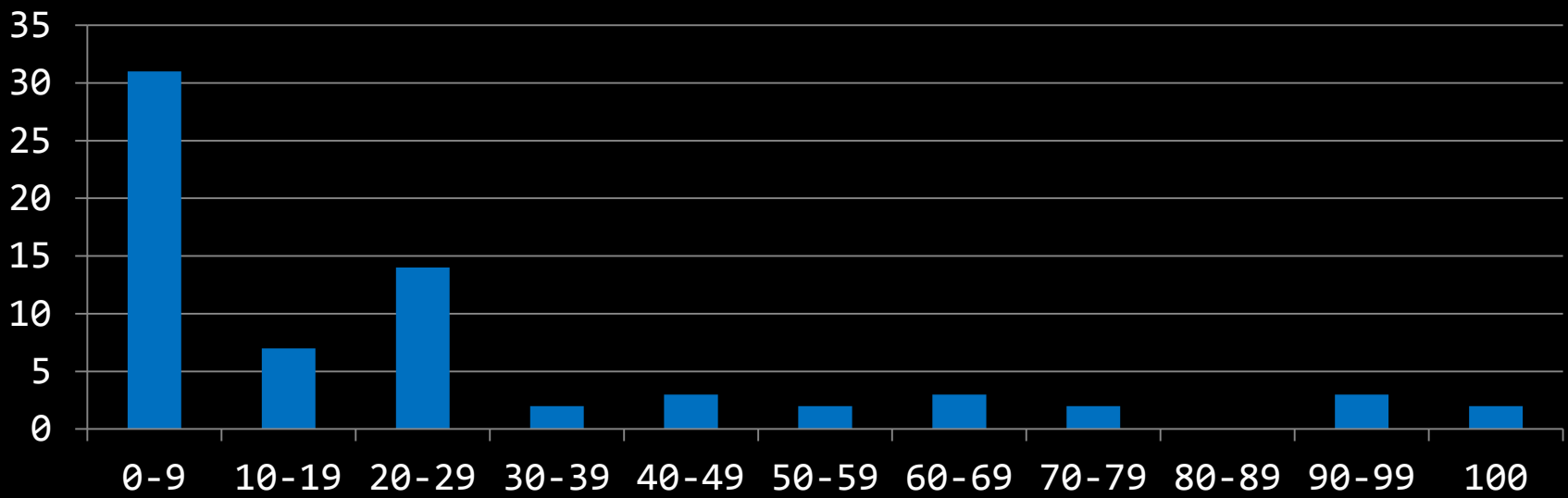


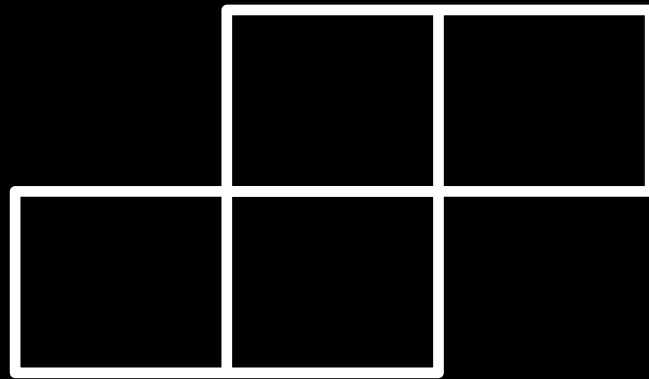
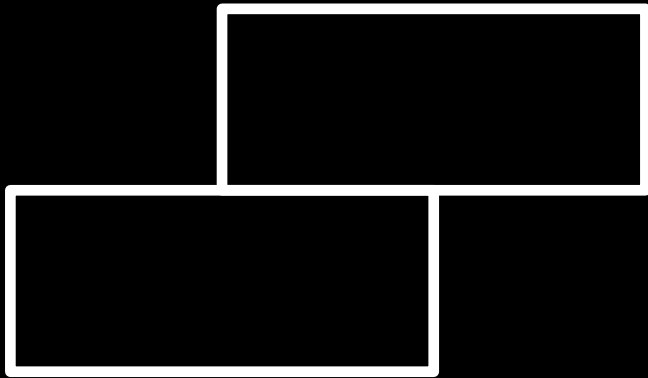
Toothpicks

HKOI 2012 Senior Q2



Problem

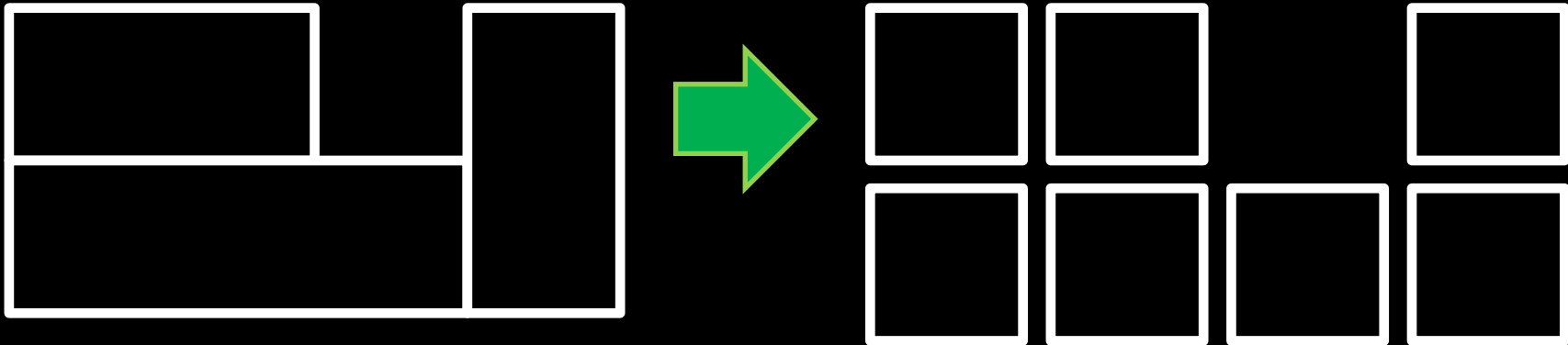
- N non-overlapping rectangles
- Using toothpicks to form the shape



Solution

- Lets view it in a simple way

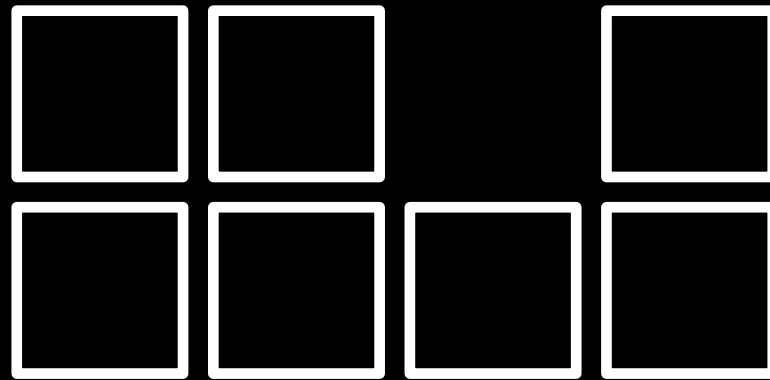
Break to 1x1
blocks



Compute the perimeters
of the squares

Solution

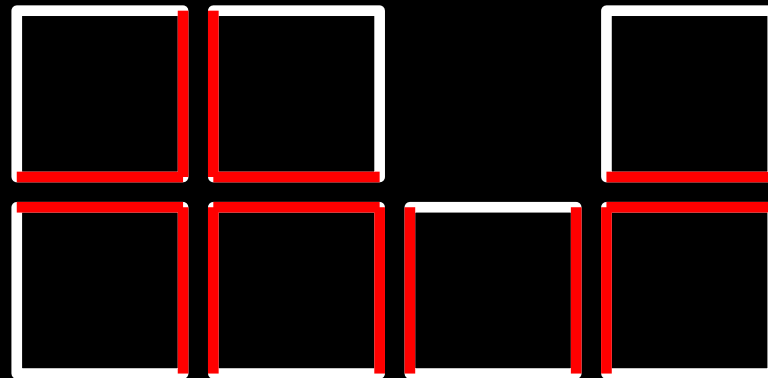
- Compute the perimeter



$4 \times (\text{number of squares}) ?$

Solution

- Compute the perimeter

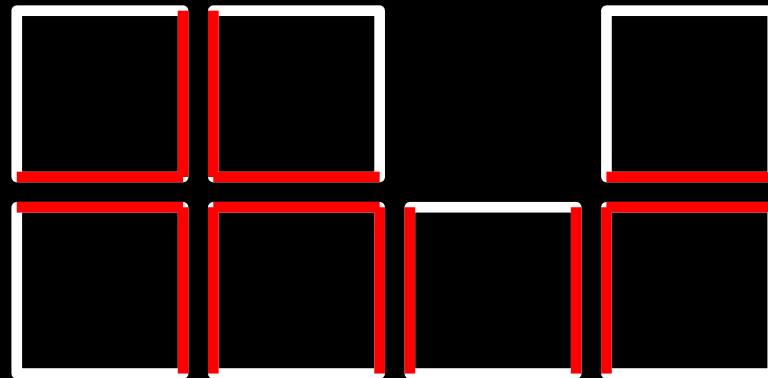


Overlapped.. We counted twice

$4 \times (\text{number of squares}) ?$

Solution

- Compute the perimeter

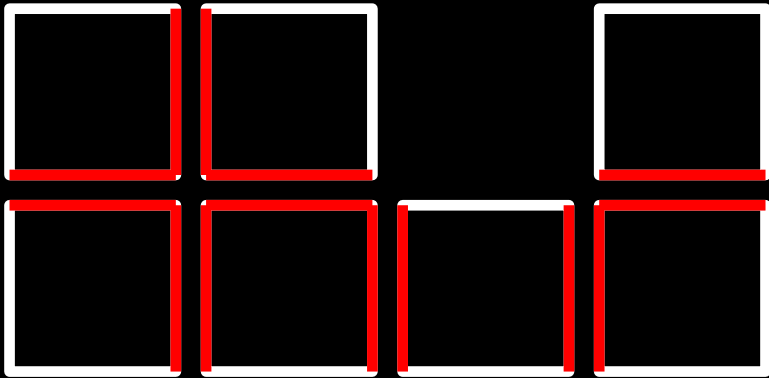


Overlapped.. We counted twice

$4 \times (\text{number of squares}) - \text{Overlapped lines}$

Solution

- Counting overlapped length



XX.X

XXXX

Draw the squares on
a 'map'

A 2D array!

Solution

- Counting overlapped length

XX.X

Count how many adjacency 'x's!

XXXX

XX.X

XX.X

XX.X

XXXX

XXXX

XXXX

XX.X

XX.X

XX.X

XX.X

XXXX

XXXX

XXXX

XXXX

Solution

- How about negative coordinates?

Pascal:

Easy.

```
var a:array[-100..100][-100..100] of boolean;
```

C/C++?

Shift all rectangles by
adding 100 to them!

Algorithm 1

- Break the rectangles to 1x1 squares
- Draw the squares on a map
- Count the overlapped length

Time complexity?

Number of 1x1 squares

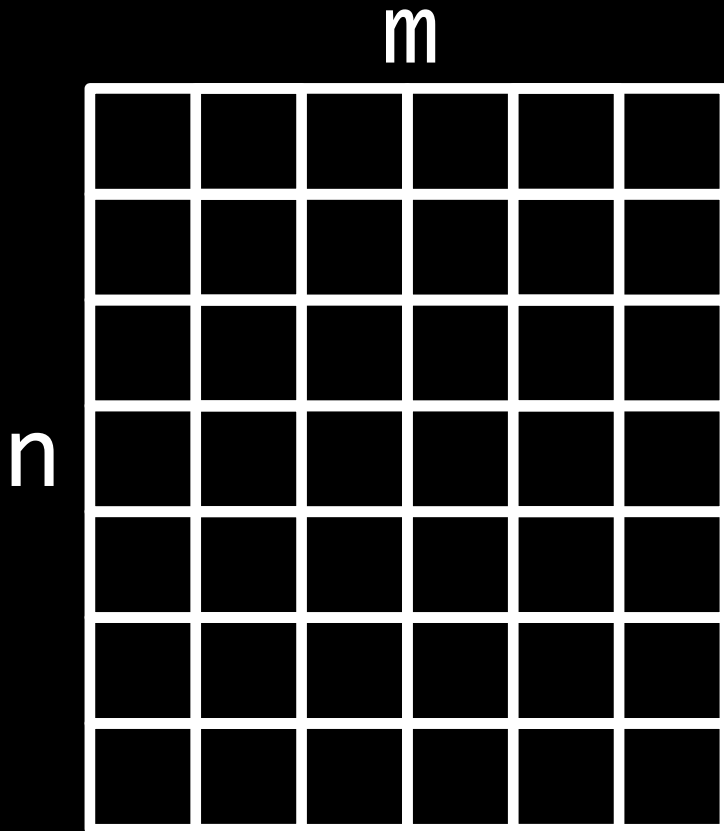
Depend on size of rectangles

Work for 50% test cases

In 50% test cases, absolute values of coordinates is at most 100

Speed up

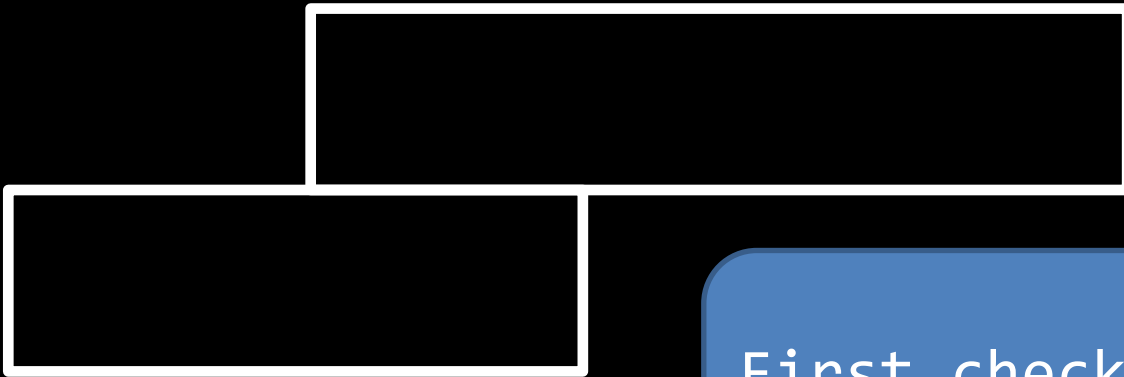
- Do the easy part first - fill the inner part of the rectangle



Number of toothpicks
needed for a $n \times m$
rectangle = $2nm + n + m$

Speed up

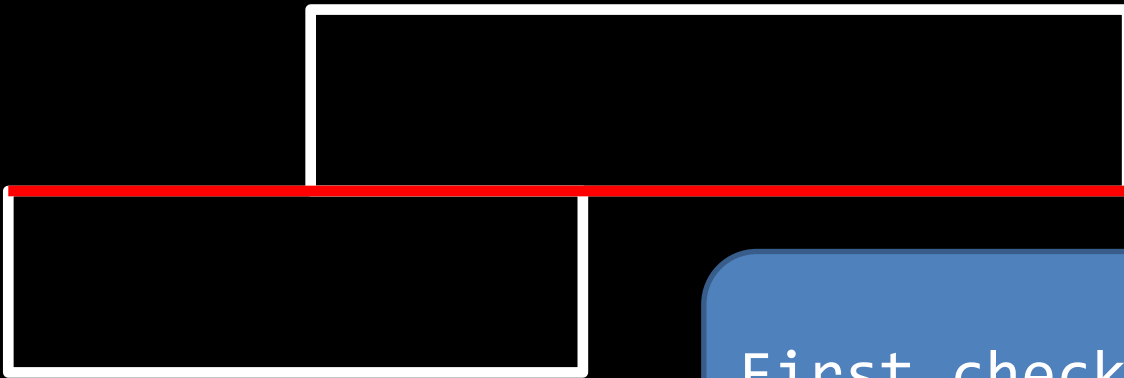
- How about the overlapped length?



First check which lines are overlapping..

Speed up

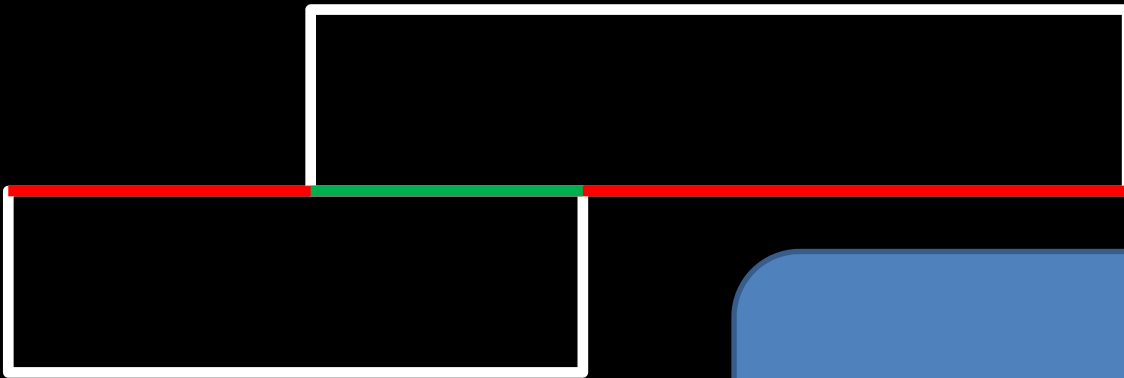
- How about the overlapped length?



First check which lines are overlapping..

Speed up

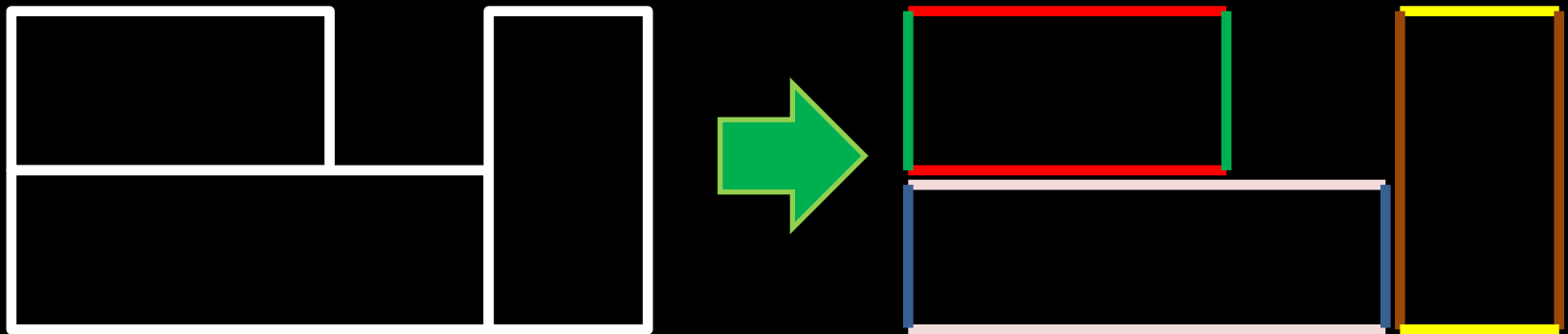
- How about the overlapped length?



Compute the overlapped length

Solution

- Break each rectangle into 2 vertical line and 2 horizontal line



For each pair of line, compute their overlapped length

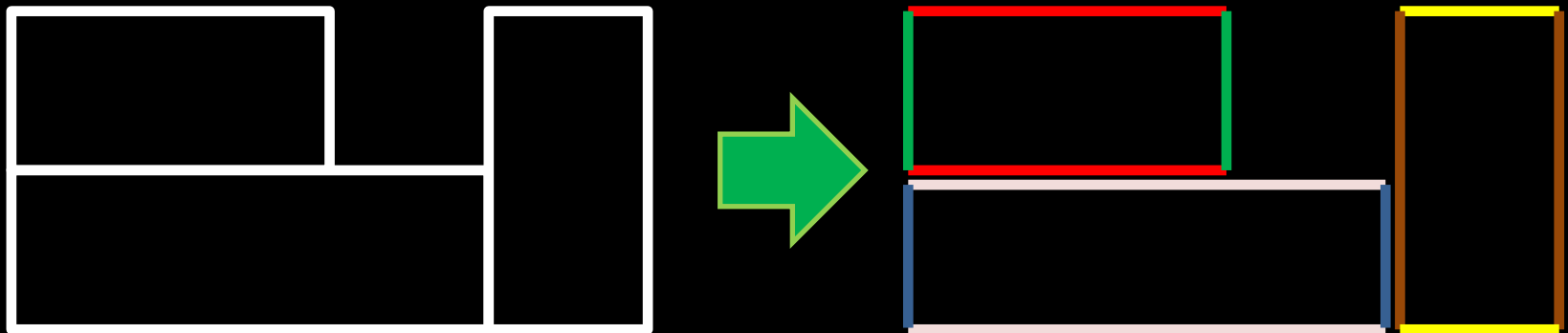
Algorithm 2

- Sum the toothpicks needed for inner part
- Break N rectangles to $2N$ horizontal lines and $2N$ vertical lines
- For each pair of lines, compute their overlapped length (if any), subtract it from answer

Time complexity: $O(N^2)$

Extra – We can do faster

For each pair of line, compute their overlapped length



Do we really need to go through all pair of lines?