HKOI 2013 SENIOR

Bacteria Research

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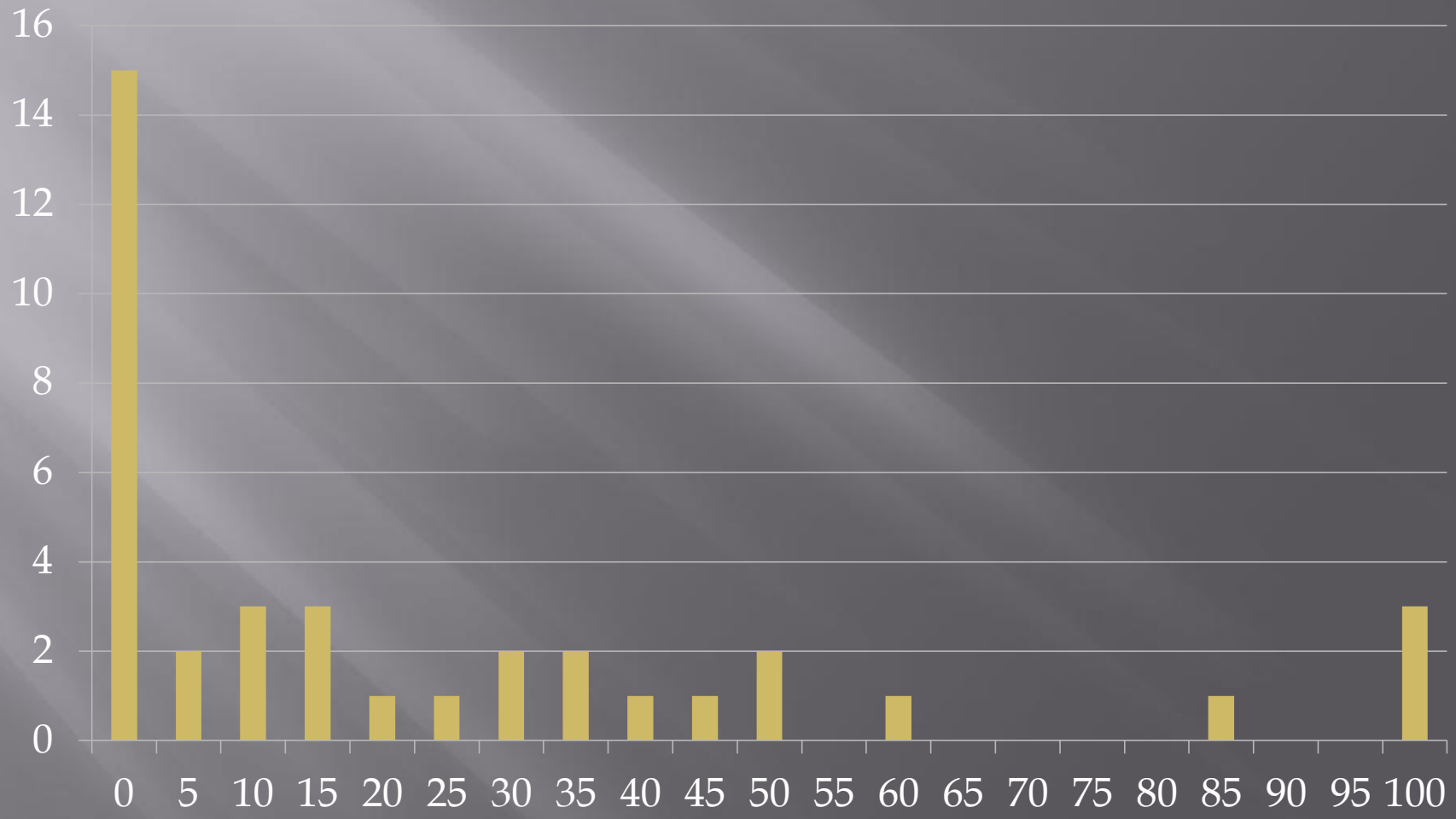
Statistics


16
14
12
10
8
6
4
2
0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100

Problem

- Given 2 groups of N 2-digit integers each
- Match them in a way such that GCD is greatest

Solution 1

- Try all possible matching
- Determine the GCD
- Complexity = O(N! * time to find GCD)
- Score <= 30

- How to determine the GCD?

Solution idea

- Fix GCD (say k)
- Determine whether a matching exists, where each number is divisible by k

Observation 1

- If k >= 100
- We can determine the last 2 digits given a fixed first 2 digits
- e.g. k = 479, given the first 2 digits are 62. We know the last 2 digits must be 27

Solution 2

- Iterate k from max possible GCD (say 9999) down to 100
- For each “first 2 digits”, find if there still exists a “last 2 digits” which is divisible by k
- If every “first 2 digits” can be matched, then k is the optimal answer
- Complexity = O(max GCD * N^2)
- Score ~ 40

Observation 2

- There can only be 100 different value of 2 digit number
- We can reduce N to 100 by counting the frequency of each occurrence

Solution 3

- Applying observation 2 to solution 2
- Complexity = max GCD * 100^2
- Score = 65

Solution 4

- What if k < 100? How to determine whether a matching is possible?
- e.g. k = 36, given "first 2 digits" is 34. The "last 2 digits" can be 20, 56, 92
- Which one of 20, 56, 92 should be used?
- Answer = anyone!!
- Why?

Solution 4 (cont.)

- Argument:
- If only some of “last 2 digits” can be used, then there exists at least a “first 2 digits” which contains only a subset of the “last 2 digits”
- However, given x*100+y is divisible by k, x*100 + y + d*k is also divisible by k (d can be negative, 0 <= y + d*k <= 99)
- Hence, there does not exist a “first 2 digits” which contain only a subset of “last 2 digits”

Solution 4 (cont.)

- Therefore, the solution is simple
- We can greedily match the digits even if k < 100
- Complexity = max GCD * 100^2
- Score = 100