Alice's Meal

YIK WAI PAN

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Statistics

- Attempts: 70
- Max: 100
- Mean: 53.071
- Subtasks:

- Relatively easy problem
- Tickets for the medals

Problem Description

- N dishes on the menu, the dish i has the delicious level of D_i
- Alice have K minutes to eat, and each minute she can finish one dish
- Eating dish i during the t^{th} minute gives Alice $D_i \times t$ happiness
- Alice needs to eat K consecutive dishes
- Find the maximum possible total happiness

- Let $f(i) = D_i \times 1 + D_{i+1} \times 2 + D_{i+2} \times 3 + \cdots + D_{i+K-1} \times K$
- Find the maximum $f(i)$, where $1 \le i \le N - K + 1$

Subtask 1 (11%)

- $1 \leq N \leq 2000,\ K = 1,\ 0 \leq D_i \leq 1000$
- Alice can only eat 1 dish
- Output the largest D_i

- Time Complexity: $O(N)$

Subtask 2 (9%)

- $1 \leq N \leq 2000$, $K = N$, $0 \leq D_i \leq 1000$
- Alice must eat all the dishes
- Just calculate the total happiness from all dishes

- Time Complexity: $O(N)$

Subtask 3 (27%)

- $1 \leq N \leq 2000$, $1 \leq K \leq N$, $0 \leq D_i \leq 1000$
- For each i, calculate the total happiness gained by adding each happiness one by one
- Choose the maximum total happiness
- Time Complexity: $O(NK)$

Subtask 4 (15%)

- $1 \leq N \leq 100000,\ 1 \leq K \leq N,\ 0 \leq D_1 \leq D_2 \leq \cdots \leq D_N \leq 1000$
- For all $i < N - K + 1$,

$$f(i) \quad = D_i \times 1 \quad + D_{i+1} \times 2 + D_{i+2} \times 3 + \cdots + D_{i+K-1} \times K$$

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$$f(i+1) = D_{i+1} \times 1 + D_{i+2} \times 2 + D_{i+3} \times 3 + \cdots + D_{i+K} \times K$$

- Choosing the last K dishes can always give the highest total happiness
- Calculate the total happiness from the last K dishes

- Time Complexity: $O(N)$

Subtask 5 (38%)

- $1 \le N \le 100000,\ 1 \le K \le N,\ 0 \le D_i \le 1000$
- Look at the formula of total happiness again

$$f(i) = D_i \times 1 + D_{i+1} \times 2 + D_{i+2} \times 3 + \cdots + D_{i+K-2} \times (K-1) + D_{i+K-1} \times K$$
$$f(i-1) = D_{i-1} \times 1 + D_i \times 2 + D_{i+1} \times 3 + D_{i+2} \times 4 + \cdots + D_{i+K-2} \times K$$

$f(i-1) - f(i) = (D_{i-1} + D_i + D_{i+1} + \cdots + D_{i+K-2}) - D_{i+K-1} \times K$, or

$$f(i) = f(i-1) - (D_{i-1} + D_i + D_{i+1} + \cdots + D_{i+K-2}) + D_{i+K-1} \times K$$

- We can calculate $f(i)$ using $f(i-1)$
- Calculation of $(D_{i-1} + D_i + D_{i+1} + \cdots + D_{i+K-2})$ is still $O(K)$

Subtask 5 (38%)

$$f(i) = f(i-1) - (D_{i-1} + D_i + D_{i+1} + \cdots + D_{i+K-2}) + D_{i+K-1} \times K$$

- Let $g(i) = D_{i-1} + D_i + D_{i+1} + \cdots + D_{i+K-2}$
- $g(i-1) = D_{i-2} + D_{i-1} + D_i + \cdots + D_{i+K-3}$
- $g(i)$ can also be calculated by using $g(i-1)$!

- $g(i) = g(i-1) - D_{i-2} + D_{i+K-2}$
- $f(i) = f(i-1) - g(i) + D_{i+K-1} \times K$
- Both can be done in constant time

Subtask 5 (38%)

- Calculate $f(1)$ and $g(1)$ by adding the terms one by one
- For i from 2 to N-K+1,
  - Calculate $g(i)$ by using $g(i-1)$
  - Calculate f(i) by using $f(i-1)$ and $g(i)$
- Choose the maximum $f(i)$ as the answer

- Time Complexity: $O(N)$

Thank You