T163 – Model Answer II

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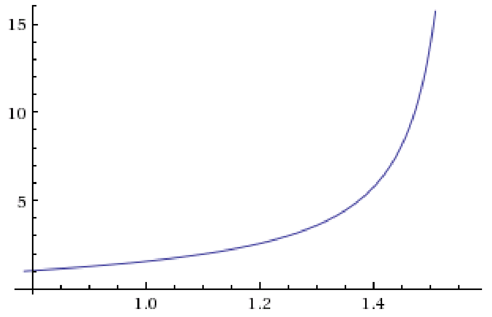
Background

- This is partly based on a real story of two students during ICT class
- Every year we have students who score less than the expected score (6.25) in HKOI Heat Event
  - Lowest score this year: Junior = 3, Senior = 2

Scoring

- **tan** gives a steep vertical curve near 90 degrees
- When s = 25000, (s + 80000) / 362500 = 0.290
- As s approaches 100000,
  (s + 80000) / 362500 approaches 0.5
- The region between 52.31 deg to 89.38 deg is used
- Take root that depends on p



$$x = \min\left(100, 36 \times \sqrt[(p+1)^{0.8}]{\tan\frac{(s+80000)\pi}{362500}} - 28\right)$$

Scoring

- Obviously, it's a tradeoff between s and p
- To achieve 100 points, p cannot be greater than 3
- p = 4, s = 100000, x = 97.487
- p = 3, s = 98784, x =100.006
- p = 2, s = 95816, x = 100.001
- p = 1, s = 88625, x = 100.001
- p = 0, s = 69615, x = 100.002
  - With no information, this is impossible

(score if cap = 100 is removed)

	25000	37500	50000	60000	70000	75000	80000	85000	90000	92000	94000	95000	96000	97000	97500	98000	98500	99000	99500	100000
40	8.47	8.90	9.40	9.86	10.44	10.80	11.24	11.79	12.56	12.97	13.48	13.80	14.18	14.64	14.91	15.23	15.60	16.05	16.62	17.40
39	8.48	8.92	9.43	9.90	10.49	10.86	11.31	11.87	12.65	13.07	13.60	13.93	14.31	14.78	15.06	15.39	15.77	16.23	16.81	17.61
38	8.49	8.94	9.45	9.94	10.55	10.92	11.38	11.95	12.75	13.18	13.72	14.06	14.45	14.93	15.22	15.55	15.94	16.42	17.01	17.83
37	8.50	8.96	9.49	9.98	10.60	10.99	11.45	12.04	12.86	13.30	13.85	14.19	14.60	15.09	15.39	15.73	16.13	16.61	17.23	18.06
36	8.51	8.98	9.52	10.03	10.66	11.05	11.53	12.13	12.97	13.42	13.99	14.34	14.75	15.26	15.56	15.91	16.32	16.82	17.45	18.31
35	8.52	9.00	9.55	10.07	10.72	11.12	11.61	12.23	13.09	13.55	14.13	14.49	14.92	15.44	15.75	16.10	16.52	17.04	17.68	18.57
34	8.53	9.03	9.59	10.12	10.79	11.20	11.70	12.33	13.21	13.69	14.29	14.65	15.09	15.62	15.94	16.31	16.74	17.27	17.93	18.84
33	8.54	9.05	9.63	10.17	10.85	11.28	11.79	12.44	13.34	13.83	14.44	14.82	15.27	15.82	16.15	16.52	16.97	17.51	18.20	19.13
32	8.56	9.08	9.67	10.23	10.93	11.36	11.88	12.55	13.48	13.98	14.61	15.00	15.46	16.03	16.36	16.75	17.21	17.77	18.48	19.44
31	8.57	9.10	9.71	10.29	11.00	11.45	11.99	12.68	13.63	14.14	14.79	15.19	15.67	16.25	16.60	17.00	17.47	18.04	18.77	19.77
30	8.59	9.13	9.76	10.35	11.08	11.54	12.10	12.80	13.79	14.32	14.98	15.40	15.88	16.48	16.84	17.26	17.74	18.34	19.09	20.12
29	8.60	9.16	9.80	10.41	11.17	11.64	12.21	12.94	13.95	14.50	15.19	15.61	16.12	16.74	17.11	17.53	18.04	18.65	19.43	20.49
28	8.62	9.20	9.85	10.48	11.26	11.74	12.33	13.08	14.13	14.69	15.40	15.84	16.36	17.00	17.39	17.83	18.35	18.98	19.79	20.89
27	8.64	9.23	9.91	10.55	11.36	11.86	12.46	13.24	14.32	14.90	15.64	16.09	16.63	17.29	17.69	18.14	18.68	19.34	20.17	21.31
26	8.65	9.27	9.97	10.63	11.46	11.98	12.60	13.40	14.52	15.12	15.88	16.35	16.91	17.60	18.01	18.48	19.04	19.72	20.59	21.78
25	8.68	9.31	10.03	10.71	11.57	12.10	12.75	13.58	14.74	15.36	16.15	16.64	17.22	17.93	18.36	18.85	19.43	20.14	21.04	22.27
24	8.70	9.35	10.09	10.80	11.69	12.24	12.91	13.77	14.97	15.62	16.44	16.95	17.55	18.29	18.73	19.24	19.85	20.59	21.52	22.81
23	8.72	9.40	10.17	10.90	11.82	12.39	13.09	13.98	15.22	15.90	16.75	17.28	17.90	18.68	19.14	19.67	20.30	21.07	22.05	23.39
22	8.75	9.45	10.24	11.00	11.96	12.55	13.28	14.20	15.50	16.20	17.09	17.64	18.29	19.10	19.58	20.14	20.80	21.60	22.63	24.03
21	8.77	9.50	10.33	11.12	12.11	12.73	13.48	14.45	15.80	16.53	17.46	18.03	18.72	19.56	20.06	20.65	21.34	22.18	23.25	24.73
20	8.80	9.56	10.42	11.24	12.27	12.92	13.71	14.71	16.13	16.89	17.86	18.46	19.18	20.06	20.59	21.20	21.93	22.82	23.95	25.50
19	8.83	9.62	10.52	11.38	12.45	13.13	13.95	15.01	16.48	17.29	18.31	18.94	19.69	20.62	21.18	21.82	22.58	23.52	24.71	26.35
18	8.87	9.69	10.63	11.52	12.65	13.36	14.22	15.33	16.88	17.73	18.80	19.46	20.26	21.23	21.82	22.50	23.31	24.30	25.56	27.30
17	8.91	9.77	10.75	11.69	12.87	13.61	14.52	15.68	17.32	18.21	19.34	20.05	20.88	21.92	22.54	23.26	24.12	25.17	26.51	28.35
16	8.95	9.85	10.88	11.87	13.11	13.90	14.85	16.08	17.81	18.75	19.95	20.70	21.59	22.69	23.35	24.12	25.03	26.15	27.58	29.55
15	9.00	9.95	11.03	12.07	13.39	14.21	15.22	16.53	18.36	19.37	20.64	21.44	22.39	23.56	24.27	25.09	26.06	27.26	28.79	30.91
14	9.05	10.05	11.20	12.30	13.69	14.57	15.64	17.03	18.99	20.06	21.43	22.27	23.29	24.55	25.31	26.19	27.24	28.52	30.18	32.46
13	9.11	10.17	11.39	12.56	14.04	14.98	16.12	17.61	19.70	20.85	22.32	23.24	24.33	25.69	26.51	27.46	28.60	29.99	31.78	34.27
12	9.18	10.31	11.60	12.86	14.44	15.45	16.68	18.27	20.53	21.77	23.36	24.35	25.54	27.02	27.91	28.95	30.18	31.70	33.66	36.39
11	9.26	10.47	11.85	13.20	14.91	15.99	17.32	19.05	21.50	22.85	24.58	25.67	26.97	28.58	29.56	30.70	32.06	33.73	35.90	38.91
10	9.36	10.65	12.15	13.60	15.45	16.63	18.08	19.96	22.65	24.13	26.04	27.23	28.67	30.46	31.54	32.81	34.32	36.18	38.60	41.97
9	9.47	10.87	12.50	14.08	16.11	17.40	18.99	21.06	24.04	25.69	27.81	29.14	30.74	32.74	33.96	35.38	37.09	39.19	41.92	45.75
8	9.60	11.13	12.92	14.66	16.90	18.33	20.10	22.42	25.75	27.61	30.00	31.51	33.33	35.60	36.99	38.62	40.57	42.98	46.12	50.55
7	9.76	11.45	13.44	15.38	17.89	19.50	21.50	24.13	27.92	30.05	32.80	34.54	36.64	39.28	40.90	42.80	45.07	47.91	51.61	56.85
6	9.96	11.86	14.10	16.31	19.17	21.01	23.31	26.35	30.77	33.26	36.50	38.55	41.05	44.20	46.13	48.40	51.14	54.57	59.06	65.46
5	10.23	12.40	14.98	17.53	20.87	23.03	25.75	29.36	34.67	37.69	41.63	44.14	47.21	51.10	53.50	56.33	59.76	64.07	69.76	77.93
4	10.59	13.14	16.19	19.24	23.27	25.91	29.24	33.72	40.38	44.19	49.23	52.47	56.44	61.51	64.66	68.40	72.95	78.70	86.36	97.49
3	11.12	14.23	17.99	21.82	26.94	30.33	34.67	40.58	49.51	54.71	61.65	66.16	71.75	78.96	83.47	88.86	95.48	100.00	100.00	100.00
2	11.97	16.01	21.00	26.19	33.29	38.09	44.34	53.02	66.52	74.57	85.53	92.76	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1	13.60	19.53	27.15	35.38	47.15	55.42	66.51	82.56	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
0	18.31	30.39	47.65	68.37	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Strategy 0 (p = 0)

- Instead of trying C, try ABCD evenly
- p = 0, s = 25000, x = 18.307
- Great deal!

```
void exam() {
  for (int i = 0; i < 2500; i++) {
    for (int j = 0; j < 10; j++) answer('A');
    for (int j = 0; j < 10; j++) answer('B');
    for (int j = 0; j < 10; j++) answer('C');
    for (int j = 0; j < 10; j++) answer('D');
  }
}
```

Strategy 1A (p <= 40)

- Find out the model answers with as few incorrect answers as possible
- After that, you will get all correct for the remaining questions

- For the first 10 questions, answer(A) and score()
  - If the score increased for the question, that question's answer is A
- For the next 10 questions, (if not A) answer(B) and score()
  - If the score increased for the question, that question's answer is B
- Repeat for C and D
- Worst case p = 40, s = 99970, x = 17.343 (input = DDDDDDDDDD)

Randomization

- You may attempt to increase the score of a strategy by not guessing in the same order for every question
- Example: DBCA for question 1, BCDA for question 2…

```
  char t[4][11] = {"DBDABCDABD", "BCACCDABCA", "CDBBDABCDC", "AACDABCDAB"};
//char t[4][11] = {"AAAAAAAAAA", "BBBBBBBBBB", "CCCCCCCCCC", "DDDDDDDDDD"};
```

- When p = 31, s = 99979, x = 19.717 (+2.374)
- Is this effective?
- However, with the 1000 test cases, the chance of encountering a bad case (not necessary worst) is very high
- Implemented randomized: p = 37, s = 99973, x = 18.009 (+0.666)

Strategy 1B (p <= 30)

- We should look for better strategies instead of applying tricks
- For example, if we know that the answer is not A, B, or C, then the answer must be D
- So the last round of 10 questions is not necessary

- Worst case: p = 30, s = 99970, x = 20.043
- Implemented randomized: p = 29, s = 99976, x = 20.428 (+0.385)

p	s	x
0	25000	18.307
40	99970	17.343
30	99970	20.043

Strategy 2A (p = 10)

- Even if s is very high, the penalty of p is too much to get a good score
- We need some strategies that use fewer presses
- Here is a strategy that uses exactly 10 presses
- To find out the answer for a single question: we answer A x 0 + B x 1 + C x 2 + D x 3 using 6 rounds.
  (Answer X for other questions)
- Press score after the 6 rounds
- For example, if the score increased by 2,then the answer is C
- p = 10, s = 99400, x = 38.054

B	X	X	X	X	X	X	X	X	X
C	X	X	X	X	X	X	X	X	X
C	X	X	X	X	X	X	X	X	X
D	X	X	X	X	X	X	X	X	X
D	X	X	X	X	X	X	X	X	X
D	X	X	X	X	X	X	X	X	X

p	s	x
0	25000	18.307
40	99970	17.343
30	99970	20.043
10	99400	38.054

Strategy 2B (p = 10)

- Answering known answer instead of X will result in fewer incorrect answer
- For example, when figuring out the answer for the 3rd question, we can answer the correct answer for questions 1 and 2 for all 6 rounds
- p = 10, s = 99670, x = 39.602

D	A	C	A	B	X	X	X	X	X
D	A	C	A	C	X	X	X	X	X
D	A	C	A	C	X	X	X	X	X
D	A	C	A	D	X	X	X	X	X
D	A	C	A	D	X	X	X	X	X
D	A	C	A	D	X	X	X	X	X

p	s	x
0	25000	18.307
40	99970	17.343
30	99970	20.043
10	99400	38.054
10	99670	39.602

Strategy 3A (p = 5)

- Instead of figuring out the answer of one question at a time, make it two
- Consider the base-4 number system
- 0 = AA, 1 = AB, 2 = AC, 3 = AD
  4 = BA, 6 = BB, 7 = BC, 8 = BD ….
- 24 rounds required per 2 questions
- p = 5, s = 99280, x = 67.034

B	B
C	B
C	B
D	B
D	C
D	C
X	C
X	C
X	C
X	C
X	C
X	C
X	D
X	D
X	D
X	D
X	D
X	D
X	D
X	D
X	D
X	D
X	D
X	D

p	s	x
0	25000	18.307
40	99970	17.343
30	99970	20.043
10	99400	38.054
10	99670	39.602
5	99280	67.034

Strategy 3B

- We are answering many X when doing the rounds
- Replace 4 Xs to the same question with ABCD
- The number of incorrect answers will be reduced by not more than 1/4
- p = 5, s = 99420, x = 68.720

B	B	A	A	A	A	A	A	A	A
C	B	B	B	B	B	B	B	B	B
C	B	C	C	C	C	C	C	C	C
D	B	D	D	D	D	D	D	D	D
D	C	A	A	A	A	A	A	A	A
D	C	B	B	B	B	B	B	B	B
X	C	C	C	C	C	C	C	C	C
X	C	D	D	D	D	D	D	D	D
A	C	A	A	A	A	A	A	A	A
B	C	B	B	B	B	B	B	B	B
C	C	C	C	C	C	C	C	C	C
D	C	D	D	D	D	D	D	D	D
A	D	A	A	A	A	A	A	A	A
B	D	B	B	B	B	B	B	B	B
C	D	C	C	C	C	C	C	C	C
D	D	D	D	D	D	D	D	D	D
A	D	A	A	A	A	A	A	A	A
B	D	B	B	B	B	B	B	B	B
C	D	C	C	C	C	C	C	C	C
D	D	D	D	D	D	D	D	D	D
A	D	A	A	A	A	A	A	A	A
B	D	B	B	B	B	B	B	B	B
C	D	C	C	C	C	C	C	C	C
D	D	D	D	D	D	D	D	D	D

p	s	x
0	25000	18.307
40	99970	17.343
30	99970	20.043
10	99400	38.054
10	99670	39.602
5	99280	67.034
5	99420	68.720

Strategy 4 (p = 2)

- It is logical to try the next factor of 10
- 2 groups of 5 questions
- Each group takes 4^4 x 6 = 1536 rounds

- p = 2, s = 78880, x = 42.775

- If the XXXX -> ABCD trick is applied:
  - p = 2, s = 81696, x = 46.935

p	s	x
0	25000	18.307
40	99970	17.343
30	99970	20.043
10	99400	38.054
10	99670	39.602
5	99280	67.034
5	99420	68.720
2	78880	42.775
2	81696	46.935

Strategy 5 (p = 1)

- It is not possible to extend the strategy to p = 1 since 4^9 x 6 > 100000
- We have to drop 4 questions (at least 30000 wrong answers)
- p = 1, s <= 70000, x <= 47.150
- So we are not going to discuss this strategy

p	s	x
0	25000	18.307
30	99970	20.043
10	99670	39.602
5	99420	68.720
2	81696	46.935

Strategy 6 (p = 3)

- Let's make p = 3, how?
- Group the questions in 3-3-4 or 3-4-3 or 4-3-3
- If the XXXX -> ABCD trick is applied
- 3-3-4: p = 3, s = 97432, x = 82.809
- 3-4-3: p = 3, s = 96856, x = 77.786
- 4-3-3: p = 3, s = 96280, x = 73.569

p	s	x
0	25000	18.307
30	99970	20.043
10	99670	39.602
5	99420	68.720
2	81696	46.935
3	97432	82.809

Strategy 7 (p = 3)

- Instead of getting the exact answer of one question, try to determine which questions have answer A, then B, …
  - Question 1: 1 A, 511 X
  - Question 2: 2 A, 510 X
  - Question 3: 4 A, 508 X
  - …
  - Question 10: 512 A
- Deduce the positions of A from the binary notation of score
- If the XXXX -> ABCD trick is applied
- p = 3, s = 87709, x = 44.878

p	s	x
0	25000	18.307
30	99970	20.043
10	99670	39.602
5	99420	68.720
2	81696	46.935
3	97432	82.809
3	87709	44.878

Strategy 8 (p = 2)

- Similar to the previous strategy, but we perform “binary search” instead of “linear search”
- First press (1024 rounds): determine if a question is one of A or B
- Second press (512 rounds): determine if a question is A or B (if previous result is true) / C or D (if previous result is false)
- p = 2, s = 88736, x = 62.408

p	s	x
0	25000	18.307
30	99970	20.043
10	99670	39.602
5	99420	68.720
2	81696	46.935
3	97432	82.809
3	87709	44.878
2	88736	62.408

Official solution (p = 3)

- It's a cocktail strategy (mix of the previous strategies)
- When performing the previous "binary search" strategy, the most significant bit is quite inefficient
- It causes the number of rounds to double
- Change one of the questions from "is it A or B?" to "is it A?"

64 Rounds	1	2	3	4	5	6	7	8	9	10	
A	64	32	16	8	4	2	1				
B		32	16	8	4	2	1				
C											
D											
Worst	BCD	CD	CD	CD	CD	CD	CD	ABCD	ABCD	ABCD	
Correct	0	0	8	12	14	15	15	16	16	16	**112**

Second score()

- In the second score, we would continue to binary search 5 of the questions to determine the exact answer
- Also, we ask a new question "is it A or B?"
- Also, reduce the possibility of Question 1 by asking "is it B?"

64 Rounds	1	2	3	4	5	6	7	8	9	10	
A											
B	64		16	8	4	2	1	32			
C								32			
D											
Worst	CD	CD	D	D	D	D	D	CD	ABCD	ABCD	
Correct	0	32	24	28	30	31	31	0	16	16	**208**

Third and last score()

- Lastly, binary search questions 1, 2, 8
- For questions 9 and 10, use the base-4 number method (refer to strategy 3A/B)
- Worst case: p = 3, s = 98784
- (is this number familiar to you?)

64 Rounds	1	2	3	4	5	6	7	8	9	10	
A											
B									4	1	
C	64	32						16	8	2	
D									12	3	
Worst	D	D	D	D	D	D	D	D	A	A	
Correct	0	16	64	64	64	64	64	24	10	14	384
										9808 x 10	98080
											98784

Summary

p = 3, s = 98784

x = 100.006

(if cap is removed)

Note: There exist better solutions

p	s	x
0	25000	18.307
30	99970	20.043
10	99670	39.602
5	99420	68.720
2	81696	46.935
3	97432	82.809
3	87709	44.878
2	88736	62.408
3	98784	100.006