[i * 2 + 1]
  - parent: array[i / 2]



Binary Heap

- C++ STL: priority_queue
- Max-heap,
  - all nodes are **less than or equal to** parent

```
#include <queue>
...
std::priority_queue<int> q;
q.push(4); q.push(7); q.push(3);
int x = q.top(); //x = 7
q.pop(); x = q.top(); //x = 4
```

Binary Heap

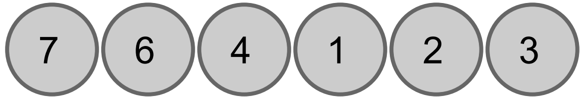
- Heapsort
  - insert everything into heap: O(Nlog(N))
  - repeatedly remove minimum: O(Nlog(N))
  - overall: O(Nlog(N))

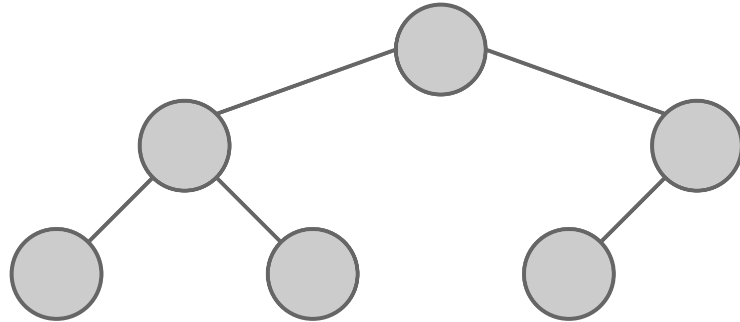
Binary Heap

- Heapsort
  - build heap: O(N)
  - repeatedly remove minimum: O(Nlog(N))
  - overall: O(Nlog(N))

Binary Heap

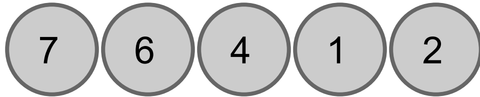
- O(N) build heap
- insert from the lowest level, then sift-down

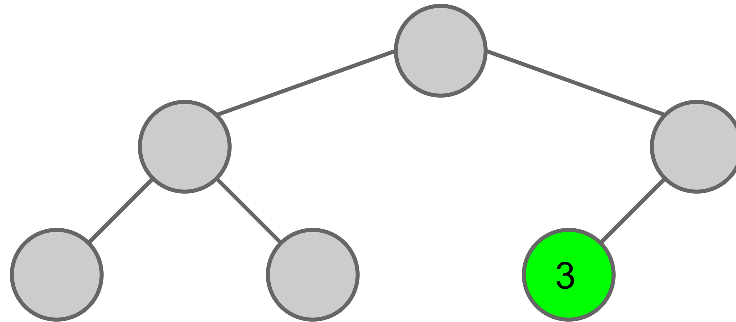




Binary Heap

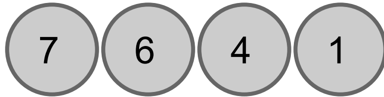
- O(N) build heap
- insert from the lowest level, then sift-down

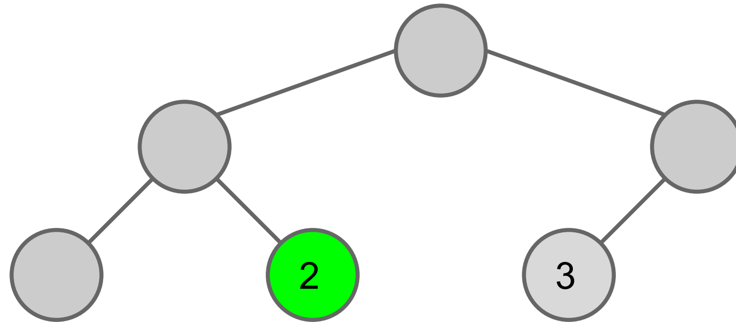




Binary Heap

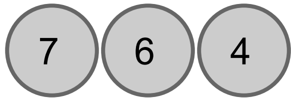
- O(N) build heap
- insert from the lowest level, then sift-down

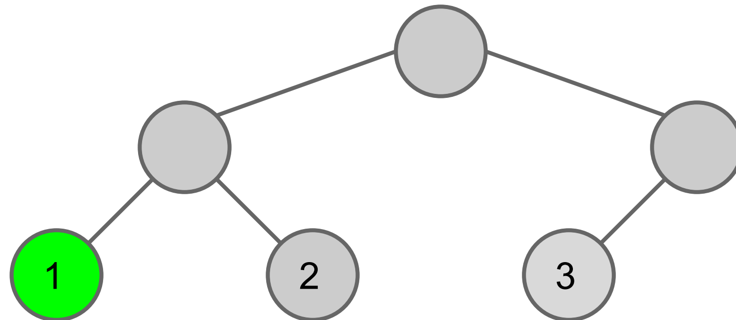




Binary Heap

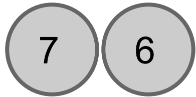
- O(N) build heap
- insert from the lowest level, then sift-down

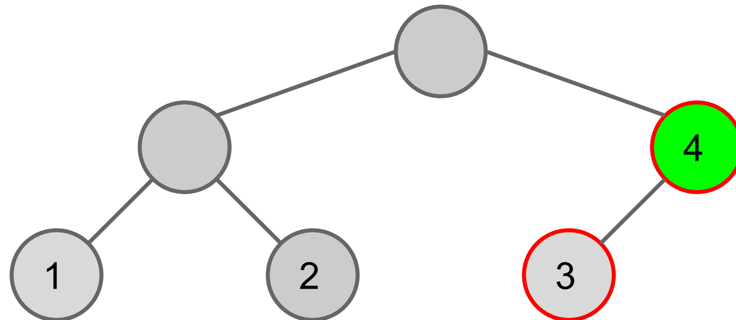




Binary Heap

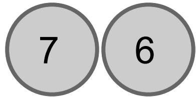
- O(N) build heap
- insert from the lowest level, then sift-down

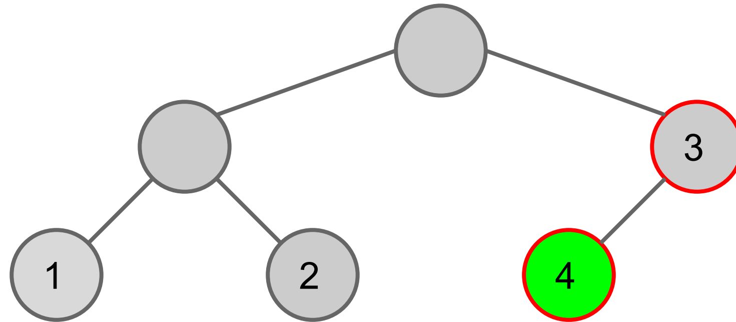




Binary Heap

- O(N) build heap
- insert from the lowest level, then sift-down

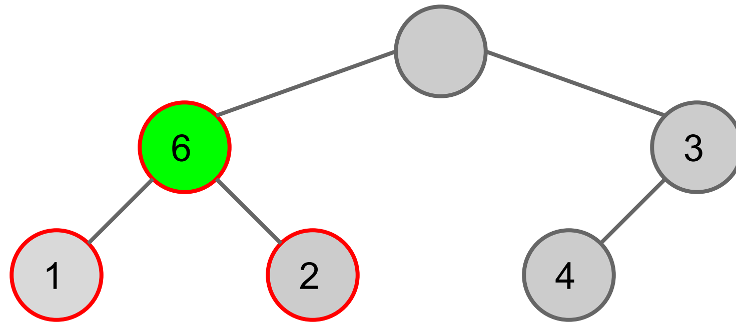




Binary Heap

- O(N) build heap
- insert from the lowest level, then sift-down

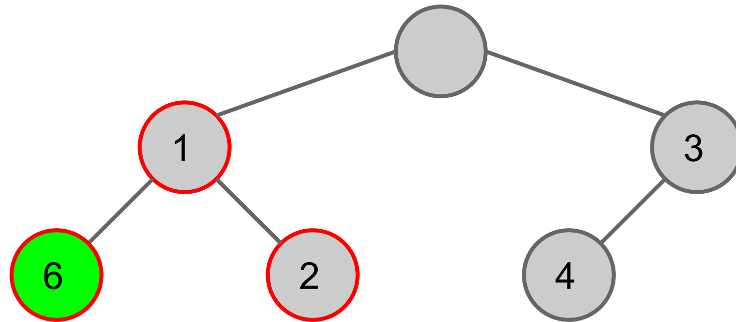
7



Binary Heap

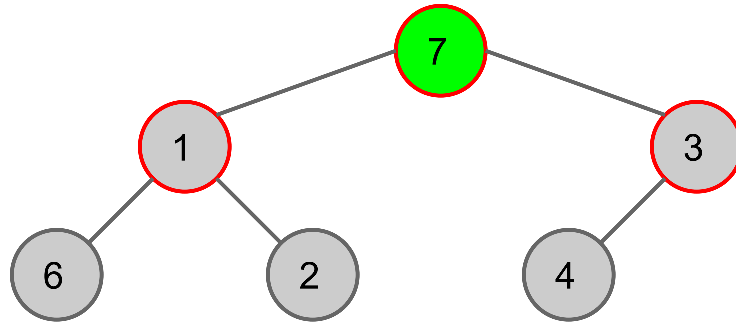
- O(N) build heap
- insert from the lowest level, then sift-down

7



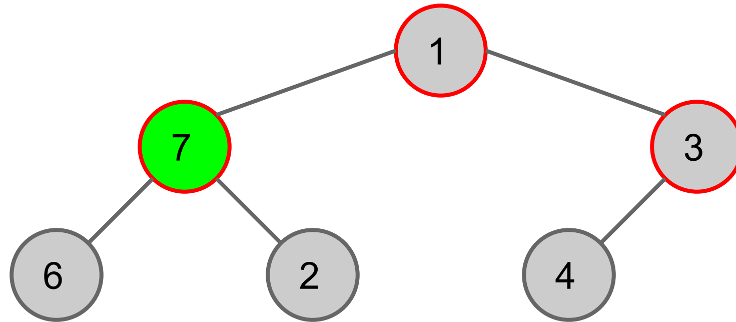
Binary Heap

- O(N) build heap
- insert from the lowest level, then sift-down



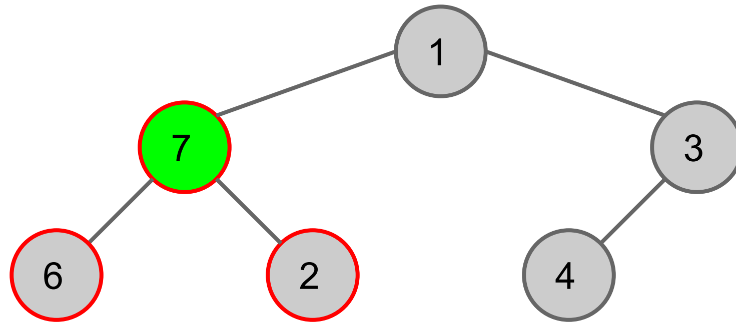
Binary Heap

- O(N) build heap
- insert from the lowest level, then sift-down



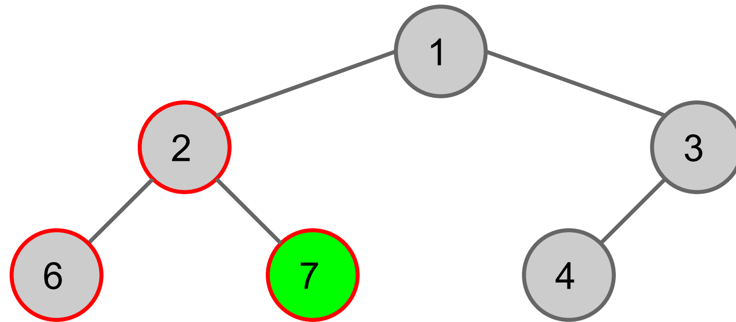
Binary Heap

- O(N) build heap
- insert from the lowest level, then sift-down



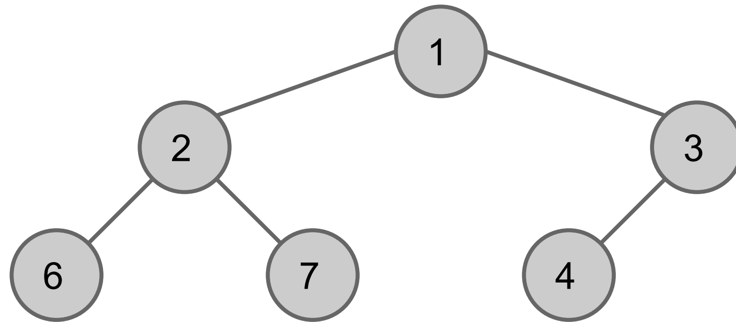
Binary Heap

- O(N) build heap
- insert from the lowest level, then sift-down



Binary Heap

- O(N) build heap
- insert from the lowest level, then sift-down
- proof?



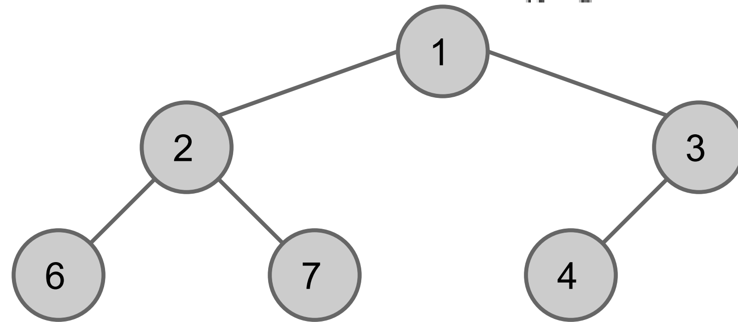
Binary Heap

- O(N) build heap
- insert from the lowest level, then sift-down
- proof from wikipedia

$$\sum_{h=0}^{\lceil \log n \rceil} \frac{n}{2^{h+1}} O(h) = O\left(n \sum_{h=0}^{\lceil \log n \rceil} \frac{h}{2^{h+1}}\right)$$

$$= O\left(n \sum_{h=0}^{\infty} \frac{h}{2^{h}}\right)$$

$$= O(n)$$



Binary Heap - summary

- Time complexities:
  - insert : O(log(N))
  - remove minimum : O(log(N))
  - build heap : O(N)
- Space complexity:
  - O(N)

	Binary heap	Leftist heap	Binomial heap
merge heap	O(N)	O(log(N))	O(log(N))

Data Structures (II)

- Binary Heap
- Binary Search Tree
- Hash Table

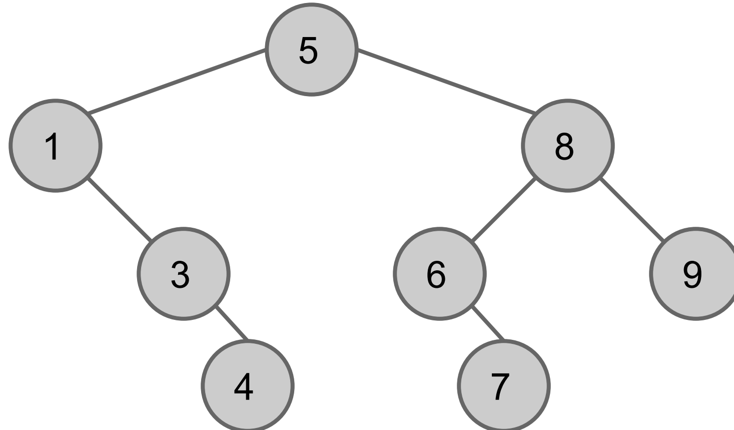
Binary Search Tree

- binary search is so fast

	Insert	Find	Remove
Plain array	O(1)	O(N)	O(N)
Sorted array	O(N)	O(log(N))	O(N)
Binary search tree	O(log(N)) average	O(log(N)) average	O(log(N)) average

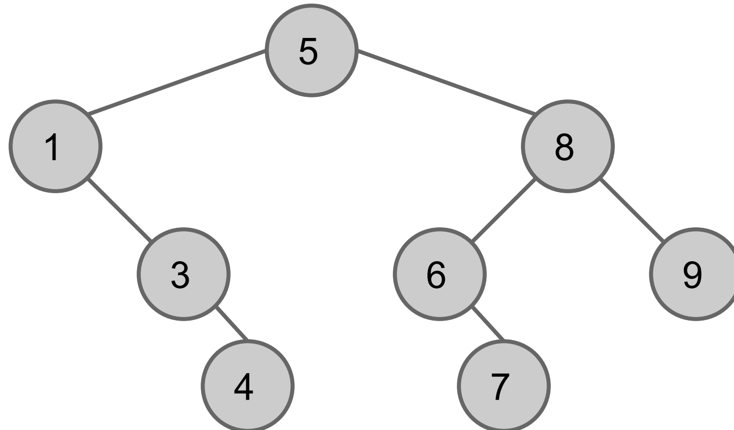
Binary Search Tree

- Binary tree
- Each node
  - > all left descendants
  - < all right descendants



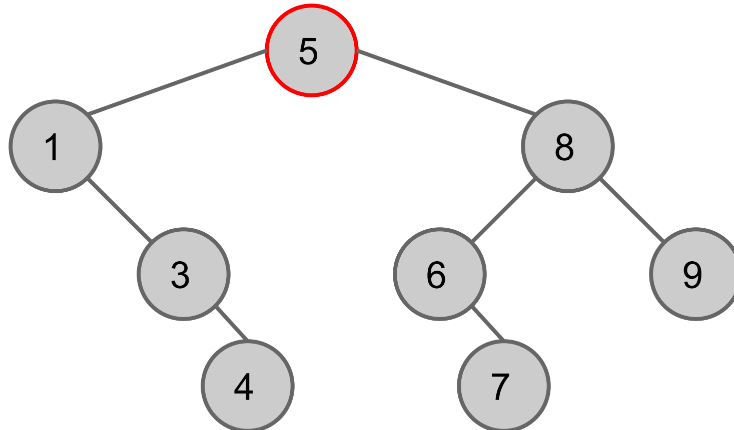
Binary Search Tree

- find a node
  - is ‘6’ in the tree?
  - binary search



Binary Search Tree

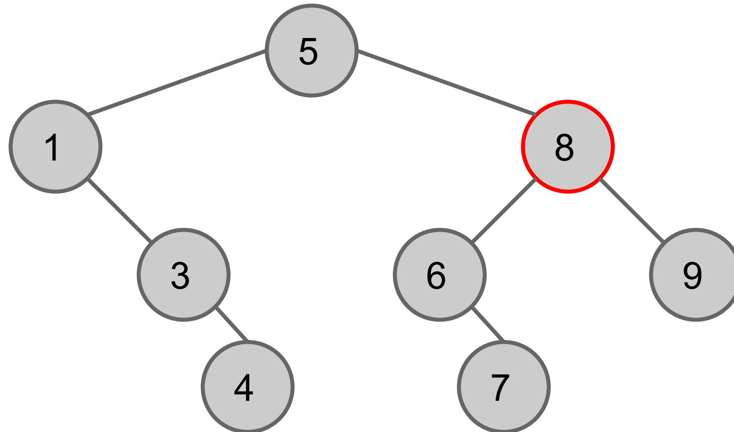
- find a node
  - is ‘6’ in the tree?
  - binary search



‘6’ is not here.
‘6’ won’t be on the left, so let’s go to the right

Binary Search Tree

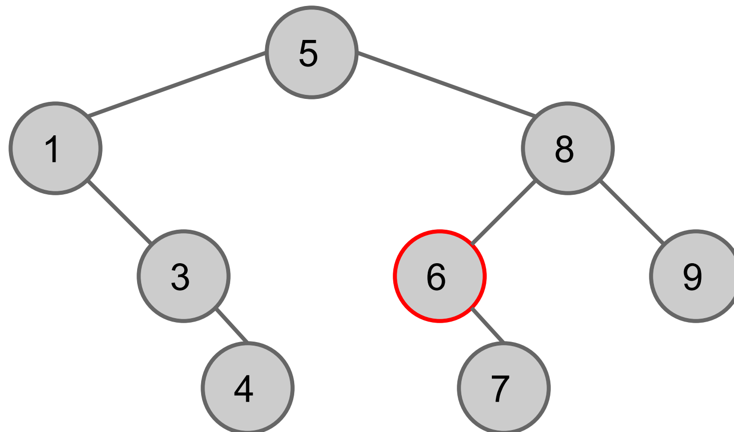
- find a node
  - is ‘6’ in the tree?
  - binary search



‘6’ is not here either.
‘6’ won’t be on the right, so let’s go to the left

Binary Search Tree

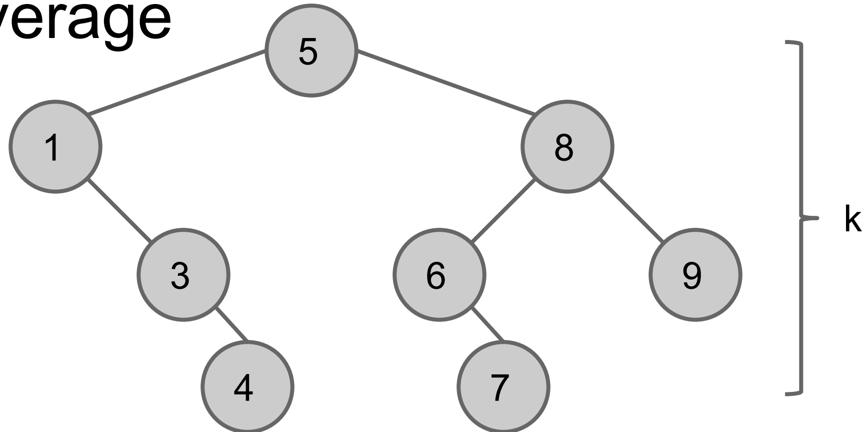
- find a node
  - is ‘6’ in the tree?
  - binary search



‘6’ is in the tree!

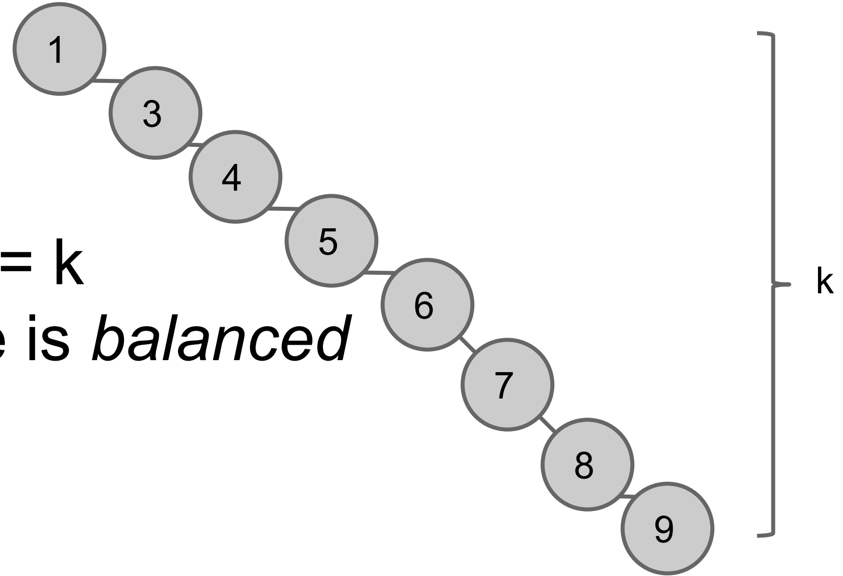
Binary Search Tree

- find a node
  - Time complexity?
  - max. # of nodes visited = k
  - k ≈ log(N) when the tree is *balanced*
  - O(log(N)) average



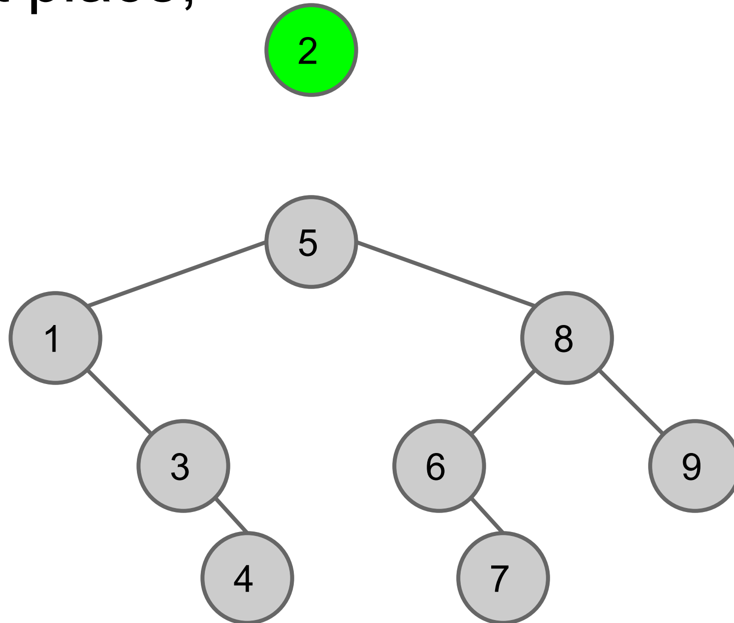
Binary Search Tree

- find a node
  - Time complexity?
  - max. # of nodes visited = k
  - k ≈ log(N) when the tree is *balanced*
  - O(log(N)) average
  - O(N) worst case



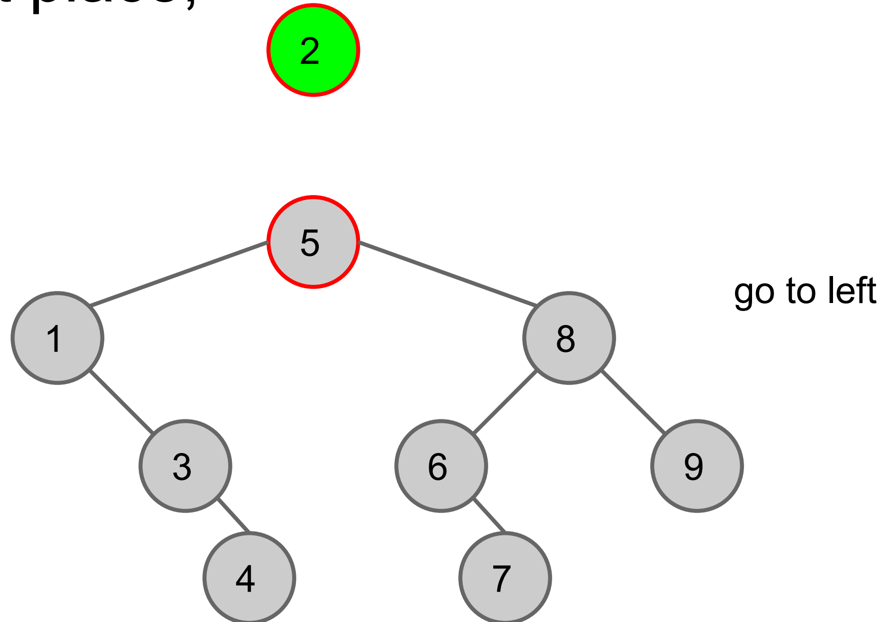
Binary Search Tree

- Insert a node
  - find the right place,
  - insert.



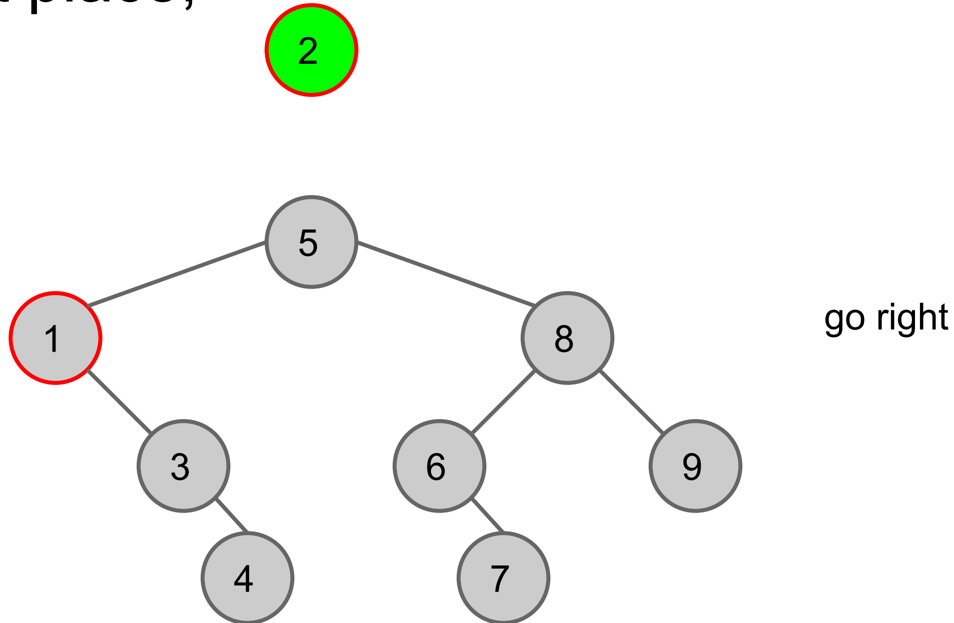
Binary Search Tree

- Insert a node
  - find the right place,
  - insert.



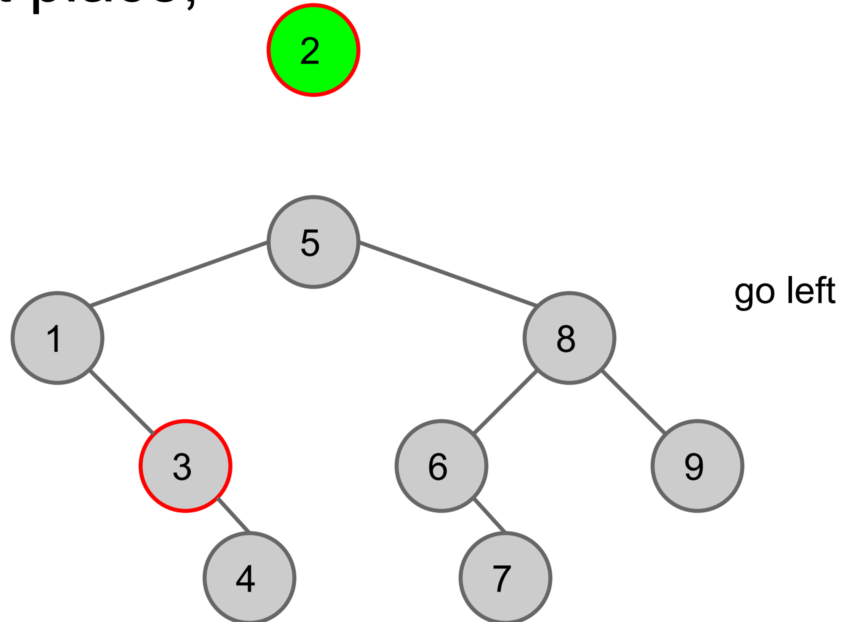
Binary Search Tree

- Insert a node
  - find the right place,
  - insert.



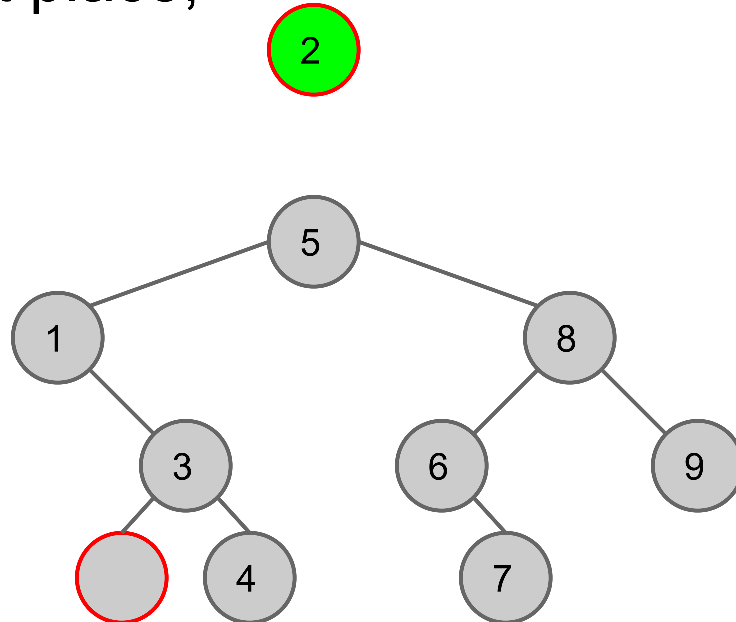
Binary Search Tree

- Insert a node
  - find the right place,
  - insert.



Binary Search Tree

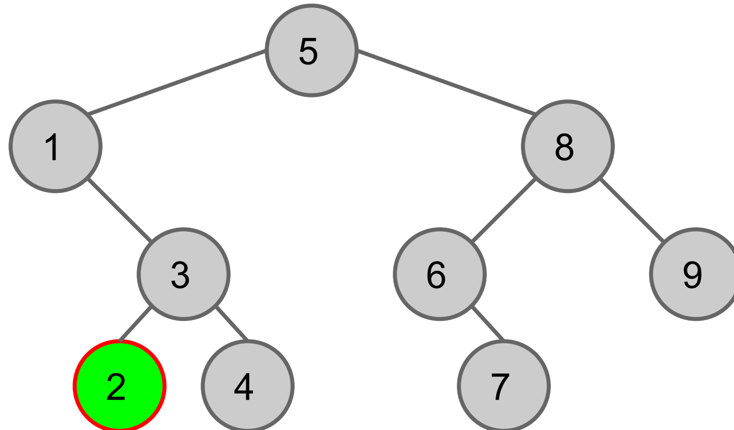
- Insert a node
  - find the right place,
  - insert.



left child is empty,
create new node

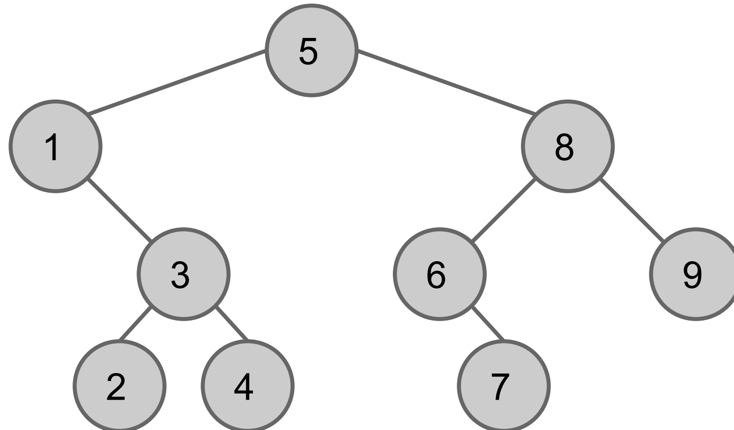
Binary Search Tree

- Insert a node
  - find the right place,
  - insert.



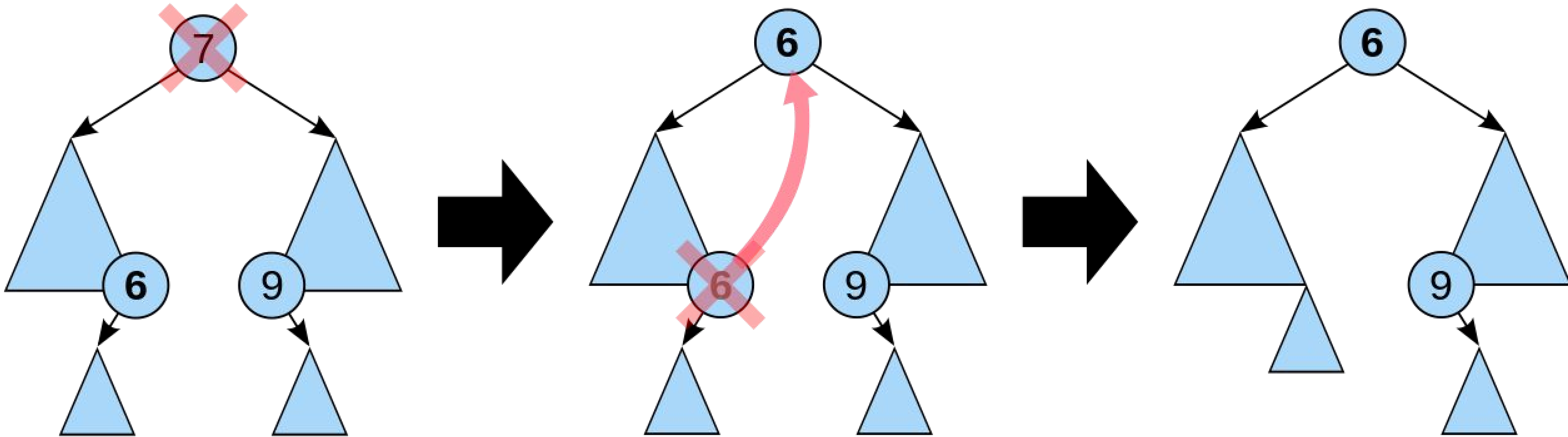
Binary Search Tree

- Insert a node
  - Time complexity
  - O(log(N)) average
  - O(N) worst case



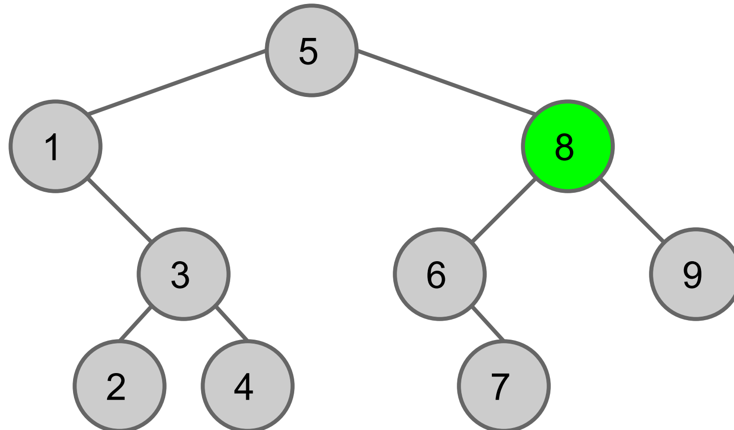
Binary Search Tree

- Remove a node
  - replace that node with the node *just* less than it
  - (image from wikipedia)



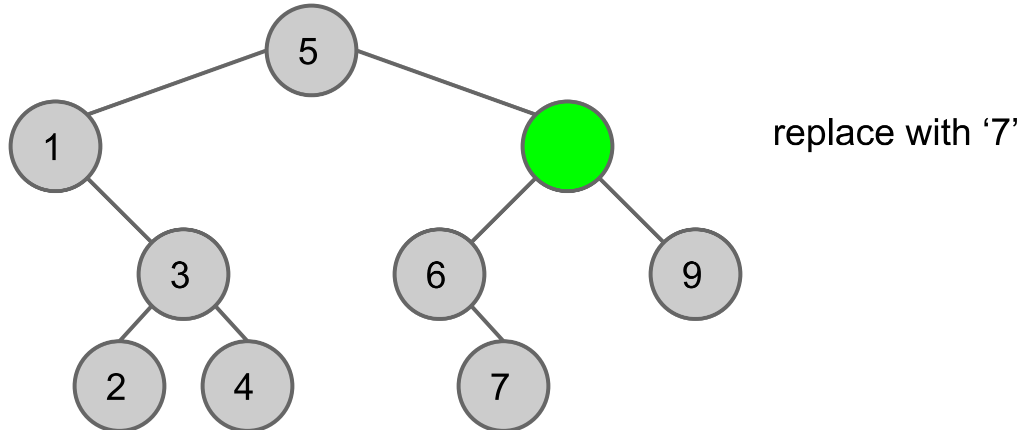
Binary Search Tree

- Remove a node
  - replace that node with the node *just* less than it



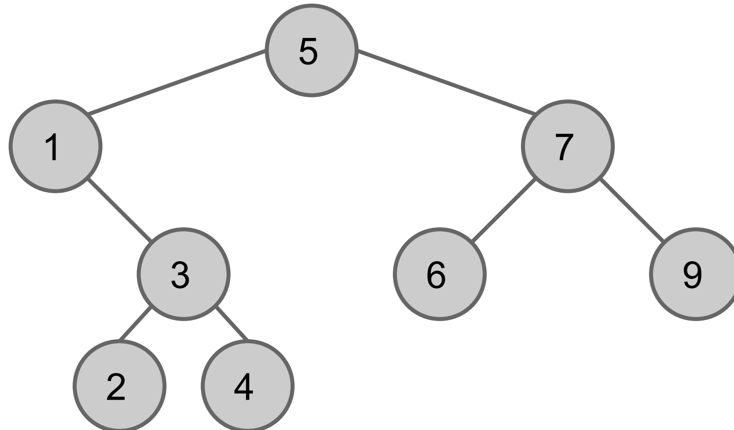
Binary Search Tree

- Remove a node
  - replace that node with the node *just* less than it



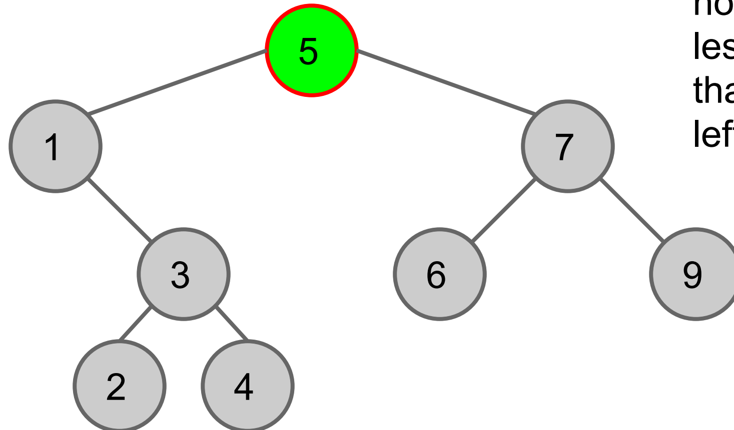
Binary Search Tree

- Remove a node
  - replace that node with the node *just* less than it



Binary Search Tree

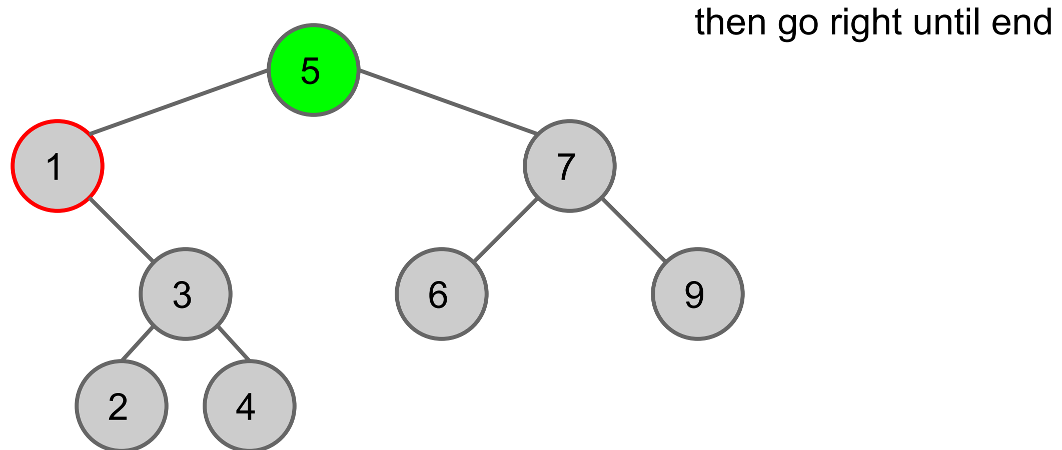
- Remove a node
  - replace that node with the node *just* less than it



how to find the node just less than ‘5’?
that node must be on the left. go left first

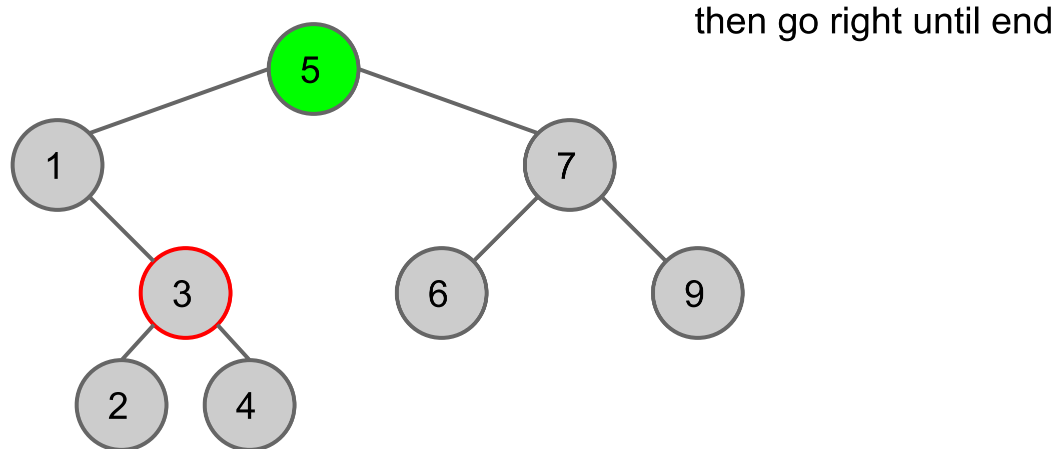
Binary Search Tree

- Remove a node
  - replace that node with the node *just* less than it



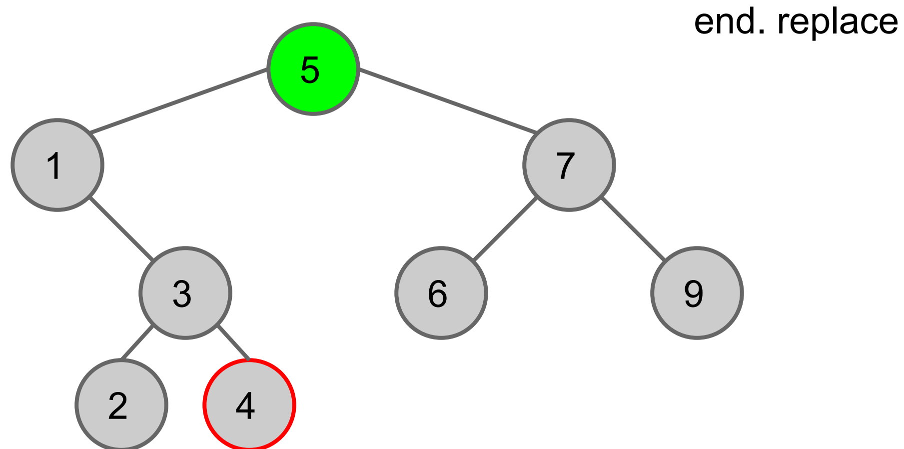
Binary Search Tree

- Remove a node
  - replace that node with the node *just* less than it



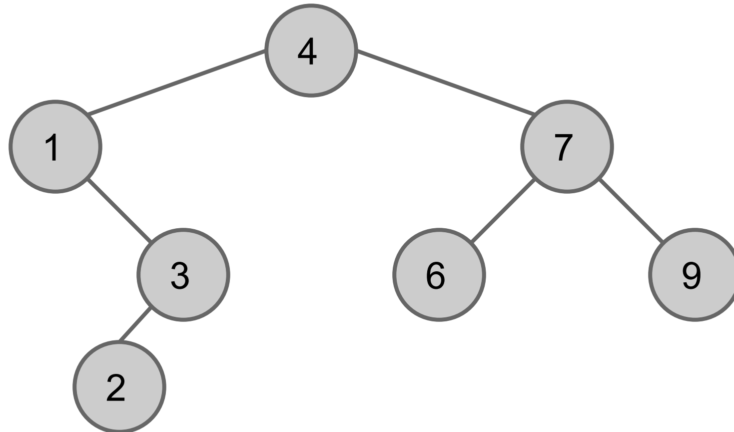
Binary Search Tree

- Remove a node
  - replace that node with the node *just* less than it



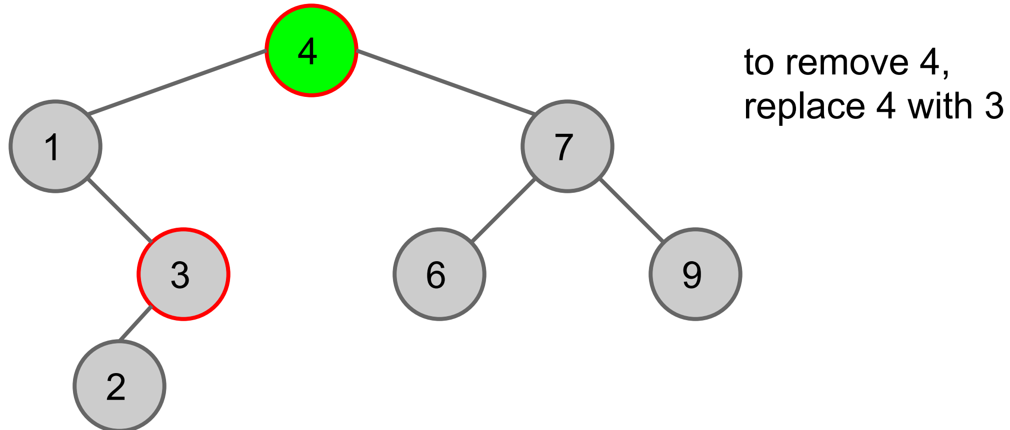
Binary Search Tree

- Remove a node
  - replace that node with the node *just* less than it



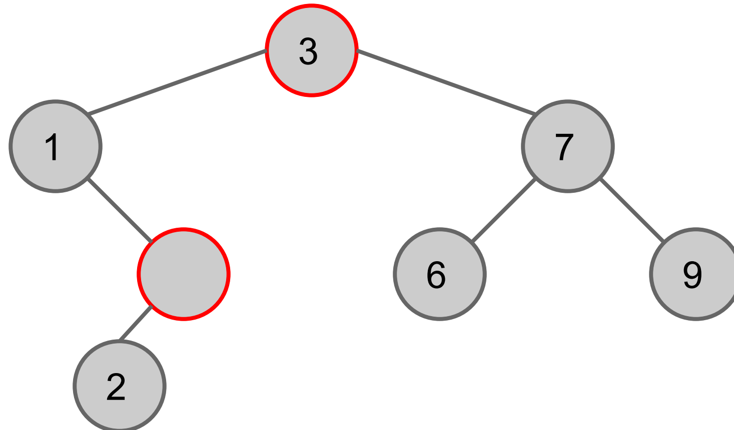
Binary Search Tree

- Remove a node
  - replace that node with the node *just* less than it



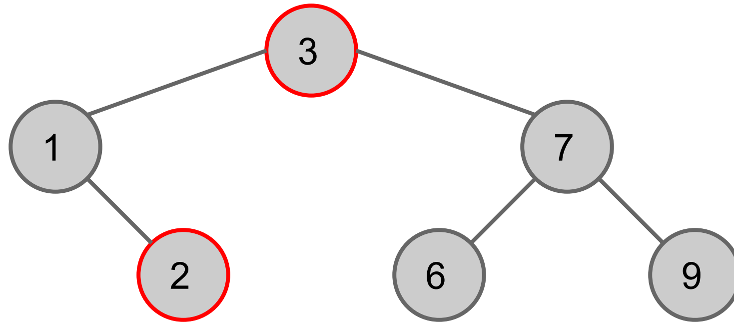
Binary Search Tree

- Remove a node
  - replace that node with the node *just* less than it



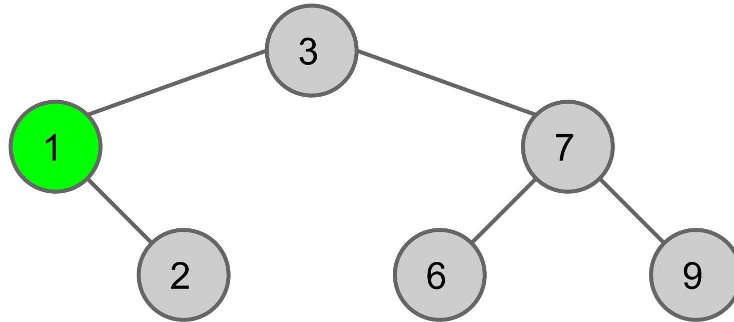
Binary Search Tree

- Remove a node
  - replace that node with the node *just* less than it



Binary Search Tree

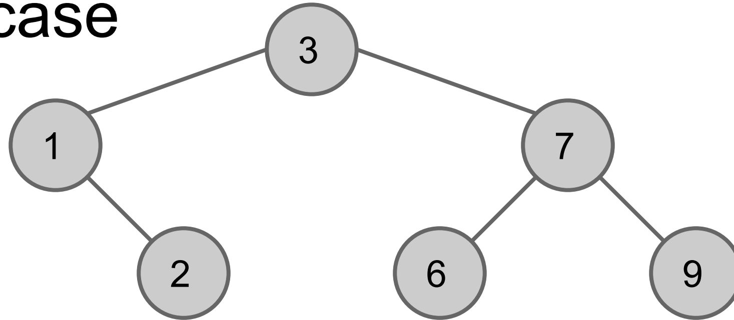
- Remove a node
  - replace that node with the node *just* less than it



how to remove 1?

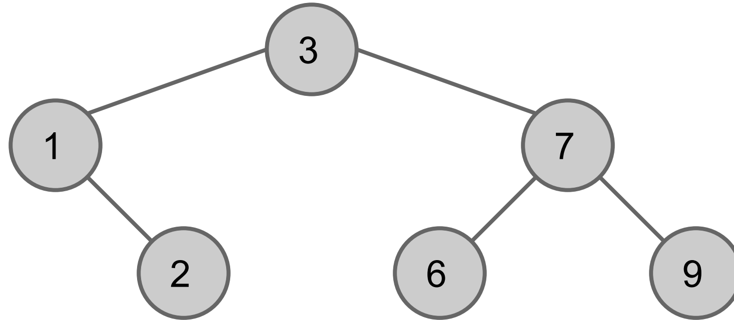
Binary Search Tree

- Remove a node
  - locate the node to be removed: O(log(N)) average
  - locate the node *just* less than it: O(log(N)) average
  - overall, O(log(N)) average
  - O(N) worst case



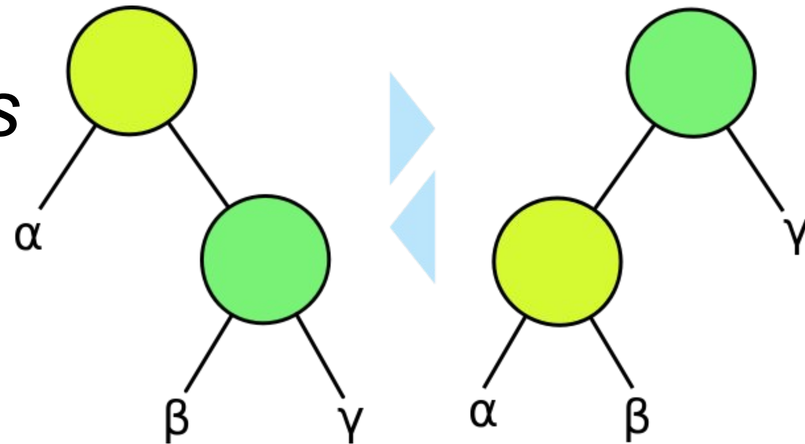
Binary Search Tree

- Summary
- Find, Insert, Remove
    - O(log(N)) average
    - O(N) worst case



Binary Search Tree

- How to get rid of O(N) worst case?
- self-balancing binary search trees
    - AVL tree, Red-black tree, AA tree, Treap, Size-balanced tree, Splay tree
- make use of *tree rotations*
- O(log(N)) worst case



Binary Search Tree

- Implementation

```
struct BST {
  int value;
  BST *left, *right;
} *root, memory[100000], memory_i;
```

Binary Search Tree

- C++ STL: set, map
- Red-black tree internally, so worst case O(log(N))

```
#include <set>
...
std::set<int> s;
s.insert(456);
int x = s.count(123); //0
s.erase(456);
```

Data structures (II)

- Binary Heap
- Binary Search Tree
- Hash Table

Hash Table

- Binary search tree is too complicated
- Hash table is much simpler, it is just an array

	Insert	Find	Remove
Plain array	O(1)	O(N)	O(N)
Sorted array	O(N)	O(log(N))	O(N)
Binary search tree	O(log(N)) average	O(log(N)) average	O(log(N)) average
Hash table	O(1) average	O(1) average	O(1) average

Hash Table

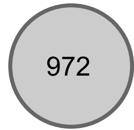
- Array too short, value too big

972

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Hash Table

- Array too short, value too big
- *Hash* it



972 % 17 = 3

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Hash Table

- Array too short, value too big
- *Hash* it

257

257 % 17 = 2

			972													
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Hash Table

- Array too short, value too big
- *Hash* it

339

339 % 17 = 16

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
		257	972													

Hash Table

- Array too short, value too big
- *Hash* it

376

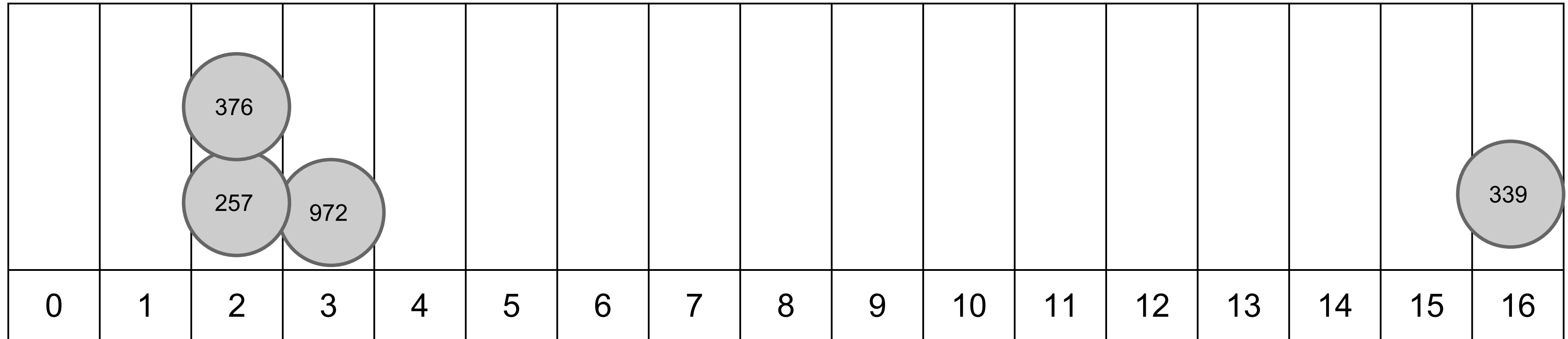
376 % 17 = 2

oops occupied

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
		257	972													339

Hash Table

- Hash collision resolution 1:
  - *separate chaining*
  - array of linked lists



Hash Table

- Hash collision resolution 2:
  - *open addressing*
  - find the first empty slot on the right
  - usually slower

		257	972	376												339
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Hash Table

- Insert
  - Time complexity
  - O(1) average
  - O(N) worst case
  - a good hash function gets better time

		257	972	376												339
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Hash Table

- Find (separate chaining)
  - traverse the linked list on the hashed slot

376
257
972
339

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

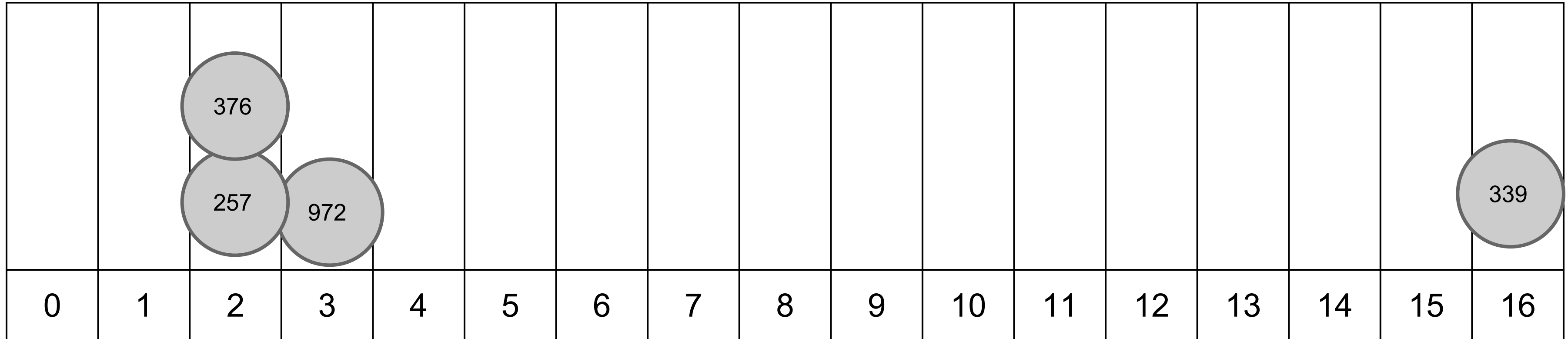
Hash Table

- Find (open addressing)
  - scan from the hashed slot to the right until the required value or an empty slot is found

		257	972	376												339
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Hash Table

- Remove (separate chaining)
  - remove from the linked list on the hashed slot



Hash Table

- Remove (open addressing)
  - set the slot value to -1 (or other dummy value)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
		257	972	-1												339

Hash Table

- Summary
- Insert, Find, Remove
    - O(1) average
    - O(N) worst case