Zoo Planning HKOI 2007 Junior Question 1

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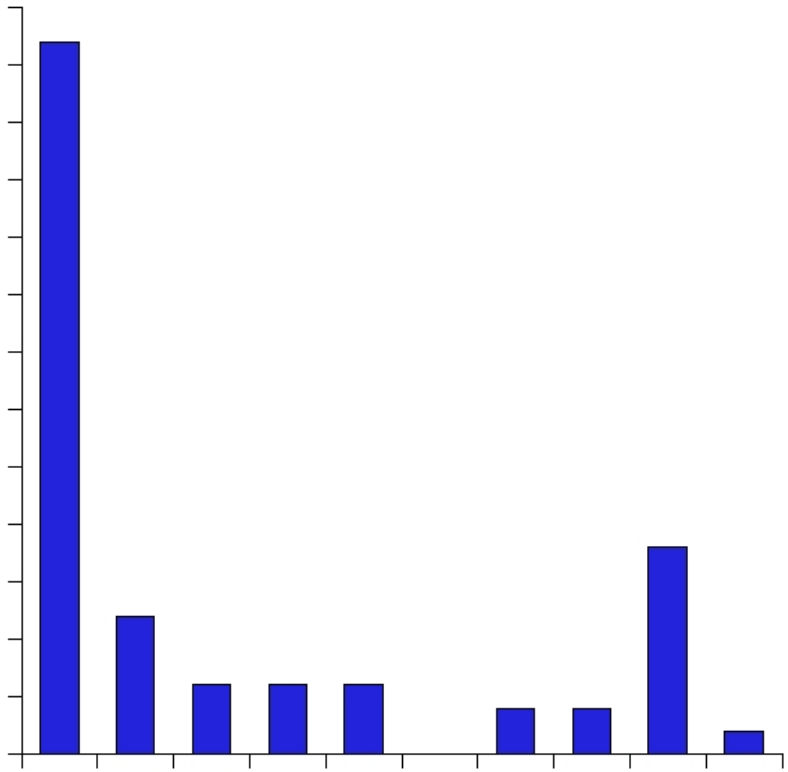
January 13, 2006

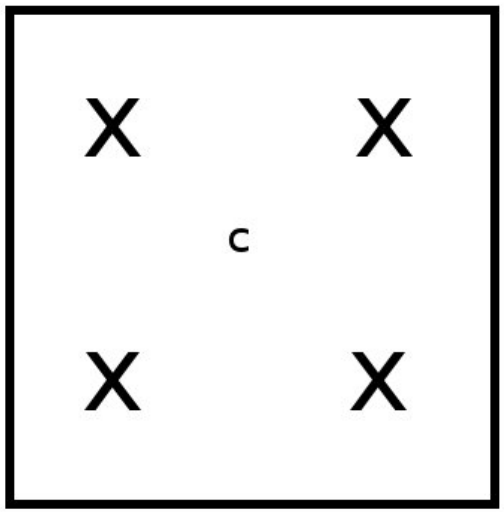
Statistics

Summary

# Attempts	60
Max Score	90
Min Score	0
Mean	40.82
Median	0
Mode	0



Problem



Story

Build the largest zoo with a fountain at the center

Constraints

100% Constraints
$1 \leq W, H \leq 10^7, 1 \leq K \leq 3000$
50% Constraints
$1 \leq W, H \leq 10^3, 1 \leq K \leq 1000$

Naïve Solutions

$O(KW^2H^2)$ Solution Expected Score:20

Pseudocode of NaïveSolution #1

for $i = 1$ to W **do**
  for $j = 1$ to H **do**
    for $a = 1$ to W **do**
      for $b = 1$ to H **do**
        if only one cross in the rectangle (i+1,j+1) , (a-1,b-1) **then**
          update the maximum area when neccessary
        end if
      end for
    end for
  end for
end for
output the maximum area

Observations

Observation #1

Optimization can be indepedent of the size of the map(W, H)

Naïve Solutions (Continued)

Improving Naïve Solution #1
$O(K^3)$ Solution Expected Score:50

Pseudocode of Naïve Solution #2

for $i = 1$ to K **do**
  for $j = i + 1$ to K **do**
    $x_1 \leftarrow Crosses[i] \rightarrow x$
    $y_1 \leftarrow Crosses[i] \rightarrow y$
    $x_2 \leftarrow Crosses[j] \rightarrow x$
    $y_2 \leftarrow Crosses[j] \rightarrow y$
    if only one cross in the rectangle $(x_1 + 1, y_1 + 1), (x_2 - 1, y_2 - 1)$ **then**
      update the maximum area when neccessary
    end if
  end for
end for
output the maximum area

Efficient Solutions

Redefining the problem

Change the problem into "For a given cross (x_i, y_i), what are a, b, c, d s.t.
a=min$\{x_i \ s.t. \ x < x_i\}$
b=min$\{y \ s.t. \ y < y_i\}$
c=max$\{x \ s.t. \ x > x_i\}$
d=max$\{y \ s.t. \ y > y_i\}$"

Efficient Solutions (Continued)

$O(K^2)$ Solution Expected Score:100

Pseudocode of Efficient Algorithm #1

for $i = 1$ to K **do**
  a=min$\{x \ s.t. \ x < x_i\}$
  b=min$\{y \ s.t. \ y < y_i\}$
  c=max$\{x \ s.t. \ x > x_i\}$
  d=max$\{y \ s.t. \ y > y_i\}$
  update the maximum area when neccessary
end for
output the maximum area

Efficient Solutions (Continued)

Improving efficient algorithm #1 by using binary search
$O(K^2)/O(KlgK)$ Solution Expected Score:100

Pseudocode of Efficient Algorithm #2

sort(X)
sort(Y)
for $i = 1$ to K **do**
  a=min$\{x\ s.t.\ x < x_i\}$
  b=min$\{y\ s.t.\ y < y_i\}$
  c=max$\{x\ s.t.\ x > x_i\}$
  d=max$\{y\ s.t.\ y > y_i\}$
  update the maximum area when neccessary
end for
output the maximum area

Observations (Continued)

Observation #2

Optimization of width and height are two independent processes.

$\therefore$ All roads are either running from the north boundary of the city to the south boundary or running from the west boundary to the east boundary.

Efficient Solutions (Continued)

By observation #2, the maximum width and maximum height can be determined separately.

Redefining the problem again

What are the maximum height and maximum width?

Efficient Solutions (Continued)

$O(K)$ Solution Expected Score:100

Pseudocode of Efficient Solution #3

remove duplicated x_i
remove duplicated y_i
sort X
sort Y
width $\leftarrow \max\{y_i - y_{i-2} - 1 \quad s.t.\ 3 \leq i \leq |Y|\}$
height $\leftarrow \max\{x_i - x_{i-2} - 1 \quad s.t.\ 3 \leq i \leq |X|\}$
output the maximum area

Common Mistakes

- Trivial Case
- Boundary Case
- Using too many memeories

Question?

No, of course.