

Unfair Santa Claus

Solution

The final absolute difference can never be greater than n . This can be shown by induction: if the current absolute difference does not exceed n , after giving a chocolate piece with at most n bars to the one who has less chocolate bars, the absolute difference is still bounded by n . Therefore Alice can receive at most n chocolate bars more than Bob after all.

To achieve the difference n , we must give out the n -bar chocolate piece at last. Right before that, Alice and Bob should have exactly same total number of chocolate bars. However, $1 + 2 + \dots + n - 1$ is not always divided by 2. Which is the case when the remainder of n divided by 4 equals 2 or 3. Then it is not possible to distribute the 1 to $n - 1$ -bar chocolate to Alice and Bob evenly. Therefore, the maximum difference upper bound reduced to $n - 1$ in this case.

The rest is to find a sequence results in the maximum difference. There are many approaches, we give one here:

When the remainder of n divides 4 = 0 or 1: $n-1$ $n-2$ $n-3$... 3 2 1 n

When the remainder of n divides 4 = 2: $n-1$ $n-2$ $n-3$... 3 2 n 1

When the remainder of n divides 4 = 3: $n-1$ $n-2$ $n-3$... 3 1 2 n