

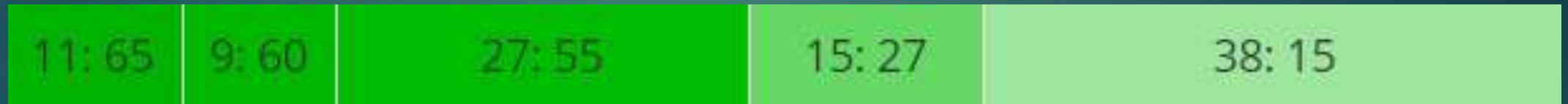
Alice's Meal

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21 - 1 - 2016

Statistics

- ▶ Attempts: 70
- ▶ Max: 100
- ▶ Mean: 53.071
- ▶ Subtasks:



- ▶ Relatively easy problem
- ▶ Tickets for the medals

Problem Description

- ▶ N dishes on the menu, the dish i has the delicious level of D_i
- ▶ Alice have K minutes to eat, and each minute she can finish one dish
- ▶ Eating dish i during the t^{th} minute gives Alice $D_i \times t$ happiness
- ▶ Alice needs to eat K consecutive dishes
- ▶ Find the maximum possible total happiness



- ▶ Let $f(i) = D_i \times 1 + D_{i+1} \times 2 + D_{i+2} \times 3 + \dots + D_{i+K-1} \times K$
- ▶ Find the maximum $f(i)$, where $1 \leq i \leq N - K + 1$

Subtask 1 (11%)

- ▶ $1 \leq N \leq 2000, K = 1, 0 \leq D_i \leq 1000$
- ▶ Alice can only eat 1 dish
- ▶ Output the largest D_i

- ▶ Time Complexity: $O(N)$

Subtask 2 (9%)

- ▶ $1 \leq N \leq 2000, K = N, 0 \leq D_i \leq 1000$
- ▶ Alice must eat all the dishes
- ▶ Just calculate the total happiness from all dishes
- ▶ Time Complexity: $O(N)$

Subtask 3 (27%)

- ▶ $1 \leq N \leq 2000, 1 \leq K \leq N, 0 \leq D_i \leq 1000$
- ▶ For each i , calculate the total happiness gained by adding each happiness one by one
- ▶ Choose the maximum total happiness
- ▶ Time Complexity: $O(NK)$

Subtask 4 (15%)

▶ $1 \leq N \leq 100000, 1 \leq K \leq N, 0 \leq D_1 \leq D_2 \leq \dots \leq D_N \leq 1000$

▶ For all $i < N - K + 1$,

$$f(i) = D_i \times 1 + D_{i+1} \times 2 + D_{i+2} \times 3 + \dots + D_{i+K-1} \times K$$

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$$f(i + 1) = D_{i+1} \times 1 + D_{i+2} \times 2 + D_{i+3} \times 3 + \dots + D_{i+K} \times K$$

▶ Choosing the last K dishes can always give the highest total happiness

▶ Calculate the total happiness from the last K dishes

▶ Time Complexity: $O(N)$

Subtask 5 (38%)

- ▶ $1 \leq N \leq 100000, 1 \leq K \leq N, 0 \leq D_i \leq 1000$
- ▶ Look at the formula of total happiness again

$$\begin{aligned} f(i) &= D_i \times 1 + D_{i+1} \times 2 + D_{i+2} \times 3 + \dots + D_{i+K-2} \times (K-1) + D_{i+K-1} \times K \\ f(i-1) &= D_{i-1} \times 1 + D_i \times 2 + D_{i+1} \times 3 + D_{i+2} \times 4 + \dots + D_{i+K-2} \times K \end{aligned}$$

$$f(i-1) - f(i) = (D_{i-1} + D_i + D_{i+1} + \dots + D_{i+K-2}) - D_{i+K-1} \times K, \text{ or}$$

$$f(i) = f(i-1) - (D_{i-1} + D_i + D_{i+1} + \dots + D_{i+K-2}) + D_{i+K-1} \times K$$

- ▶ We can calculate $f(i)$ using $f(i-1)$
- ▶ Calculation of $(D_{i-1} + D_i + D_{i+1} + \dots + D_{i+K-2})$ is still $O(K)$

Subtask 5 (38%)

$$f(i) = f(i - 1) - (D_{i-1} + D_i + D_{i+1} + \dots + D_{i+K-2}) + D_{i+K-1} \times K$$

- ▶ Let $g(i) = D_{i-1} + D_i + D_{i+1} + \dots + D_{i+K-2}$
- ▶ $g(i - 1) = D_{i-2} + D_{i-1} + D_i + \dots + D_{i+K-3}$
- ▶ $g(i)$ can also be calculated by using $g(i - 1)$!

- ▶ $g(i) = g(i - 1) - D_{i-2} + D_{i+K-2}$
- ▶ $f(i) = f(i - 1) - g(i) + D_{i+K-1} \times K$
- ▶ Both can be done in constant time

Subtask 5 (38%)

- ▶ Calculate $f(1)$ and $g(1)$ by adding the terms one by one
- ▶ For i from 2 to $N-K+1$,
 - ▶ Calculate $g(i)$ by using $g(i-1)$
 - ▶ Calculate $f(i)$ by using $f(i-1)$ and $g(i)$
- ▶ Choose the maximum $f(i)$ as the answer

- ▶ Time Complexity: $O(N)$

Thank You

