Competition

Chan, Siu On

January 8, 2005

Roadmap

- Problem
- Statistics
- 50% solution
- Greedy solution

Problem

- 田忌賽馬
- Two teams of students take part in a one-on-one competition
- Find the maximum number of rounds which Team A can win

Statistics

- 15 full marks
- Highest mean
- Highest standard deviation

50% solution

- There are $N!$ ways of assigning students from Team A
- Enumerate them one by one and count the number of winning rounds
- When $N \leq 10$, it takes at most $10! = 3628800$ iterations
- Implementation: recursion
- Enough to score 50 marks

Greedy solution

Idea

Keep assigning the strongest student from Team A to beat the student from Team B who is just weaker than him/her

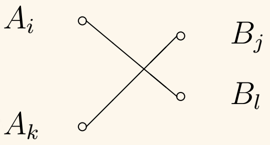
Algorithm

1. Find the strongest student from Team A, call him/her A_i
2. Find the student from Team B who is just weaker than A_i, call him/her B_j
3. If B_j does not exist, terminate
4. Increase counter, remove A_i and B_j from consideration, iterate

Proof of correctness

Let A_i be the strongest student from Team A
Let B_j be the student from Team B who is just weaker than A_i
Assume in an optimal assignment S, A_i competes with B_l and A_k competes with B_j
Further assume B_l and B_j are different students

Optimal assignment S



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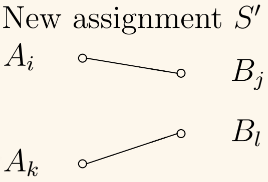
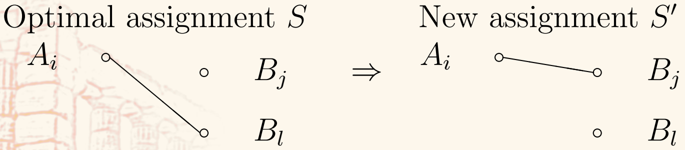


What happens if A_i and A_k exchange their competitors?

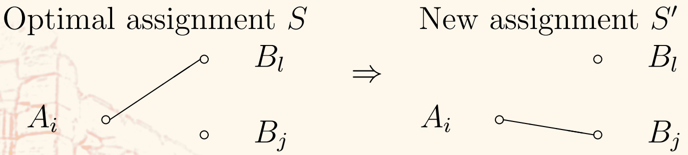
First case: $A_i > B_l$

- $A_i > B_l$ means A_i is stronger than B_l
- By swapping competitors, A_i can still win a round in S'
- A_k now competes with $B_l \leq B_j$, and the result will not be worse



Second case: $A_i \leq B_l$

- A_i and A_k together win at most one round in S
- A_i and A_k together win at least one round in S'



Conclusion

- There always exists an optimal assignment in which the strongest student from Team A competes with the one from Team B who is just weaker
- Clearly an $O(N^2)$ implementation is possible
- $O(N \log N)$ implementations also exist
- Keep it simple, stupid (KISS)