Promotion Period Solution

ALEX POON

What is Promotion Period

- Promotion Period (Pro P) is a term commonly used in university.
- In university, Pro P represents the promotion period of committees.
- In CUHK, committee members usually “dem beat” during Promotion period.

Problem Statement

- Snow white has “R” red apples and “G” green apples initially
- There is one seed in every apples
- “w” red seeds + “x” green seeds can trade a red apple
- “y” red seeds + “z” green seeds can trade a green apple
- Eating a red apple increase “P” happiness
- Eating a green apple increase “Q” happiness
- Maximize the happiness

Sample input

6 4 3 1 1 3 2 6

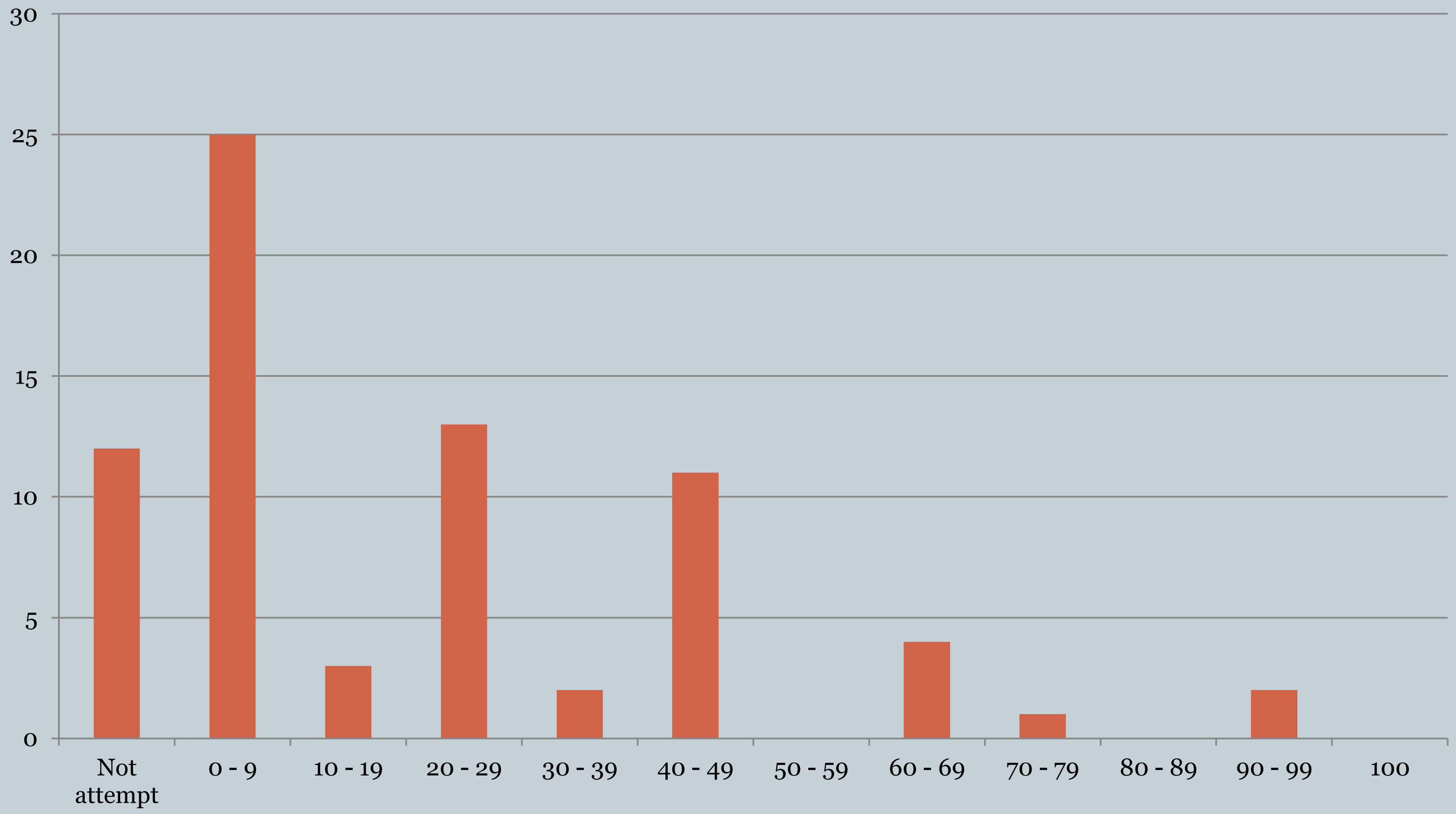
Sample output

46

Red Apple	Green Apple	Red Seed	Green Seed	Total Happiness	Action
6	4	0	0	0	Initial
0	0	6	4	36	Eat all apples she has
1	0	3	3	36	Trade for a red apple
0	0	4	3	38	Eat a red apple
0	1	3	0	38	Trade for a green apple
0	0	3	1	44	Eat a green apple
1	0	0	0	44	Trade for a red apple
0	0	1	0	46	Eat a red apple

Subtask	Max Points	R, G, W, X, Y, Z	P, Q
1	40	$1 \leq R, G, W, X, Y, Z \leq 10$	$1 \leq P, Q \leq 500$
2	25	$1 \leq R, G, W, X, Y, Z \leq 3000$	$1 \leq P, Q \leq 500$
3	35	$1 \leq R, G, W, X, Y, Z \leq 10^6$	$1 \leq P, Q \leq 500$

Statistic



30
25
20
15
10
5
0
Not attempt
0 - 9
10 - 19
20 - 29
30 - 39
40 - 49
50 - 59
60 - 69
70 - 79
80 - 89
90 - 99
100

Solution 1

- Exhaust all trading permutation
- Let f(R, G) is the maximum happiness can be achieved having R red apples and G green apples
- If we trade a red apple first, the maximum happiness will be f(R – W + 1, G - X) + P
- If we trade a green apple, the maximum happiness will be f(R – Y, G – Z + 1) + Q
- Which mean, the maximum happiness =
- Max(f(R – W + 1, G - X) + P, f(R – Y, G – Z + 1) + Q)

Solution 1

		F[2, 2]
	F[4, 3]	
		F[1, 3]
F[6, 4]		
		F[1, 3]
	F[5, 2]	
		Invalid

Solution 1

- We need to call the function for how many times?

- As W, X, Y, Z >= 1
- We need at least 2 seeds to trade an apple
- Remember we will get 1 seed after trading an apple
- So each time we trade an apple, the total number of seeds will decrease at least 1
- So we can trade at most R + G – 1 times (Why -1??)

Solution 1

- The maximum length of a trading permutation is R + G – 1 as well
- The time complexity of the solution :
- O(2^(R + G - 1))

- Expected score : 40

Solution 2

- Note that on solution 1
- We call the function f with the same parameter for many times
- F[1, 3] is calculated for twice

		F[2, 2]
	F[4, 3]	
		F[1, 3]
F[6, 4]		
		F[1, 3]
	F[5, 2]	
		Invalid

Solution 2

- Why not memorize all of them so that we need not to call as many times as before

Solution 2

- How to memorize?
- Use an array

- If ANS[R – W + 1, G – X] is not calc then
  calc f(R – W + 1, G - X)
  else return ANS[R – W + 1, G - X]

- What is the number of different states?

Solution 2

- The problem we need to find is f(R, G)
- Recall the formula:
- Max(f(R – W + 1, G - X) + P, f(R – Y, G – Z + 1) + Q)
- R and G is decreasing
- So there are at most R * G state
- So we need to call the function “f” for at most R * G times but not 2^(R + G - 1) times

Solution 2

- Time complexity ?
- There is at most R * G states
- We can calculate f(i, j) in O(1)
- So, the time complexity : O(R * G)

- Expected score : 65 - 75

Solution 3

- We may first analyze the sequence of trading apples

- If we fix the number of red and green apples we trade
- E.G. 4 red apples and 3 green apples

- Does the order of trading affect the result?

Solution 3

- Example :

- RRRGGGR and RGRGRGR
- Are they the same??

- They can achieve the same happiness :
- 4 * P + 3 * Q + original
- But, the feasibility of them may be different

Solution 3

- For example : the sample input
- 6 4 3 1 1 3 2 6
- RGR is feasible
- RRG is not feasible

- So, the order of trading the apples is important, right?
- We need to exhaust all order of trading, right?

Solution 3

- Yes, it is important, but we don't need to exhaust all order!!!!

- The fact is : If a combination is not feasible in both RRRRRGGGG and GGGGGRRRRR (Trade all Red apples first or vice versa)
- Then the combination is not feasible in any order!!!!!

- Why??

Solution 3

- Recall our observation first :
- Observation : Each time we trade an apple, the total number of seeds will decrease at least 1
- Moreover, the numbers of both kind of seeds will not increase when we trade an apple. (Maybe decrease or remain unchange)

Solution

- RGGGRRGGGR
- 1 23 4
- Therefore, If we can trade the 4th red apple, we can trade the 1st, 2nd, 3rd, red apple before as well
- First assume the last unit of red apple can be trade, then why not put all green apple transaction at the front to maximize the opportunity to complete the trade

Solution 3

- RGGGRRGGGR -> GGGGGGRRRR
- As the numbers of both kinds of seeds are more at first, so putting G at front can increase the possibility to complete the trade

- Undoubtedly, we should also assume the last unit we trade is a green apple as well

Solution 3

- We can only exhaust the numbers of red apple and green apple we trade and calculate is it feasible.
- How to calculate?
- RRRRRGGGG
- We can only calculate if the last G and last R can be traded due to the observation above
- So, we can find the number of seeds we use and we get before trading the last G/R in order to check the feasibility

Solution 3

- RRRRRGGGG
- Number of red seeds we need before the last red apple = W * 4
- Number of red seeds we get = 4
- Number of green seeds we need = X * 4
- Number of green seed we get = 0
- If (R + 4– (W * 4) - W >= 0) and (G + 0 – X * 4 – X >= 0)
- It is feasible
- Don’t forget to check the last green apple as well

Solution 3

- Time complexity :
- The Max. time of red apple and green apple we trade = 2 * R/ 2 * G
- Time complexity = O(R * G)

- Score : 65

Solution 4

- We can further improve solution 3
- After we exhaust the number of red apples we trade
- We can calculate the number of green apples directly instead of exhausting it

Solution 4

- E.g. the remaining red seeds and green seeds after trading "i" red apples is 6 and 3 where Y = 2 and Z = 2
- We can directly calculate we can at most trade 2 green apples by :
- 6 / 2 = 3
- 3 / (2 - 1) – (3 mod (2 - 1) = 0) = 2
- Min(3, 2) = 2

Solution 4

- Therefore, the algorithm becomes :
- Exhaust the red apples we trade: O(R)
- For each number of red apples, calculate the maximum number of green apples we can trade by the remaining seeds : O(1)
- Do the same thing again but exhaust the green apples this time : O(G) * O(1)
- Total time complexity O(max(R, G))
- Expected Score : 100

- Any Question??