HKOI2010 Senior

Flexibility Solution

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whh

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Stat:

#Attempts	44
Maximum	100
#Maximum	3
Mean (Attempted)	23.4
Minimum	0
Std Dev (Attempted)	26.5

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Problem Statement :

Given W and N masses a[1],a[2] … , a[N]

1) put some masses on the balance

2) weight on the left + W = weight on the right

3) a[i] * 3 <= a[i+1]

4) Constraints

100% : $1 \le N \le 35$, $1 \le Ai \le 2^{60}$

50% : $1 \le N \le 10$, $1 \le Ai \le 2^{30}$

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50% Solution

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```
flex(int k,int L,int R){
    If k>N return;
    If  L=R    done =]
    flex(k+1, L, R);
    flex(k+1, L+a[k] , R);
    flex(k+1, L, R+a[k]);
}
```

*** flex(1,W,0) ***

Complexity : O(3^N)

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100% Solution

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a[i] * 3 <= a[i+1] ?

Consider a[k] and a[k-1]+a[k-2]+...+a[1]

a[k-1]+a[k-2]+...+a[1]
<=a[k]/3 + a[k]/9 + a[k]/27+ ....
<=(a[k]/3) / (1-1/3)
=a[k]/2

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1) We have a[k], a[k-1], … , a[1]
2) D is the different between the weight of the balance on the two sides

Consider masses in descending order of their weight

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Case 1: $D > a[k] + a[k]/2$ (D is too large)

$D > a[k] + a[k]/2$

$\geq a[k] + a[k-1] + a[k-2] + \ldots + a[1]$

Hence, it is impossible !

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Case 2: $D > a[k]$ (D is large)

If we ignore a[k],

$D > a[k] > a[k]/2$

$>= a[k-1] + a[k-2] + \ldots + a[1]$

Hence,

1) $a[k-1] + a[k-2] + \ldots + a[1]$ can not generate a difference equal to D

2) Use a[k] to reduce D ! new $D' = D-a[k]$

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Case 3: $D = a[k]$

Of course, use a[k] !

New $D' = D - a[k] = 0$

Done !

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Case 4: D > a[k]/2 (D is small)

If we ignore a[k],

D > a[k]/2

>= a[k-1] + a[k-2] + … + a[1]

Hence,

1) a[k-1] + a[k-2] + … + a[1] can not generate a difference equal to D

2) Use a[k] , new D' = a[k]-D

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Case 5: $D < a[k]/2$ (D is very small)

If we use a[k],

new $D' = a[k] - D > a[k] - a[k]/2$

$\geq a[k-1] + a[k-2] + \ldots + a[1]$

Hence,

1) $a[k-1] + a[k-2] + \ldots + a[1]$ can not generate a difference equal to D

2) ignore a[k] , new $D' = D$

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Complexity : O(N)

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Thank you