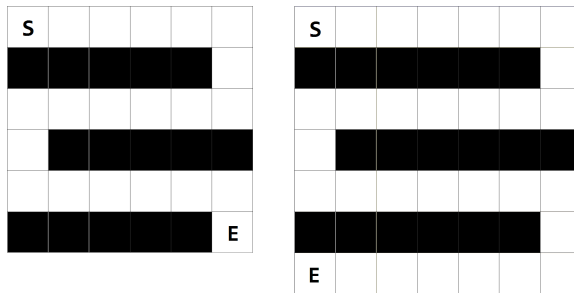# Safe Maze

Solution

Instead of trying to construct a 'maze', actually we are finding a 'path' that meets the requirement. Assume that the entrance is always placed at the upper left corner, and the exit is placed at one of the lower corners. First we try to construct a long path. If we walk as a 'S' shape such as:

Then the path length is always at least $0.5N^2$ using one of the corners. To shorten the path, we do not have to do the 'S shape walk' until the end. We can stop in the middle and build a shortest path to the exit directly.
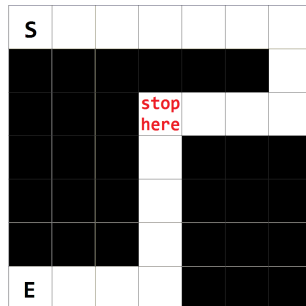
Using this method, the shortest length that can be constructed is $N - 1$, then it is $N + 1, N + 3, N + 5 \cdots$. That is, the length increased by 2 continuously until $0.5N^2$. To see that this is true, we will illustrate the idea assuming the exit is located at the lower left corner. When we are moving right during the 'S shape walk', if we stop the 'S shape walk' one step later, the maze length is increased by 2. When making turns and moving left, the maze length keeps the same. Then it increase again when we move right. The maze length increase by 2 continuously.