

Paper 1 (Compulsory Part) 卷一（必答部分）

	Format 類型	# Questions 題目數	Total Marks 佔分
Section A 甲部	Multiple Choice 多項選擇題	10	10
Section B 乙部	Short Questions 短答題	6 (A - F)	9
Section C 丙部	Long Questions 長填充題	2 (G - K)	6
Total 總分			25

- (1) The term “arrays” in Paper 1 generally refers to lists in Python.
卷一提及的「陣列」泛指 Python 語言的串列。
- (2) The following code is added to the beginning of all C++ programs.
在所有 C++ 程序的頂部加入以下程式碼:

C++

```
#include <algorithm>
#include <array>
#include <cmath>
#include <cstdlib>
#include <deque>
#include <forward_list>
#include <iostream>
#include <list>
#include <queue>
#include <stack>
#include <string>
#include <utility>
#include <vector>
using namespace std;
```

- (3) Assume all programs shall be compiled and executed in Debian 12 using the compilers and commands below.

假設所有程序都將在 Debian 12 下使用以下編譯器及指令進行編譯，然後執行。

GNU G++ (g++-12 12.2.0)

```
g++ -std=c++20 program.cpp -o program
./program
```

Python 3 (python3.11 3.11.2)

```
python3.11 -S -m py_compile program.py
python3.11 -O -S program.py
```

Section A 甲部 (10 marks 分)

For each question, choose the **most appropriate** answer and mark the corresponding box (A, B, C, or D) on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.

請為下列每題各選一個**最適合**的答案，然後把答題紙對應的空格（A、B、C、或 D）填滿。

答對得一分，答錯不扣分。

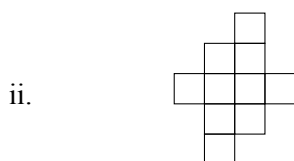
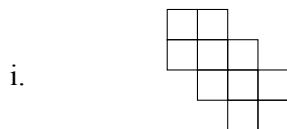
1. Alice has an infinite amount of identical 1×2 tiles shown as below:

愛麗絲有無限個相同的 1×2 方塊，如下圖所示：



If all tiles can be rotated, which of the following shapes can be completely covered by tiles without any tiles extending outside of the shape or overlapping with each other?

若所有方塊都可以被旋轉，以下那些圖形能在沒有方塊凸出圖形外或互相重疊的情況下被完全覆蓋？



- A. i only 只有 i
- B. iii only 只有 iii
- C. ii and iii only 只有 ii 和 iii
- D. None of them 無
2. Which of the following statements about sorting algorithms is true?
- 以下哪個有關排序算法的描述為真？
- A. Selection Sort selects the median element and splits the array into two halves every iteration.
選擇排序每次迭代都選擇中位數元素，並將陣列分成兩個部分。
- B. Insertion Sort inserts the first unsorted element into the frontmost position of the sorted section.
插入排序將第一個未排序的元素插入到已排序區域的最前面位置。
- C. Merge Sort merges sorted subarrays by repeatedly taking the frontmost element from either subarray.
合併排序通過反覆從兩個子陣列中選取最開頭的元素來合併已排序的子陣列。
- D. Quicksort compares adjacent elements quickly and repeatedly swaps them until the array is sorted.
快速排序快速比較相鄰元素，並反覆交換它們，直到陣列排序完成。

3. Alice, Bob, and 10 other students participated in both Chinese and English exams. After the exam, their teacher made the following claim: “All students who got 50 marks or above in the Chinese exam also got 50 marks or above in the English exam.”

愛麗絲、鮑伯和另外 10 個學生參加了中文和英文兩場考試。考試後，他們的老師作出了以下聲言：「所有在中文考試中取得 50 分或以上的學生，在英文考試中也取得 50 分或以上。」

Which of the following students can use only their exam papers to prove or disprove the teacher's claim?

哪個／哪些學生僅用自己的試卷便能證明或反證上述老師的聲言？

Student 學生	Score of Chinese exam 中文考試分數	Score of English exam 英文考試分數
Alice 愛麗絲	60	60
Bob 鮑伯	80	40

- A. Alice only 只有愛麗絲
B. Bob only 只有鮑伯
C. Alice and Bob 愛麗絲和鮑伯
D. None of them 無
4. Which of the following statements about undirected, simple graphs (i.e. no self loop and no duplicate edge) are true?

對於以下關於無向簡單圖（即沒有自環且沒有重複邊）的陳述，有哪些是正確的？

- i. A graph of 5 nodes must have at least 4 edges.
一個有 5 個節點的圖必須至少有 4 條邊。
ii. A graph of 5 nodes must have at most 10 edges.
一個有 5 個節點的圖至多只能有 10 條邊。
iii. A graph with no edges must not be a connected graph.
一個沒有邊的圖一定不是連通圖。

- A. i only 只有 i
B. ii only 只有 ii
C. iii only 只有 iii
D. None of them 無
5. There are 10 numbers arranged from left to right in a row. A number is *interesting* if it is larger than all numbers to the left of it, and a number is *exciting* if it is larger than or equal to all numbers to the right of it. How many numbers are both *interesting* and *exciting*?

有 10 個數字從左到右排成一列。如果某個數字大於它左邊所有的數字，則稱它為 *有趣* 的；如果某個數字大於或等於它右邊所有的數字，則稱它為 *刺激* 的。有多少個數字同時既 *有趣* 又 *刺激*？

- A. 0
B. 1
C. 2
D. Cannot be determined 無從判斷

6. Alice has two data structures R and S , both of which are either a stack or a queue. Initially, both data structures are empty. The following operations are then executed in order on R and S :

愛麗絲有兩個數據結構 R 和 S ，它們皆為堆疊（棧）或隊列。最初，兩個數據結構都是空的。接下來，在 R 和 S 上按順序執行以下操作：

- | | |
|---|---------------------------|
| 1. Push 1 into R . | 將 1 推入 R 。 |
| 2. Push 2 into R . | 將 2 推入 R 。 |
| 3. Push 3 into R . | 將 3 推入 R 。 |
| 4. Pop an element from R and push this element into S . | 從 R 彈出一個元素並將該元素推入 S 。 |
| 5. Pop an element from S . | 從 S 彈出一個元素。 |
| 6. Pop an element from R and push this element into S . | 從 R 彈出一個元素並將該元素推入 S 。 |
| 7. Pop an element from R and push this element into S . | 從 R 彈出一個元素並將該元素推入 S 。 |
| 8. Pop an element from S . | 從 S 彈出一個元素。 |
| 9. Pop an element from S . | 從 S 彈出一個元素。 |

It is known that the order in which values are popped from S is 3, 1, 2. What are the data structures R and S ?

如果從 S 彈出的值的順序是 3、1、2，那麼 R 和 S 是什麼數據結構？

- | | | |
|----|----------------|----------------|
| A. | R : Queue 隊列 | S : Queue 隊列 |
| B. | R : Queue 隊列 | S : Stack 棧 |
| C. | R : Stack 棧 | S : Queue 隊列 |
| D. | R : Stack 棧 | S : Stack 棧 |

7. A palindrome is a string that remains the same when read forward or backward. For example, ABBA is a palindrome but ABAB is not.

迴文字串是正讀反讀都一樣的字串。舉例來說，ABBA 是迴文字串，ABAB 則不是。

We denote $s[l..r]$ to be the substring of s from the l -th letter to r -th letter. For example, if s is ABBAB, then $s[3..5]$ is BAB.

我們以 $s[l..r]$ 來表示字串 s 中從第 l 個字元到第 r 個字元的子字串。舉例來說，假如 s 是 ABBAB，則 $s[3..5]$ 為 BAB。

Given a palindrome s of length 8. Which of the following statements must be true?

給定一個長度為 8 的迴文字串 s 。以下哪項陳述必定為真？

- | | | |
|-----|--|--------|
| i. | If $s[1..4]$ is palindrome, then $s[5..8]$ is also palindrome. | |
| | 若 $s[1..4]$ 為迴文字串，則 $s[5..8]$ 亦為迴文字串。 | |
| ii. | If $s[1..3]$ is palindrome, then $s[4..8]$ is also palindrome. | |
| | 若 $s[1..3]$ 為迴文字串，則 $s[4..8]$ 亦為迴文字串。 | |
| A. | i only | 只有 i |
| B. | ii only | 只有 ii |
| C. | i and ii | i 和 ii |
| D. | None of them | 無 |

8. Consider the following program: 考慮以下程序：

Python

```
remain = initial value 初始值
p = [10, 6, 4, 5, 2, 3]
v = [False for _ in range(6)]
for i in range(6):
    if remain >= p[i]:
        remain -= p[i]
        v[i] = True
```

C++

```
int main(){
    int remain = initial value 初始值;
    vector<int> p = {10, 6, 4, 5, 2, 3};
    vector<bool> v(6, false);
    for (int i = 0; i < 6; ++i) {
        if (remain >= p[i]) {
            remain -= p[i];
            v[i] = true;
        }
    }
}
```

The initial value of remain is between 0 and 1000. After executing the program segment, which of the following statements is always true for any initial value?

remain 的初始值在 0 到 1000 之間。執行該程序段後，對於任意初始值，以下哪項陳述必定為真？

- i. If v[1] and v[2] are both False (C++: false), v[0] must be False (C++: false).
如果 v[1] 和 v[2] 都為 False (C++: false)，v[0] 必定為 False (C++: false)。
- ii. If v[4] and v[5] are both True (C++: true), v[3] must be True (C++: true).
如果 v[4] 和 v[5] 都為 True (C++: true)，v[3] 必定為 True (C++: true)。

- A. i only 只有 i
- B. ii only 只有 ii
- C. i and ii i 和 ii
- D. None of them 無

9. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
def f(n: int) -> None:
    if n:
        f(n - 1)
        print(n, end=' ')
        f(n - 1)

f(4)
```

C++

```
void f(int n) {
    if (n) {
        f(n - 1);
        cout << n << ' ';
        f(n - 1);
    }
}

int main() {
    f(4);
}
```

- A. 1 2 1 3 1 2 1 4 1 2 1 3 1 2 1
- B. 1 2 3 4 3 2 1
- C. 4
- D. The program does not terminate 該程序不會終止

10. Consider a cyclic network of 12 nodes as shown in the figure on the right. There are 6 communication requests (A, B, C, D, E and F), each consisting of two ends (nodes). For each request, you can either route the communication via the clockwise route or the anticlockwise route to connect the two ends.

考慮一個由 12 個節點組成的環狀網路，如右圖中所示。現有 6 個通訊請求（A、B、C、D、E 和 F），每個請求都包含兩個端點（節點）。對於每個請求，您可以選擇通過順時針或逆時針的路徑路由，以連接該兩個端點。

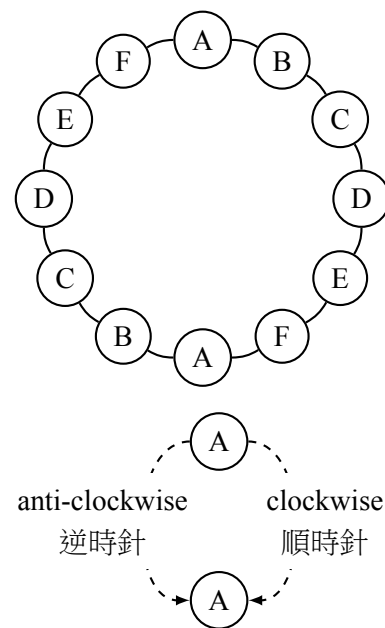
The *load* on an edge is the number of communication requests whose chosen route passes through that edge. The *maximum load* of the system is the maximum load of any edge. For example, when all requests are routed clockwise, the maximum load of the system is 6.

定義一條邊的 *負載* 為穿過該邊的通訊請求的數量。整個系統的 *最大負載* 為任意邊上的最大負載。例如，當所有請求都選擇順時針路徑時，系統的最大負載為 6。

What is the smallest possible maximum load of the system if the requests are routed optimally?

如果所有請求都以最佳方式路由，則系統最大負載的最小可能值是多少？

- A. 1
- B. 2
- C. 3
- D. 4



END OF SECTION A 甲部完

Section B 乙部 (9 marks 分)

The blanks are labeled from A to F. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 A 至 F，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Answers can be in either C++ or Python. Completed programs shall be compiled and executed according to the procedure specified on Page 1.
您可以使用 C++ 或 Python 作答。完成的程序將以第 1 頁之步驟進行編譯及執行。
- (2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。

1. There is a screen and two buttons in front of you. Initially, the screen shows the number 0. You can perform the following actions in any arbitrary order:

在你面前有一個螢幕和兩個按鈕。螢幕上最初顯示的數字是 0。你可以用任意次序執行以下操作：

- Click the red button: Increase the number on the screen by 1.
點擊紅色按鈕：螢幕上的數字會增加 1。
- Click the blue button: Multiply the number on the screen by 3.
點擊藍色按鈕：螢幕上的數字會乘以 3。

At least how many button clicks are required such that the screen shows the number 330?

至少需要點擊多少次按鈕，才能讓螢幕顯示 330 這個數字？

Answer 答案: A (1.5 marks 分)

2. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
a = 1
b = 1
c = 1

for i in range(2025):
    a ^= b
    b ^= c
    c ^= a

print(a, b, c, sep=",")
```

C++

```
int main() {
    int a = 1;
    int b = 1;
    int c = 1;
    for (int i = 0; i < 2025; ++i) {
        a ^= b;
        b ^= c;
        c ^= a;
    }
    cout << a << "," << b << "," << c;
}
```

Answer 答案: B (1.5 marks 分)

3. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
ans = 0
for i in range(56):
    ans += (i % 7) - (i % 8)
print(ans)
```

C++

```
int main() {
    int ans = 0;
    for (int i = 0; i < 56; ++i)
        ans += (i % 7) - (i % 8);
    cout << ans;
}
```

Answer 答案: _____ C _____ (1.5 marks 分)

4. 12 seats are arranged around a circular table, with each accommodating at most one person. In how many ways can people be seated, so that every 6 consecutive seats contains exactly 4 occupied seats? Two seating arrangements are considered different if and only if there is at least one seat that is occupied in one arrangement but empty in the other.

一張圓桌周圍有 12 個座位，每個座位最多容納一個人。有多少種坐法，使得每 6 個連續的座位之中，都恰好有 4 個有人坐的座位？當且僅當至少有一個座位在一種坐法中有人坐，但在另一種坐法中空著時，這兩種坐法才算不同。

Answer 答案: _____ D _____ (1.5 marks 分)

5. Given an array $A = [6, 0, -8, 4, 8, 6, -4, 2]$. How many pairs (L, R) are there, such that $0 \leq L \leq R \leq 7$ and sum of elements in the subarray $A[L], A[L + 1], \dots, A[R]$ is between 10 and 14, inclusive? For example, $(3, 4)$ is a valid pair, since the sum of $A[3]$ and $A[4]$ is 12, while $(1, 3)$ is not a valid pair, since the sum of $A[1], A[2]$ and $A[3]$ is -4 .

給定一個陣列 $A = [6, 0, -8, 4, 8, 6, -4, 2]$ 。有多少個數對 (L, R) 使得 $0 \leq L \leq R \leq 7$ 並且子陣列 $A[L], A[L + 1], \dots, A[R]$ 中元素的總和在 10 到 14（含）之間？例如， $(3, 4)$ 是一個有效的數對，因為 $A[3]$ 和 $A[4]$ 的總和是 12，而 $(1, 3)$ 不是一個有效的數對，因為 $A[1]$ 、 $A[2]$ 和 $A[3]$ 的總和是 -4 。

Answer 答案: _____ E _____ (1.5 marks 分)

6. Complete the program $f(a, b, c)$ so that, for integers $1 \leq a, b, c \leq 3$, it returns True (C++: true) if and only if a, b, c are either all the same or all different.

完成程式 $f(a, b, c)$ ，對於滿足 $1 \leq a, b, c \leq 3$ 的整數，當且僅當 a 、 b 、 c 全部相同或全部不同時回傳 True (C++: true)。

Python

```
def f(a: int, b: int, c: int) -> bool:
    return bool(_____ F _____ == 0)
```

C++

```
bool f(int a, int b, int c) {
    return bool(_____ F _____ == 0);
}
```

Answer 答案: _____ F _____ (1.5 marks 分)

END OF SECTION B 乙部完

Section C 丙部 (6 marks 分)

The blanks are labeled from G to K. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 G 至 K，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Answers can be in either C++ or Python. Completed programs shall be compiled and executed according to the procedure specified on Page 1.
您可以使用 C++ 或 Python 作答。完成的程序將以第 1 頁之步驟進行編譯及執行。
- (2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。
- (5) If blank X is divided into N parts X1, X2, ..., XN, it means that marks will only be given when X1, X2, ..., XN are all correct.
如果空格 X 分為 N 部份 X1、X2、...、XN，那麼 X1、X2、...、XN 皆為正確才會給分。

1. Consider the following program: 考慮以下程序：

Python

```
a = [
    [1, 2, 3, 4],
    [5, 6, 7, 8],
    [9, 10, 11, 12],
    [13, 14, 15, 16]
]

for i in range(4):
    for j in range(4):
        print(a[___ G1, H1 ___][___ G2, H2 ___],
              end=" ")

    print()
```

C++

```
vector<vector<int>> a = {
    {1, 2, 3, 4},
    {5, 6, 7, 8},
    {9, 10, 11, 12},
    {13, 14, 15, 16}
};

int main() {
    for (int i = 0; i < 4; ++i) {
        for (int j = 0; j < 4; ++j) {
            cout << a[___ G1, H1 ___][___ G2, H2 ___]
                  << " ";
        }
        cout << endl;
    }
}
```

(a) Complete the program so that it outputs the following:

完成程序使其輸出：

```
13 9 5 1
14 10 6 2
15 11 7 3
16 12 8 4
```

Answer 答案: _____ G1 _____ G2 _____ (1.5 marks 分)

(b) Complete the program so that it outputs the following:

完成程序使其輸出：

1 3 1 3

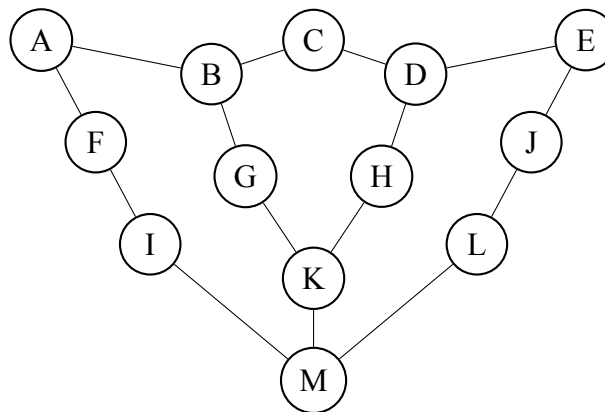
1 3 1 3

5 7 5 7

5 7 5 7

Answer 答案: H1 H2 (1.5 marks 分)

2. Consider the following undirected graph. 考慮以下無向圖。



There are multiple ways to walk from A to E in this graph. For example, $A \rightarrow B \rightarrow G \rightarrow K \rightarrow H \rightarrow D \rightarrow H \rightarrow D \rightarrow E$ is a walk that passes through 8 edges.

圖上有若干方法由 A 走到 E。例如， $A \rightarrow B \rightarrow G \rightarrow K \rightarrow H \rightarrow D \rightarrow H \rightarrow D \rightarrow E$ 是一個經過 8 條邊的走法。

(a) What is the minimum number of edges you must remove from the graph, such that it is impossible to walk from A to E passing through exactly 5 edges?

你至少需要從圖中移除多少條邊，使得不能從 A 經過恰好 5 條邊走到 E？

Answer 答案: I (1 mark 分)

(b) What is the minimum number of edges you must remove from the graph, such that it is impossible to walk from A to E passing through exactly 6 edges?

你至少需要從圖中移除多少條邊，使得不能從 A 經過恰好 6 條邊走到 E？

Answer 答案: J (1 mark 分)

(c) What is the minimum number of edges you must remove from the graph, such that it is impossible to walk from A to E passing through exactly 9 edges?

你至少需要從圖中移除多少條邊，使得不能從 A 經過恰好 9 條邊走到 E？

Answer 答案: K (1 mark 分)

END OF PAPER 1 卷一完

Paper 2 卷二

Important Instructions 重要指示

- (1) Choose ONE programming language: Python OR C++.
請從 Python 或 C++ 中選擇一款編程語言。
- (2) Indicate your choice by marking the corresponding box on the answer sheet.
將答題紙對應的空格填滿，以表示你的選擇。
- (3) If you choose the Python version, attempt all questions from this page to page 10, and put your answers in the Python sections of the answer sheet.
如選擇 Python 版本，應作答此頁起至第 10 頁之所有題目，並將你的答案填寫在答題紙的 Python 部分。
- (4) If you choose the C++ version, attempt all questions from page 11 to page 18, and put your answers in the C++ sections of the answer sheet.
如選擇 C++ 版本，應作答第 11 至 18 頁之所有題目，並將你的答案填寫在答題紙的 C++ 部分。

Paper 2 卷二 (Python)

	Format 類型	# Questions 題目數	Total Marks 佔分
Section A 甲部	Multiple Choice 多項選擇題	5	5
Section B 乙部	Short Questions 短答題	6 (A - F)	9
Section C 丙部	Long Questions 長填充題	2 (G - J)	6
Total 總分			20

Assume all programs shall be compiled and executed in Debian 12 using the compilers and commands below.
假設所有程序都將在 Debian 12 下使用以下編譯器及指令進行編譯，然後執行。

```
Python 3 (python3.11 3.11.2)
python3.11 -S -m py_compile program.py
python3.11 -O -S program.py
```

Section A 甲部 (5 marks 分)

For each question, choose the **most appropriate** answer and mark the corresponding box (A, B, C, or D) on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.

請為下列每題各選一個**最適合**的答案，然後把答題紙對應的空格（A、B、C、或 D）填滿。
答對得一分，答錯不扣分。

1. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
s = 0
a = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
for i in range(10):
    s += a[i]
print(s)
```

- A. 44
- B. 45
- C. 54
- D. 55

2. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
a = [3, 2, 7, 6, 8]
for i in range(4):
    a[i], a[3 - i] = a[3 - i], a[i]
for i in range(5):
    print(a[i], end='')
```

- A. 32768
- B. 67238
- C. 72308
- D. 86723

3. Given that `rand()` returns a uniformly and independently random integer between 0 and $m - 1$ inclusive, what is the **expected** value of the output of the following program?

已知 `rand()` 會在 0 到 $m - 1$ （含）之間以相等的概率隨機和獨立地傳回一個整數，請問下列程式的輸出的 **期望值** 為何？

Python

```
p = [1, 5, 10, 100]
s = [14, 17, 19, 20]
m = s[3]
n = 10
gain = 0

for i in range(n):
    x = rand()
    for j in range(4):
        if x < s[j]:
            gain += p[j]
            break

print(gain)
```

- A. 74.5
B. 100
C. 114.45
D. 290
4. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
def f(x: int, y: list[int]) -> None:
    x = 1
    y[1] = 1

a = [0, 0]
f(a[0], a)
print(a[0], a[1])
```

- A. 0 0
B. 0 1
C. 1 0
D. 1 1

5. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
n = 7
a = [-1 for _ in range(n)]

def p(k: int) -> None:
    i = k % n
    while a[i] != -1:
        i = (i + 1) % n
    a[i] = k

p(8)
p(15)
p(3)
p(1)
p(10)

print(a)
```

- A. [-1, 8, 15, 3, 1, 10, -1]
- B. [-1, 8, 1, 3, 10, -1, -1]
- C. [-1, 1, -1, 10, -1, -1, -1]
- D. [10, 1, 8, 15, 3, -1, -1]

END OF SECTION A 甲部完

Section B 乙部 (9 marks 分)

The blanks are labeled from A to F. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 A 至 F，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Answers must be in Python. Completed programs shall be compiled and executed according to the procedure specified on Page 1.
您必須使用 Python 作答。完成的程序將以第 1 頁之步驟進行編譯及執行。
- (2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。

1. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
a = [3, 1, 3, 5, 7, 4]
x = 0
for i in range(6):
    if (i + x) % 2 == 0:
        x += a[i]
    else:
        x -= a[i]
    x %= 2
print(x)
```

Answer 答案: A (1.5 marks 分)

2. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
def f(a: int, b: int) -> int:
    if a <= 0:
        return b
    if b <= 0:
        return a
    return f(a + 1, b - 2) - f(a - 2, b + 1)

print(f(3, 2))
```

Answer 答案: B (1.5 marks 分)

3. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
state = 0
inputs = [2, 1, 3, 1, 1, 2, 1, 1, 3, 2]

for input_val in inputs:
    if state == 0:
        if input_val == 1:
            state = 1
    elif state == 1:
        if input_val == 1:
            state = 2
        elif input_val == 2:
            state = 0
    elif state == 2:
        if input_val == 3:
            state = 0
        else:
            state = 1

print(state)
```

Answer 答案: C (1.5 marks 分)

4. Complete the program such that it outputs 721398.
完成以下程序，使其輸出 721398。

Python

```
a = [3, 8, 9, 7, 1, 2]
for i in range(6):
    ____D____
for i in range(6):
    print(a[i], end="")
```

Answer 答案: D (1.5 marks 分)

5. Complete the function `duration(a_h, a_m, b_h, b_m)`, such that given four non-negative integers `a_h`, `a_m`, `b_h` and `b_m`, the function returns the time difference of `a_h:a_m` and `b_h:b_m` (in HHMM format). It is guaranteed that $0 \leq a_h, b_h \leq 23$, $0 \leq a_m, b_m \leq 59$, and `a_h:a_m` is earlier than `b_h:b_m` in a day.

完成函數 `duration(a_h, a_m, b_h, b_m)`，使得給定四個非負整數 `a_h`，`a_m`，`b_h` 和 `b_m`，該函數傳回 `a_h:a_m` 和 `b_h:b_m` 之間的時間差（以 HHMM 格式表示）。保證 $0 \leq a_h, b_h \leq 23$, $0 \leq a_m, b_m \leq 59$ ，且在同一天中，`a_h:a_m` 比 `b_h:b_m` 早。

For example, `duration(10, 30, 14, 15)` should return 345 and `duration(7, 15, 20, 45)` should return 1330.

舉例而言，`duration(10, 30, 14, 15)` 應傳回 345，而 `duration(7, 15, 20, 45)` 應傳回 1330。

Python

```
def duration(a_h: int, a_m: int, b_h: int, b_m: int) -> int:
    a = a_h * 100 + a_m
    b = b_h * 100 + b_m
    return ____ E ____
```

Answer 答案: ____ E ____ (1.5 marks 分)

6. Complete the program such that it outputs HKOI.

完成以下程序，使其輸出 HKOI。

Python

```
s = "HIOK"

for i in range(4):
    print(s[____ F ____], end="")
```

Answer 答案: ____ F ____ (1.5 marks 分)

END OF SECTION B 乙部完

Section C 丙部 (6 marks 分)

The blanks are labeled from G to J. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 G 至 J，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Answers must be in Python. Completed programs shall be compiled and executed according to the procedure specified on Page 1.
您必須使用 Python 作答。完成的程序將以第 1 頁之步驟進行編譯及執行。
- (2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。
- (5) If blank X is divided into N parts $X_1, X_2, \dots, X_N$, it means that marks will only be given when $X_1, X_2, \dots, X_N$ are all correct.
如果空格 X 分為 N 部份 $X_1, X_2, \dots, X_N$ ，那麼 $X_1, X_2, \dots, X_N$ 皆為正確才會給分。

1. Consider boxes with balls inside. Each ball is marked with a number from between 0 and $K - 1$, inclusive. We represent a box as a list X of length K , where $X[i]$ is the number of balls in the box with i written on it for all $0 \leq i < K$.

考慮一些裝有球的箱子。每個球上標有一個由 0 到 $K - 1$ (含) 之間的數字。我們將一個箱子表示為長度為 K 的串列 X ，其中對於所有 $0 \leq i < K$ ， $X[i]$ 是箱子中標記為 i 的球之數量。

(a) Given a box, represented by a list X of length K . Complete the function `cumulative_count(X)` that modifies the list X , such that $X[i]$ is the number of balls inside the box with written number not exceeding i for all $0 \leq i < K$. Assume $1 \leq K \leq 10$ and $0 \leq X[i] \leq 1000$ for all i .

給定一個由長度為 K 的串列 X 所表示的箱子。完成函數 `cumulative_count(X)`，以修改串列 X ，使得對於所有 $0 \leq i < K$ ， $X[i]$ 是箱子中標記不超過 i 的球的總數。假設 $1 \leq K \leq 10$ ，且對於所有 i 都有 $0 \leq X[i] \leq 1000$ 。

Python

```
def cumulative_count(X: list[int]) -> None:
    K = len(X)
    for i in range(1, K):
        X[i] = _____ G _____
```

Answer 答案: _____ G _____ (1 mark 分)

In a lottery, there are K distinct prizes. The prizes are sorted according to their rarity, where prize 0 is the rarest, while prize $K - 1$ is the most common.

在一個抽獎中，有 K 種不同的獎品。獎品依照稀有度排序，其中獎品 0 最為稀有，而獎品 $K - 1$ 最為常見。

Now, two boxes are given, represented by two lists A and B , each of length K .

現在，給定兩個箱子，分別以長度為 K 的串列 A 和 B 來表示。

The lottery works as follows: Alice picks one ball from each of the two boxes, separately. If Alice picks a ball representing prize i from a box and a ball representing prize j from another box, then Alice gets the prize $\max(i, j)$. If there are N balls in a box and M balls in another box, then there are in total $N \times M$ ways of picking balls.

抽獎的運作方式如下：愛麗絲分別從兩個箱子中各抽取一個球。如果愛麗絲從一個箱子中抽到標記為 i 的球，又從另一個箱子中抽到標記為 j 的球，那麼愛麗絲將獲得獎品 $\max(i, j)$ 。如果一個箱子中有 N 個球，而另一個箱子中有 M 個球，則總共有 $N \times M$ 種抽球方法。

(b) Complete the function `prize_count(A, B)` such that it returns a list C , where $C[i]$ contains the number of ways Alice can pick balls to get prize i for all $0 \leq i < K$. Assume $1 \leq K \leq 10$, $0 \leq A[i], B[i] \leq 1000$ for all i , and `cumulative_count(X)` has been correctly implemented.

請完成函數 `prize_count(A, B)`，使其傳回一個串列 C ，其中對於所有 $0 \leq i < K$ ， $C[i]$ 是愛麗絲獲得獎品 i 的抽球方法數量。假設 $1 \leq K \leq 10$ ，對於所有 i 都有 $0 \leq A[i], B[i] \leq 1000$ ，並且 `cumulative_count(X)` 已經被正確實現。

Python

```
def prize_count(A: list[int], B: list[int]) -> list[int]:
    cumulative_count(A)
    cumulative_count(B)
    K = len(A)
    C = [0] * K
    C[0] = A[0] * B[0]
    for i in range(1, K):
        C[i] = _____ H _____
    return C
```

Answer 答案: _____ H _____ (2 marks 分)

2. Let x , y and z be integer variables that take the values 0 or 1. Alice wants to design a function such that it returns 1 if and only if at least one of the following conditions is met. Otherwise, it should return 0.

設 x 、 y 和 z 為取值 0 或 1 的整數變量。愛麗絲想設計一個函數，當且僅當下列條件中至少一項成立時回傳 1，否則回傳 0。

1. $x = 1$ and $y = 1$ $x = 1$ 及 $y = 1$
2. $x = 0$ and $z = 1$ $x = 0$ 及 $z = 1$
3. $y = 1$ and $z = 1$ $y = 1$ 及 $z = 1$

(a) Her first attempt is as follows: Consider the function $f(x, y, z)$ below. The function checks each of the three conditions once, and returns 1 immediately if some condition is met. Finally, the function returns 0 if none of the conditions is met. Complete the program such that the function $f(x, y, z)$ returns the correct value for all $0 \leq x, y, z \leq 1$.

她的第一個嘗試如下：考慮以下函數 $f(x, y, z)$ 。該函數會依序檢查三個條件，若任一條件成立便立刻回傳 1，若無任何條件成立，則最終回傳 0。完成以下程序，使函數 $f(x, y, z)$ 對所有 $0 \leq x, y, z \leq 1$ 均能回傳正確的數值。

Python

```
def condition(a: int, b: int, c: int, d: int) -> bool:
    return bool(_____ I _____)

def f(x: int, y: int, z: int) -> int:
    if condition(x, 1, y, 1):
        return 1
    if condition(x, 0, z, 1):
        return 1
    if condition(y, 1, z, 1):
        return 1
    return 0
```

Answer 答案: _____ I _____ (1 mark 分)

(b) Let's assume $f(x, y, z)$ is implemented correctly. Complete the function $g(x, y, z)$, such that $f(x, y, z) = g(x, y, z)$ for $0 \leq x, y, z \leq 1$.

假設我們已正確地實現函數 $f(x, y, z)$ 。完成函數 $g(x, y, z)$ ，使得對所有 $0 \leq x, y, z \leq 1$ ， $f(x, y, z) = g(x, y, z)$ 。

Python

```
def g(x: int, y: int, z: int) -> int:
    return (____ J1 ____ ) | (____ J2 ____ )
```

Answer 答案: _____ J1 _____ J2 _____ (2 marks 分)

END OF PAPER 2 卷二 (Python) 完

Paper 2 卷二 (C++)

	Format 類型	# Questions 題目數	Total Marks 佔分
Section A 甲部	Multiple Choice 多項選擇題	5	5
Section B 乙部	Short Questions 短答題	6 (K - P)	9
Section C 丙部	Long Questions 長填充題	2 (Q - T)	6
Total 總分			20

- (1) Assume that the size of the `int` data type is 32 bits.
假設 `int` 資料類型的長度為 32 位元。
- (2) The following code is added to the beginning of all C++ programs.
在所有 C++ 程序的頂部加入以下程式碼:

C++

```
#include <algorithm>
#include <array>
#include <cmath>
#include <cstdlib>
#include <deque>
#include <forward_list>
#include <iostream>
#include <list>
#include <queue>
#include <stack>
#include <string>
#include <utility>
#include <vector>
using namespace std;
```

- (3) Assume all programs shall be compiled and executed in Debian 12 using the compilers and commands below.

假設所有程序都將在 Debian 12 下使用以下編譯器及指令進行編譯，然後執行。

GNU G++ (g++-12 12.2.0)

```
g++ -std=c++20 program.cpp -o program
```

```
./program
```

Section A 甲部 (5 marks 分)

For each question, choose the **most appropriate** answer and mark the corresponding box (A, B, C, or D) on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.

請為下列每題各選一個**最適合**的答案，然後把答題紙對應的空格（A、B、C、或 D）填滿。

答對得一分，答錯不扣分。

1. What is the output of the following program? 以下程序的輸出是什麼？

C++

```
int main() {  
    int a[5] = {2, 4, 6, 8, 10};  
    cout << a[1] << ' ' << a[4];  
}
```

- A. 2 8
- B. 2 10
- C. 4 8
- D. 4 10

2. Choose the correct answer to fill into the line such that it converts a positive integer between 1 and 10000 to its hexadecimal form. For example, if the input is 2026, the output should be 7ea.

選出正確的答案填入空格，使得程式能將介於 1 到 10000 之間的正整數轉換為其十六進位表示形式。例如，若輸入為 2026，則輸出應為 7ea。

C++

```
int main() {  
    list<char> a;  
    int x;  
    cin >> x;  
    while (x > 0) {  
        int y = x % 16;  
        char z = (y >= 10 ? 'a' : '0') + y % 10;  
        _____;  
        x /= 16;  
    }  
    for (char c : a) {  
        cout << c;  
    }  
}
```

- A. a.back() = z
- B. a.front() = z
- C. a.push_back(z)
- D. a.push_front(z)

3. What is the output of the following program? 以下程序的輸出是什麼？

C++

```
int main() {  
    int i;  
    for (i = 1; i <= 7; ++i) {  
        int j = i;  
        if (++i == j) break;  
        if (i++ == j) break;  
    }  
    cout << i;  
}
```

- A. 2
- B. 8
- C. 9
- D. 10

4. What is the output of the following program? 以下程序的輸出是什麼？

C++

```
int main() {  
    int a = 3, b = 5;  
    a, b = b, a;  
    cout << a << " " << b;  
}
```

- A. 3 3
- B. 3 5
- C. 5 3
- D. 5 5

5. What is the output of the following program? 以下程序的輸出是什麼？

C++

```
void f(int &a, int b, int &c) {  
    b = a * 2;  
    c = b - a;  
    a = c + b;  
}  
  
int main() {  
    int a = 2, b = 3, c = 1;  
    f(c, a, a);  
    cout << a << " " << b << " " << c;  
}
```

- A. 1 3 2
- B. 1 3 3
- C. 2 3 1
- D. 3 3 7

END OF SECTION A 甲部完

Section B 乙部 (9 marks 分)

The blanks are labeled from K to P. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 K 至 P，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Answers must be in C++. Completed programs shall be compiled and executed according to the procedure specified on Page 9.
您必須使用 C++ 作答。完成的程序將以第 9 頁之步驟進行編譯及執行。
- (2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。

1. What is the output of the following program? 以下程序的輸出是什麼？

C++

```
int main() {  
    int ans = 0;  
    for (int i = 0; i < 10; ++i)  
        for (int j = i % 4; j < (8 - i) % 4; ++j)  
            ++ans;  
    cout << ans;  
}
```

Answer 答案: K (1.5 marks 分)

2. What is the output of the following program? 以下程序的輸出是什麼？

C++

```
int f(int a, int b) {  
    if (a <= 0)  
        return b;  
    if (b <= 0)  
        return a;  
    return f(a - 2, b + 1) - f(a + 1, b - 2);  
}  
  
int main() {  
    cout << f(3, 2);  
}
```

Answer 答案: L (1.5 marks 分)

3. What is the output of the following program? 以下程序的輸出是什麼？

C++

```
int main() {
    int a[] = {13, 9, 5, 8, 6, 3, 0};
    int ans = 0;
    while (a[0] > 0) {
        for (int i = 0; a[i] > 0; ++i) {
            --a[i];
            ++ans;
        }
    }
    cout << ans;
}
```

Answer 答案: M (1.5 marks 分)

4. Complete the following program so that it outputs hkoj-uoj-joj.
完成以下程序，使得其輸出 hkoj-uoj-joj。

C++

```
int main() {
    string s = "hkoi-uoi-ioi";
    for (int i = 0; i < s.length(); ++i) {
        if (___N___) {
            ++s[i];
        }
    }
    cout << s;
}
```

Answer 答案: N (1.5 marks 分)

5. Let x be an integer between 1 and 10 inclusive. Complete the following function `IsValidTyphoonSignal(x)` such that it returns `true` if and only if x is a valid typhoon signal. The valid typhoon signals are 1, 3, 8, 9 and 10, while other numbers are invalid.

設 x 為一個介乎 1 和 10 之間（含）的整數。完成以下函數 `IsValidTyphoonSignal(x)`，使其傳回 `true` 當且僅當 x 是一個有效的颱風信號。有效的颱風信號為 1、3、8、9 和 10，其他數字則為無效。

C++

```
bool IsValidTyphoonSignal(int x) {
    if (x == 2)
        return false;
    else
        return ___0___;
}
```

Answer 答案: 0 (1.5 marks 分)

6. Let $a[0..n-1]$ be an array of integers sorted in ascending order. Assume that $f(a, x)$ is a function that returns the smallest index i such that $a[i] \geq x$, or returns n if no such index exists.

設 $a[0..n-1]$ 為一個已依遞增順序排序的整數陣列。假設函數 $f(a, x)$ 傳回滿足 $a[i] \geq x$ 的最小索引 i ，若不存在此索引，則傳回 n 。

Complete the function $g(a, x)$ so that it returns $|x - a[0]| + |x - a[1]| + \cdots + |x - a[n-1]|$. You may assume that no integer overflow occurs.

請完成函數 $g(a, x)$ ，使其傳回 $|x - a[0]| + |x - a[1]| + \cdots + |x - a[n-1]|$ 。可假設不會發生整數溢位。

Here, $|x|$ is defined as the absolute value of x : if $x \geq 0$, then $|x| = x$; if $x < 0$, then $|x| = -x$. For example, $|3| = 3$, $|-1| = 1$.

這裡的 $|x|$ 定義為 x 的絕對值：若 $x \geq 0$ ，則 $|x| = x$ ；若 $x < 0$ ，則 $|x| = -x$ 。舉例來說， $|3| = 3$ ， $|-1| = 1$ 。

C++

```
int g(vector<int> a, int x) {
    int n = a.size();
    vector<int> b(n + 1, 0);
    b[0] = 0;
    for (int i = 1; i <= n; ++i) {
        b[i] = b[i - 1] + a[i - 1];
    }
    int u = f(a, x);
    return ____P____;
}
```

Answer 答案: ____P____ (1.5 marks 分)

END OF SECTION B 乙部完

Section C 丙部 (6 marks 分)

The blanks are labeled from Q to T. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 Q 至 T，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Answers must be in C++. Completed programs shall be compiled and executed according to the procedure specified on Page 9.
您必須使用 C++ 作答。完成的程序將以第 9 頁之步驟進行編譯及執行。
- (2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。
- (5) If blank X is divided into N parts $X_1, X_2, \dots, X_N$, it means that marks will only be given when $X_1, X_2, \dots, X_N$ are all correct.
如果空格 X 分為 N 部份 $X_1, X_2, \dots, X_N$ ，那麼 $X_1, X_2, \dots, X_N$ 皆為正確才會給分。

1. Consider the following function: 考慮以下函數：

C++

```
int f(int n, int k) {  
    vector<int> a(n + 1);  
    for (int i = 0; i < k; ++i)  
        a[i] = 1;  
    for (int i = k; i <= n; ++i) {  
        a[i] = 0;  
        for (int j = i - k; j < i; ++j)  
            a[i] += a[j];  
    }  
    return a[n];  
}
```

- (a) Write down the return value of $f(7, 3)$. 寫下 $f(7, 3)$ 的傳回值。

Answer 答案: _____ Q _____ (1 mark 分)

(b) Without calling the function $f(n, k)$, complete the following function $g(n, k)$, so that $g(n, k)$ has the same return value as $f(n, k)$ for $1 \leq k \leq n \leq 30$.

在不使用函數 $f(n, k)$ 的情況下，完成以下函數 $g(n, k)$ ，使得對於 $1 \leq k \leq n \leq 30$ ， $g(n, k)$ 和 $f(n, k)$ 的傳回值相同。

C++

```
int g(int n, int k) {  
    vector<int> a(n+1);  
    for (int i = 0; i < k; ++i)  
        a[i] = 1;  
    a[k] = __R1__;  
    for (int i = k + 1; i <= n; ++i)  
        a[i] = __R2__;  
    return a[n];  
}
```

Answer 答案: R1 R2 (2 marks 分)

2. Consider the following function: 考慮以下函數：

C++

```
vector<int> p = {1, 10, 100, 1000, 10000, 100000, 1000000,  
                10000000, 100000000, 1000000000};  
  
int r(int x, int n) {  
    if (n == 0)  
        return x;  
    else if (n == 1)  
        return (x + 5) / 10 * 10;  
    else  
        return r(r(x, n - 1) / p[n - 1], 1) * p[n - 1];  
}
```

(a) Write down the return value of $r(1122444669, 6)$. 寫下 $r(1122444669, 6)$ 的傳回值。

Answer 答案: S (1 mark 分)

(b) Complete the following function $rnd(x, n)$, so that $rnd(x, n)$ returns the value of x rounded off to the nearest 10^n . It is guaranteed that $0 \leq x \leq 10^9$ and $0 \leq n \leq 9$. You may use the variable p and/or the function r .
完成以下函數 $rnd(x, n)$ ，使得 $rnd(x, n)$ 傳回 x 四捨五入至最接近的 10^n 後的值。保證 $0 \leq x \leq 10^9$ 及 $0 \leq n \leq 9$ 。你可以使用變量 p 和/或函數 r 。

C++

```
int rnd(int x, int n) {  
    return r(__T__, n);  
}
```

Answer 答案: T (2 marks 分)

END OF PAPER 2 卷二 (C++) 完