

HKOI 2015/16 Solution Junior Q3 (Stone Game)

Alex Tung

23/1/2016

Task Description

- Given N piles of stones
- The i^{th} pile has a_i stones

- Two-player game
- In a player's turn, he/she has to
 - 1) Remove a stone from pile p , if $a_p > V$, or
 - 2) Add a stone to pile p , if $a_p < V$

- The first player who cannot make a move loses

Task Description

- Alice and Robo play the stone game for Q rounds
- For each round, different values of V may be given
- Output the winner

Key Observation

- Given V , we know how many turns the game will last
- T (number of turns) = $|a_1 - V| + |a_2 - V| + \dots + |a_N - V|$

How is T useful?

- If $T = 0$, Alice loses immediately \rightarrow Output “Robo”
- If $T = 1$, Alice makes a move $\rightarrow T = 0 \rightarrow$ Robo loses \rightarrow Output “Alice”
- If $T = 2$, Alice makes a move $\rightarrow T = 1 \rightarrow$ Robo makes a move $\rightarrow T = 0 \rightarrow$ Alice loses \rightarrow Output “Robo”
- ...

How is T useful?

- In general,
 - If T is even, output “Robo”
 - If T is odd, output “Alice”

Partial solution

- for each given V
 - set $T = 0$
 - for i from 1 to N
 - add $|a_i - V|$ to T
 - if T is even, output “Robo”
 - else output “Alice”

- Too slow to get 100 points

Can we calculate T faster?

- Yes
 - Partial sum
 - Find $s[x] := \text{sum}(a_i \mid a_i \leq x)$ for each x from 0 to 10^6
 - Some contestants used this method

- But...

Do we need the exact value of T?

- No!
- We only need to know the *parity* of T

T'

- $T' := (a_1 - V) + (a_2 - V) + \dots + (a_N - V)$
- $T = |a_1 - V| + |a_2 - V| + \dots + |a_N - V|$
- The parity of T and T' are the same
- Meaning: we can calculate T' instead

T'

- $T' = (a_1 - V) + (a_2 - V) + \dots + (a_N - V)$
- $= (a_1 + a_2 + \dots + a_N) - N*V$

Full Solution

- Pre-calculate $S = a_1 + a_2 + \dots + a_N$
- for each given V
 - if $(S - N * V)$ is even, output “Robo”
 - else output “Alice”

Thank you

- Any questions?