HKOI Training (Constructive Algorithm)

Problem Set

Group Number: ________

Group Members: ____

Note:

1. There are 11 problems in total.
2. You have 45 minutes to solve the problems.
3. Let your group number be g. You are required to solve and present problem (g^3 + 7) mod 10.
4. Problems are marked with asterisks, with * the easiest and ***** the hardest.
5. Problems 8 and 10 require basic knowledge in graph theory.
6. The use of computer is allowed.
7. If you think a problem statement is unclear, you may ask for clarification.
8. If a problem can be found in an online judge, you can submit your solution to the problem.

Problem 1 (Little Frog) [CF 53C]

Difficulty: *

Statement: Given N, find a permutation of the N integers from 1 to N, such that $|a_i - a_{i+1}|$ are pairwise distinct.

Example: N = 3. {1, 3, 2} satisfies the constraints, since |1 – 3| = 2, |3 – 2| = 1, which are distinct.

Problem 2 (Getting rid of the digit '2') [Self-Composed]

Difficulty: *

Statement: Given a positive integer S, which contains the digit '2', write S as the sum of two positive integers A and B, such that A and B do not contain the digit '2'.

Example: N = 29. A = 14, B = 15 satisfy the constraints.

Problem 3 (Non-crossing Roads) [Self-Composed]

Difficulty: **

Statement: Given 2N distinct points on the Cartesian coordinate plane. Pair the points up so that the N line segments connecting the point pairs do not intersect.

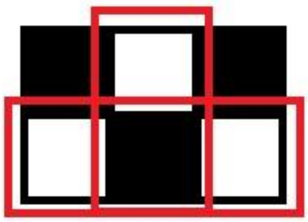
Example: N = 2, points = {(0, 0), (0, 1), (1, 0), (1, 1)}. The pairing 1-2, 3-4 is valid, but the pairing 1-4, 2-3 is not. (Draw on a paper to verify.)

Problem 4 (Alter Board) [NEERC 2014/15 A] [In CF Gym]

Difficulty: **

Statement: Given a N*M chessboard (i.e. adjacent squares are coloured differently). For each operation, you can select a sub-rectangle of the board and toggle the colours of the squares (black → white, white → black) inside the sub-rectangle. Find the minimal number of operations required to make the colours of all squares the same. Also provide the sequence of operations.

Example: N = 2, M = 3. It requires two operations.



Problem 5 (Square Snake) [(Greatly) Simplified; CF 198D]

Difficulty: **

Statement: Given an N x N grid, you are to number the squares, such that:

(1) Each number from 1 to N^2 is used exactly once,

(2) The two squares numbered i and (i+1) are neighbouring, and

(3) For each 0 <i< N, there are at least two different sub-squares made with consecutive numbers.

Example: N = 3. The following is a solution:

1	2	9
4	3	8
5	6	7

We check condition (3) only.

i = 1: {1} and {2} are two size-1 sub-squares.

i = 2: {1,2,3,4} and {3,4,5,6} are two size-2 sub-squares.

Problem 6 (Frog and String) [Simplified; ACM-ICPC 2015 Hefei Regional H]

Difficulty: **

Statement:Given N, M, find a string of length N, consisting of 'A', 'B', and 'C', such that it contains exactly M distinct palindromic substrings. (Hint: it is not always possible.)

Example: N = 5, M = 4, then "AABCA" satisfies the constraints. The palindromic substrings are "A", "B", "C", and "AA".

Problem 7 (Snake Carpet) [ACM-ICPC 2015 Beijing Regional I]

Difficulty: ***

Statement: Given N, you are to form a carpet (a H * W rectangular grid) with special patterns. For each integer K from 1 to N, the carpet should contain a "path" with K integer 'K's written. If K is odd and bigger than 1, the path should contain an odd number of turns. Otherwise, the path should contain an even number of turns.

You can decide what H and W are. (Hint: H * W = 1 + 2 + 3 + … + N)

Examples:

N = 4. The following is a possible solution (H = 2, W = 5):

3 4 4 1 2

3 3 4 4 2

N = 5. The following is a possible solution (H = 3, W = 5):

5 5 5 5 2

5 4 4 3 2

4 4 3 3 1

Problem 8 (Fox and Minimal Path) [CF 388B]

Difficulty: ****

Statement: Given a positive integer C (<= 10^9), form a simplegraph with no more than 1000 nodes, such that there are exactly C shortest paths between nodes 1 and 2.

Example: C = 2. A 1—3—2—4—1 *cycle* will do. The shortest paths are 1—3—2 and 1—4—2.

Problem 9 (Yaroslav and Algorithm) [CF 301C]

Difficulty: ****

Statement: Design an algorithm with specified format, such that it increases the value of any input positive integer by 1 (when it is input as a string).

(1) The algorithm first receives a string S.

(2) The algorithm consists of at most 50 commands in the form s_i>>w_i or s_i<>w_i, where s_i and w_i are strings with length 0 – 7, consisting of digits and '?'s.

(3) At each iteration, the algorithm looks for a command with the minimum index i, such that s_i is a substring of S. If such a command is not found, the algorithm terminates.

(4) Let the index found in (3) be k. Replace the *first* occurrence of s_k in S by w_k. If the command is of the form s_k>>w_k, return to step (3). If the command is of the form s_k<>w_k, terminate the program.

Example of analgorithm with correct format:

23<>

29 >> 333????

1 >> 2

Input 1: "1" → "2"

Input 2: “1119” → “2119” → “2219” → “2229” → “22333????”→ “233????”

Problem 10 (Graph Cutting) [CF 406C]

Difficulty: *****

Statement: Given a connected simple graph with N nodes and 2M edges, you are to partition the edges into M pairs, such that the edges in a single pair are adjacent, i.e. one edge is (a, b) and the other is (b, c).

Example: N = 4, M = 2, edges = {(1, 2), (2, 3), (3, 4), (4, 1)}.

Then, {(1, 2), (4, 1)}, {(2, 3), (3, 4)} is a valid partition.

Problem 11 (Utopia Divided) [Simplified; IOI 2002]

Difficulty: *****

Statement: Given N distinct positive integers and a booleanarray, sign[]. You need to use the N integers to fill a size-N array A[].For a number T, you can either fill T, or fill –T, into an array cell. The requirement is that:

If sign[i] = 0,A[1] + A[2] + … + A[i] < 0

If sign[i] = 1, A[1] + A[2] + … + A[i] > 0

Example:

N = 4, numbers = {1, 3, 4, 5}, sign[ ] = {0, 1, 1, 0}.

Then A[ ] = {-1, 5, -3, -4} satisfies the constraints.