

Paper 1 (Compulsory Part) 卷一（必答部分）

	Format 類型	# Questions 題目數	Total Marks 佔分
Section A 甲部	Multiple Choice 多項選擇題	10	10
Section B 乙部	Short Questions 短答題	6 (A - F)	9
Section C 丙部	Long Questions 長填充題	2 (G - J)	6
Total 總分			25

- (1) The term “arrays” in Paper 1 generally refers to lists in Python.

卷一提及的「陣列」泛指 Python 語言的串列。

- (2) The following code is added to the beginning of all C++ programs.

在所有 C++ 程序的頂部加入以下程式碼：

C++

```
#include <algorithm>
#include <array>
#include <cmath>
#include <cstdlib>
#include <deque>
#include <forward_list>
#include <iostream>
#include <list>
#include <queue>
#include <stack>
#include <string>
#include <utility>
#include <vector>
using namespace std;
```

- (3) Assume all programs shall be compiled and executed in Debian 12 using the compilers and commands below.

假設所有程序都將在 Debian 12 下使用以下編譯器及指令進行編譯，然後執行。

GNU G++ (g++-12 12.2.0)

```
g++ -std=c++20 program.cpp -o program
./program
```

Python 3 (python3.11 3.11.2)

```
python3.11 -S -m py_compile program.py
python3.11 -O -S program.py
```

Section A 甲部 (10 marks 分)

For each question, choose the **most appropriate** answer and mark the corresponding box (A, B, C, or D) on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.

請為下列每題各選一個**最適合**的答案，然後把答題紙對應的空格（A、B、C、或 D）填滿。

答對得一分，答錯不扣分。

1. Which of the following statements about control structures are true?

以下哪個有關控制結構的描述為真？

- i. An `if` must be followed by an `else`.
一個 `if` 後面必須跟着一個 `else`。
- ii. `while` loop always executes its loop body at least once.
`while` 迴圈至少執行迴圈主體一次。
- iii. `for` loop always executes its loop body at least once.
`for` 迴圈至少執行迴圈主體一次。

- A. i only 只有 i
- B. ii only 只有 ii
- C. iii only 只有 iii
- D. None of them 無

2. Four teams (A, B, C, D) play in a single-elimination tournament, where a team is immediately eliminated upon losing a single match. In the first round, A plays B and C plays D. The winners then play in the final.

有四個隊伍 (A, B, C, D) 進行單敗淘汰賽，即輸掉一場比賽的隊伍將立即被淘汰。在第一輪中，A 對 B，C 對 D。獲勝者隨後進入決賽。

- Team C has a 100% chance of winning against Team D.
隊伍 C 對隊伍 D 有 100% 的獲勝機會。
- Team A has a 100% chance of winning against Team C.
隊伍 A 對隊伍 C 有 100% 的獲勝機會。
- In a match between any two teams not explicitly mentioned above, either team has an equal chance of winning (50% chance for each).
在任何兩個未明確提及的隊伍之間的比賽中，任何一方獲勝的機率均等（各 50% 的機會）。

How many teams can possibly be a winner of the tournament?

有多少隊伍有可能成為本次比賽的冠軍？

- A. 1
- B. 2
- C. 3
- D. 4

3. Alice and Bob know each other, and Bob and Charlie also know each other. Alice is a student, and Charlie is not a student. Which of the following **must** be true?

愛麗絲和鮑伯互相認識，而鮑伯和查理互相認識。愛麗絲是一名學生，而查理不是學生。以下哪項 **必定** 為真？

- | | | |
|------|---|--------------|
| i. | A student knows a non-student. | 一位學生認識一位非學生。 |
| ii. | There exists 2 students knowing each other. | 存在兩位學生互相認識。 |
| iii. | There exists 2 non-students knowing each other. | 存在兩位非學生互相認識。 |
| A. | i only | 只有 i |
| B. | ii only | 只有 ii |
| C. | iii only | 只有 iii |
| D. | None of them | 無 |

4. The integers 1, 3, 5, 3, 2, 4 are pushed into a queue in this order. Which of the following statements about the queue is correct?

整數 1、3、5、3、2、4 依此順序被推入一個隊列。下列哪一項關於此隊列的敘述是正確的？

- A. The queue currently contains five integers, since the queue only keeps one instance of 3.
隊列現時包含五個整數，因為它只會保留其中一個 3。
- B. If a pop operation is executed on the queue, it will pop 5 out of the queue.
若在此隊列上執行一次彈出操作，5 會從隊列彈出。
- C. If the elements in the queue are popped out one by one, 2 is popped before 4.
若隊列中的數字被逐一彈出，2 會在 4 之前被彈出。
- D. If 6 is pushed into the queue and a pop operation is executed afterwards, 6 will be popped out of the queue.
若把 6 推入隊列，接著執行一次彈出操作，6 會從隊列彈出。

5. You are given 4 cards, each card has two sides and each side is written with a number:

你有四張紙牌，每張牌都有兩面，而且每一面都寫著一個數字：

- | | |
|-----------------------------------|---------------------|
| • Card 1 has numbers '1' and '2'. | 第一張牌上有數字 '1' 和 '2'。 |
| • Card 2 has numbers '2' and '3'. | 第二張牌上有數字 '2' 和 '3'。 |
| • Card 3 has numbers '3' and '4'. | 第三張牌上有數字 '3' 和 '4'。 |
| • Card 4 has numbers '4' and '1'. | 第四張牌上有數字 '4' 和 '1'。 |

The cards are shuffled and flipped randomly. The first three cards show '1', '2', and '3' on their front sides. Given this information, how many different numbers can appear on the front side of the last card?

這些牌被隨機交換並翻轉。前三張牌的正面分別是 '1'、'2' 和 '3'。根據以上資訊，有多少個數字有可能出現在最後一張牌的正面？

- A. 1
B. 2
C. 3
D. 4

6. Which of the following Boolean expressions is equivalent to $(A \text{ AND } B) \text{ OR } (\text{NOT } A \text{ AND } C) \text{ OR } (A \text{ AND NOT } B)$?

以下哪個布爾表達式等價於 $(A \text{ AND } B) \text{ OR } (\text{NOT } A \text{ AND } C) \text{ OR } (A \text{ AND NOT } B)$?

- A. $A \text{ AND } C$
- B. $A \text{ OR } B$
- C. $A \text{ OR } C$
- D. $B \text{ AND } C$

7. Given a data structure D , which is either a stack or a queue, and is initially empty. We execute the following operations:

給定一個數據結構 D ，它是堆疊（棧）或隊列兩者之一，且最初為空。我們執行以下操作：

Python

```
D.push(5)
D.pop()
D.push(7)
D.pop()
```

C++

```
D.push(5);
D.pop();
D.push(7);
D.pop();
```

It is known that the value popped in the second `pop()` call is 7. Which of the following statement is true?

已知第二次呼叫 `pop()` 彈出的值為 7，以下哪項敘述為真？

- A. D must be a stack.
D 必定是堆疊。
- B. D must be a queue.
D 必定是隊列。
- C. It is impossible to infer whether D is a stack or a queue.
無法推斷 D 是堆疊或隊列。
- D. The value popped in the first `pop()` call is also 7.
第一次呼叫 `pop()` 彈出的值亦是 7。

8. Given an array a of n positive integers. In one operation, you first pick two indices i and j , such that $0 \leq i < j < n$. Then, you increase each element in a whose indices are between i and $j - 1$ (inclusive) by 1, and also increase each element in a whose indices are between j and $n - 1$ (inclusive) by 2. For which arrays is it possible to make all elements of the array equal after some operations?

給定一個有 n 個正整數的陣列 a 。在一次操作中，你先選定兩個索引 i 和 j ，使得 $0 \leq i < j < n$ 。然後，你將 a 中索引從 i 到 $j - 1$ （含）的元素加 1，你亦將 a 中索引從 j 到 $n - 1$ （含）的元素加 2。以下哪些陣列，存在若干次的操作，使其在操作後所有的元素均為相等？

i. $[7, 6, 7, 6, 5, 4, 5]$

ii. $[7, 6, 4, 5, 4, 2, 3]$

- A. i only 只有 i
- B. ii only 只有 ii
- C. i and ii only i 和 ii
- D. None of them 無

9. Given the following two facts:

給定以下兩個事實：

- Alice and Bob are friends. If one of them goes to a party, the other will also go to the party.
愛麗絲和鮑伯是朋友。若他們其中一個人參加一個派對，另一個人也會參加那個派對。
- Charlie hates Bob. If Bob goes to a party, Charlie will not go to the party.
查理不喜歡鮑伯。若鮑伯參加一個派對，查理便不會參加那個派對。

Which of the following statements must be true?

以下哪些描述必定為真？

- i. If Charlie goes to a party, then Alice will not go to the party.
若查理參加一個派對，愛麗絲便不會參加那個派對。
- ii. If Alice does not go to a party, then Charlie will go to the party.
若愛麗絲不參加一個派對，查理便會參加那個派對。

- A. i only 只有 i
- B. ii only 只有 ii
- C. i and ii i 和 ii
- D. None of them 無

10. You have a pile of 15 identical coins. In each step, you may split one pile of coins into two non-empty piles of coins. You continue until all piles have exactly 1 coin.

你有一個由 15 枚相同硬幣組成的硬幣堆。在每一步中，你可以將一個硬幣堆分成兩個不為空的硬幣堆。你需要不斷地進行這步驟，直到所有的硬幣堆都只剩下 1 枚硬幣為止。

Which of the following statements must be true?

以下哪些描述必定為真？

- i. The number of steps performed is always 14.
執行的步驟數總是 14 步。
- ii. At some step, there must exist a pile of size 2.
在某個步驟中，必定會存在一個大小為 2 的硬幣堆。
- iii. At some step, there must exist a pile of size 3.
在某個步驟中，必定會存在一個大小為 3 的硬幣堆。

- A. i and ii only i 和 ii
- B. i and iii only i 和 iii
- C. ii and iii only ii 和 iii
- D. i, ii and iii i, ii 和 iii

END OF SECTION A 甲部完

Section B 乙部 (9 marks 分)

The blanks are labeled from A to F. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 A 至 F，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (2) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (3) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。

1. You have three pendulums. They are released from their respective top positions simultaneously at the 0th second. They would return to their starting positions every 6, 8, and 12 seconds respectively. What is the smallest positive time (in seconds) will they all be back to their starting positions?

你有三個單擺。它們同時於第 0 秒從各自的最高點被釋放。它們分別會每 6 秒、8 秒和 12 秒回到各自的起始位置。求它們再次同時回到各自起始位置的最短正時間（以秒為單位）。

Answer 答案: _____ A _____ (1.5 marks 分)

2. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
a = [1, 2, 3, 4, 5]
x = 0

for i in range(2025):
    x = (x + a[x]) % 5
print(a[x])
```

C++

```
int main() {
    int a[5] = {1, 2, 3, 4, 5};
    int x = 0;
    for (int i = 0; i < 2025; ++i)
        x = (x + a[x]) % 5;
    cout << a[x];
}
```

Answer 答案: _____ B _____ (1.5 marks 分)

3. Consider the following program: 考慮以下程序：

Python

```
def f() -> int:
    # Implementation omitted 實現省略

print(f() & 26)
```

C++

```
int f() {
    // Implementation omitted 實現省略
}

int main() {
    cout << (f() & 26);
}
```

If $f()$ returns a random integer between 0 and 31 inclusive uniformly and independently, what is the probability that the above program outputs 0? Please provide the answer as a fraction in its simplest form.

若 $f()$ 會以相等的概率隨機和獨立地傳回 0 到 31 之間（含）的整數，上述程序輸出 0 的概率是多少？請以最簡分數作答。

Answer 答案: _____ C _____ (1.5 marks 分)

4. Complete the program such that it outputs 4.
完成以下程序，使其輸出 4。

Python

```
def f(x: int) -> int:
    a = [5, 3, 6, 2, 8]
    for i in range(5):
        x = x ^ a[i]
    return x

y = ____D____
print(f(y))
```

C++

```
int f(int x) {
    int a[5] = {5, 3, 6, 2, 8};
    for (int i = 0; i < 5; ++i) {
        x = x ^ a[i];
    }
    return x;
}

int main() {
    int y = ____D____;
    cout << f(y);
}
```

Answer 答案: ____D____ (1.5 marks 分)

5. Consider all arithmetic expressions of length between 1 and 7 (inclusive) that consists of 1 and + only. The arithmetic expressions should not start or end with +, or have consecutive +. For example, 1, 11, 111+111 and 1+1+11 are valid arithmetic expressions, while +1+1+ and 1++1 are not. How many different values can be evaluated from these arithmetic expressions?

考慮所有由 1 和 + 組成，且長度為 1 和 7 之間（含）的算術表達式。算術表達式不能以 + 開頭或結尾，亦不能有連續的 +。例如，1、11、111+111 和 1+1+11 為合法的算術表達式，而 +1+1+ 和 1++1 則不是。有多少種不同的值能夠由這些算術表達式算出？

Answer 答案: ____E____ (1.5 marks 分)

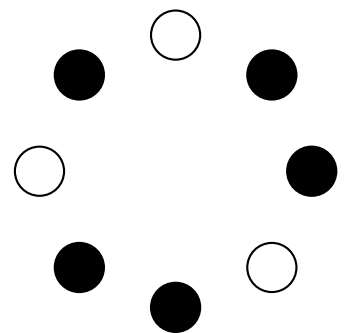
6. 8 balls are arranged in a circular formation as shown in the figure on the right. Each ball is colored black or white. For every pair of balls of the **same color**, draw a straight line segment connecting their centers. The color of the line segment is the same as that of the balls.

8 個球如右圖沿圓周排列，每個球為黑色或白色。對於每一對 **相同顏色** 的球，畫出一條連接其圓心的直線線段。該線段的顏色與兩球相同。

In one operation, you may remove a single line segment, the cost of removing it equals the number of opposite-color line segments it intersects. Repeat until all the line segments are removed; the total cost is the sum of costs of all operations. What is the minimum total cost needed to remove all line segments?

在一次操作中，你可以移除一條線段，移除該線段的代價等於與其相交的異色線段數量。重複此操作直至所有線段皆被移除；總代價為各次操作代價之和。移除所有線段所需的最小總代價是多少？

Answer 答案: ____F____ (1.5 marks 分)



END OF SECTION B 乙部完

Section C 丙部 (6 marks 分)

The blanks are labeled from G to J. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 G 至 J，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Answers can be in either C++ or Python. Completed programs shall be compiled and executed according to the procedure specified on Page 1.
您可以使用 C++ 或 Python 作答。完成的程序將以第 1 頁之步驟進行編譯及執行。
- (2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。
- (5) If blank X is divided into N parts $X_1, X_2, \dots, X_N$, it means that marks will only be given when $X_1, X_2, \dots, X_N$ are all correct.
如果空格 X 分為 N 部份 $X_1, X_2, \dots, X_N$ ，那麼 $X_1, X_2, \dots, X_N$ 皆為正確才會給分。

1. A *repdigit* is a positive integer whose decimal representation consists of the same digit repeated (e.g., 1, 11, 7, 777 are *repdigits*, while 15, 23, 998 are not).

純位數是以十進制表示時，由相同的數字重複組成的正整數（例如 1, 11, 7, 777 是純位數，而 15, 23, 998 則不是）。

(a) Complete the function $f(x)$ to return the smallest *repdigit* which consists of only digit 1 and is greater than x . For example, when x is 111, f should return 1111. It is guaranteed that x is a *repdigit* consisting of only digit 1, and $1 \leq x \leq 11111$.

完成函數 $f(x)$ 以傳回大於 x ，且只由數字 1 組成的最小純位數。例如當 x 是 111 時， f 應該傳回 1111。已知 x 是一個只由數字 1 組成的純位數，且 $1 \leq x \leq 11111$ 。

Python

```
def f(x: int) -> int:
    return x *   G1   +   G2  
```

C++

```
int f(int x) {
    return x *   G1   +   G2  ;
}
```

Answer 答案: G1 G2 (1.5 marks 分)

(b) Assuming $f(x)$ is correctly implemented, complete the function $g(x)$ to return the smallest *repdigit* greater than x . For example, when x is 222, g should return 333. It is guaranteed that x is a *repdigit*, and $1 \leq x \leq 99999$.

假設 $f(x)$ 已被正確地實現，完成函數 $g(x)$ 以傳回大於 x 的最小純位數。例如當 x 是 222 時， g 應該傳回 333。已知 x 是一個純位數，且 $1 \leq x \leq 99999$ 。

Python

```
def g(x: int) -> int:
    d = x % 10
    r = x //   H1  

    if d < 9:
        return   H2  
    else:
        return f(r)
```

C++

```
int g(int x) {
    int d = x % 10;
    int r = x /   H1  ;
    if (d < 9) {
        return   H2  ;
    } else {
        return f(r);
    }
}
```

Answer 答案: H1 H2 (1.5 marks 分)

2. According to the triangle inequality, three positive integers may only form the side lengths of a valid triangle if and only if the sum of any two integers is greater than the third number.

根據三角形不等式，三個正整數當且僅當在任意兩數之和大於第三個數時，才能構成有效三角形的邊長。

(a) Complete the function $f(a, b, c)$ such that it returns `True` (C++: `true`) if and only if the three integers a, b, c form the side lengths of a valid triangle. It is guaranteed that $1 \leq a < b < c \leq 1000$.

完成函數 $f(a, b, c)$ ，使其當且僅當三個整數 a, b, c 能構成有效三角形的邊長時傳回 `True` (C++ : `true`)。已知 $1 \leq a < b < c \leq 1000$ 。

Python

```
def f(a: int, b: int, c: int) -> bool:
    return ____I____
```

C++

```
bool f(int a, int b, int c) {
    return ____I____;
}
```

Answer 答案: ____I____ (1 mark 分)

(b) Assuming f is correctly implemented. Let a be an array of length n of distinct positive integers in ascending order. Complete the function $g(a)$ to return the number of valid triples in a that form the side lengths of a valid triangle. The triples are unordered, for example, $(2, 3, 4)$ is considered the same triple as $(4, 3, 2)$.

假設 f 已被正確地實現。設 a 為一個長度 n 的陣列，由兩兩不同且按遞增排序的正整數組成。完成函數 $g(a)$ 以傳回 a 內能構成有效三角形邊長的三元組數量。三元組是無序的，例如， $(2, 3, 4)$ 和 $(4, 3, 2)$ 會視為同一個三元組。

Python

```
def g(a: list[int]) -> int:
    n = len(a)
    ans = 0

    for i in range(n - 1):
        k = i + 1
        for j in range(i):
            while ((____J1____) and
                  f(a[j], a[i], a[k])):
                k += 1

            ans += ____J2____

    return ans
```

C++

```
int g(vector<int> a) {
    int n = a.size();
    int ans = 0;
    for (int i = 0; i < n - 1; ++i) {
        int k = i + 1;
        for (int j = 0; j < i; ++j) {
            while ((____J1____) &&
                  f(a[j], a[i], a[k])) {
                k += 1;
            }
            ans += ____J2____;
        }
    }
    return ans;
}
```

Answer 答案: ____J1____ ____J2____ (2 marks 分)

END OF PAPER 1 卷一完

Paper 2 卷二

Important Instructions 重要指示

- (1) Choose ONE programming language: Python OR C++.
請從 Python 或 C++ 中選擇一款編程語言。
- (2) Indicate your choice by marking the corresponding box on the answer sheet.
將答題紙對應的空格填滿，以表示你的選擇。
- (3) If you choose the Python version, attempt all questions from this page to page 9, and put your answers in the Python sections of the answer sheet.
如選擇 Python 版本，應作答此頁起至第 9 頁之所有題目，並將你的答案填寫在答題紙的 Python 部分。
- (4) If you choose the C++ version, attempt all questions from page 10 to page 18, and put your answers in the C++ sections of the answer sheet.
如選擇 C++ 版本，應作答第 10 至 18 頁之所有題目，並將你的答案填寫在答題紙的 C++ 部分。

Paper 2 卷二 (Python)

	Format 類型	# Questions 題目數	Total Marks 佔分
Section A 甲部	Multiple Choice 多項選擇題	5	5
Section B 乙部	Short Questions 短答題	6 (A - F)	9
Section C 丙部	Long Questions 長填充題	2 (G - K)	6
Total 總分			20

Assume all programs shall be compiled and executed in Debian 12 using the compilers and commands below.
假設所有程序都將在 Debian 12 下使用以下編譯器及指令進行編譯，然後執行。

```
Python 3 (python3.11 3.11.2)
python3.11 -S -m py_compile program.py
python3.11 -O -S program.py
```

Section A 甲部 (5 marks 分)

For each question, choose the **most appropriate** answer and mark the corresponding box (A, B, C, or D) on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.

請為下列每題各選一個**最適合**的答案，然後把答題紙對應的空格（A、B、C、或 D）填滿。
答對得一分，答錯不扣分。

1. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
print(['ab', 'c'], sep='#')
```

- A. ['ab', 'c']
- B. ['ab' #'c']
- C. a#b#c
- D. ab#c

2. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
print(2 << 4)
```

- A. 0
- B. 16
- C. 24
- D. 32

3. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
a = [1, 2, 3, 4, 5]
for i in range(10):
    a[i % 5] += a[(i + 1) % 5]

print(a)
```

- A. [8, 12, 16, 17, 16]
- B. [8, 12, 17, 19, 17]
- C. [9, 12, 16, 18, 18]
- D. [9, 12, 15, 16, 17]

4. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
print(-3 // 2, -5 % 3)
```

- A. -1 -2
- B. -1 1
- C. -2 -2
- D. -2 1

5. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
a = [4, 2, 7, 8, 4, 1, 5, 3]
num = 0
for i in range(len(a)):
    for j in range(len(a) - 1):
        if a[j] > a[j + 1]:
            a[j], a[j + 1] = a[j + 1], a[j]
            num += 1
print(num)
```

- A. 12
- B. 13
- C. 14
- D. 15

END OF SECTION A 甲部完

Section B 乙部 (9 marks 分)

The blanks are labeled from A to F. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 A 至 F，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Answers must be in Python. Completed programs shall be compiled and executed according to the procedure specified on Page 1.
您必須使用 Python 作答。完成的程序將以第 1 頁之步驟進行編譯及執行。
- (2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。

1. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
a = [2, 6, 2, 4, 1, 8, 3, 9, 7]
s = 0
for i in range(1, len(a), 2):
    s += a[i]
print(s)
```

Answer 答案: A (1.5 marks 分)

2. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
a = [28, 18, 12, 37, 23, 39, 15]
b = [0] * 7

for i in range(7):
    for j in range(i + 1):
        if (i - j) % 2 == 0:
            b[i] += a[j]
        else:
            b[i] -= a[j]

s = 0
for i in range(7):
    s += b[i]
print(s)
```

Answer 答案: B (1.5 marks 分)

3. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
a = [[0] * 8 for _ in range(8)]
x, y = 0, 0
dx, dy = 0, 1

for i in range(4, 9):
    for j in range(i):
        a[x][y] = 1
        x = (x + dx) & 7
        y = (y + dy) & 7
    dx ^= 1
    dy ^= 1

ans = 0
for i in range(8):
    for j in range(8):
        ans += a[i][j]

print(ans)
```

Answer 答案: C (1.5 marks 分)

4. Assume a function `is_cube(x)` is correctly implemented. It takes in a positive integer x , and returns `True` if and only if x is a perfect cube.

假設函數 `is_cube(x)` 已被正確地實現。該函數接受一個正整數 x ，如果 x 是個完全立方數，函數會傳回 `True`，否則，函數會傳回 `False`。

Complete the following function `f` such that for all positive integers $n \leq 10^9$, it returns the number of pairs of positive integers (a, b) such that $a^2 + b^3 = n$.

完成以下函數 `f(n)`，使其對於所有正整數 $n \leq 10^9$ ，均傳回滿足 $a^2 + b^3 = n$ 的正整數對 (a, b) 的數量。

Python

```
def is_cube(x: int) -> bool:
    # Implementation omitted 實現省略

def f(n: int) -> int:
    s, i = 0, 1
    while i * i < n:
        if     D    :
            s += 1
        i += 1
    return s
```

Answer 答案: D (1.5 marks 分)

5. Complete the following program such that it outputs 101101101.
請完成以下程式，使其輸出 101101101。

Python

```
a, b = ____ E ____  
print(a * b)
```

Answer 答案: ____ E ____ (1.5 marks 分)

6. The months of the year are encoded in binary, where the N -th month of the year ($1 \leq N \leq 12$) is represented by a 4-digit binary number $ABCD_2$, such that $0 \leq A, B, C, D \leq 1$ and $N = 8A + 4B + 2C + D$. For example, January is represented by $A = 0, B = 0, C = 0, D = 1$, February by $A = 0, B = 0, C = 1, D = 0$, ... and December by $A = 1, B = 1, C = 0, D = 0$.

一年中的月份以二進制編碼，其中第 N 個月份 ($1 \leq N \leq 12$) 以四位二進制數 $ABCD_2$ 表示，其中 $0 \leq A, B, C, D \leq 1$ ，且 $N = 8A + 4B + 2C + D$ 。例如，一月為 $A = 0, B = 0, C = 0, D = 1$ ，二月為 $A = 0, B = 0, C = 1, D = 0$ ，如此類推，十二月為 $A = 1, B = 1, C = 0, D = 0$ 。

Complete the function `has_r(A, B, C, D)`, so that given four integers A, B, C, D representing a valid month of the year, it returns **Yes if and only if** that month's English name contains the letter r .

完成函數 `has_r(A, B, C, D)`，使得給定代表有效月份的四個整數 A, B, C, D ，這個函數傳回 **Yes 當且僅當** 該月份的英文名稱包含字母 r 。

Hint: The months that contain the letter r are January, February, March, April, September, October, November and December.

提示：包含字母 r 的月份為一月、二月、三月、四月、九月、十月、十一月和十二月。

Python

```
def has_r(A: int, B: int, C: int, D: int) -> str:  
    if ____ F ____:  
        return "Yes"  
    else:  
        return "No"
```

Answer 答案: ____ F ____ (1.5 marks 分)

END OF SECTION B 乙部完

Section C 丙部 (6 marks 分)

The blanks are labeled from G to K. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 G 至 K，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Answers must be in Python. Completed programs shall be compiled and executed according to the procedure specified on Page 1.
您必須使用 Python 作答。完成的程序將以第 1 頁之步驟進行編譯及執行。
- (2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。
- (5) If blank X is divided into N parts $X_1, X_2, \dots, X_N$, it means that marks will only be given when $X_1, X_2, \dots, X_N$ are all correct.
如果空格 X 分為 N 部份 $X_1, X_2, \dots, X_N$ ，那麼 $X_1, X_2, \dots, X_N$ 皆為正確才會給分。

1. We denote a non-empty list of integers of length n as *beautiful* if and only if it consists of $x, x+1, x+2, \dots, x+n-1$ in this order for some integer x .

對於一個長度為 n 的非空整數串列，我們稱之為美麗的當且僅當對於某整數 x ，該串列由 $x, x+1, x+2, \dots, x+n-1$ 順序組成。

(a) Complete the function `is_beautiful1(a)` to return `True` if and only if list `a` is *beautiful*. It is guaranteed that the list `a` has at least one element.

完成函數 `is_beautiful1(a)`，使其傳回 `True` 當且僅當串列 `a` 是美麗的。保證串列 `a` 至少有一個元素。

Python

```
def is_beautiful1(a: list[int]) -> bool:
    n = len(a)
    for i in range(1, n):
        if ____G____ != 1:
            return False
    return True
```

Answer 答案: ____G____ (1 mark 分)

(b) Complete the function `is_beautiful2(a)` to return `True` if and only if list `a` is *beautiful*. It is guaranteed that the list `a` has at least one element.

完成函數 `is_beautiful2(a)`，使其傳回 `True` 當且僅當串列 `a` 是美麗的。保證串列 `a` 至少有一個元素。

Python

```
def is_beautiful2(a: list[int]) -> bool:
    n = len(a)
    for i in range(1, n):
        if a[i] != ____H____:
            return False
    return True
```

Answer 答案: ____H____ (1 mark 分)

(c) Complete the function `is_beautiful3(a)` to return `True` if and only if list `a` is *beautiful*. It is guaranteed that the list `a` has at least one element.

完成函數 `is_beautiful3(a)`，使其傳回 `True` 當且僅當串列 `a` 是美麗的。保證串列 `a` 至少有一個元素。

Python

```
def is_beautiful3(a: list[int]) -> bool:
    n = len(a)
    for i in range(1, n):
        if a[i - 1] >= a[i]:
            return False
    return bool(____I____)
```

Answer 答案: ____I____ (1 mark 分)

2. Consider the following program: 考慮以下程序：

Python

```
def f(x: str) -> any:
    return __ J, K __

s, t = input().split()
if f(s) == f(t):
    print("Yes")
else:
    print("No")
```

- (a) Complete the program $f(x)$ such that it produces the correct output for these inputs:
完成程序 $f(x)$ ，使其在以下輸入時產生正確的輸出結果：

Input	Output
python marathon	Yes
pair repair	Yes
class glass	Yes
crate rate	Yes
return burn	No
closet set	No
butter button	No

Answer 答案: J (1.5 marks 分)

- (b) Complete the program $f(x)$ such that it produces the correct output for these inputs:
完成程序 $f(x)$ ，使其在以下輸入時產生正確的輸出結果：

Input	Output
secure rescue	Yes
state taste	Yes
night thing	Yes
loop pool	Yes
time mine	No
write white	No
violet violin	No

Answer 答案: K (1.5 marks 分)

END OF PAPER 2 卷二 (Python) 完

Paper 2 卷二 (C++)

	Format 類型	# Questions 題目數	Total Marks 佔分
Section A 甲部	Multiple Choice 多項選擇題	5	5
Section B 乙部	Short Questions 短答題	6 (L - Q)	9
Section C 丙部	Long Questions 長填充題	2 (R - V)	6
Total 總分			20

- (1) Assume that the size of the `int` data type is 32 bits.
假設 `int` 資料類型的長度為 32 位元。
- (2) The following code is added to the beginning of all C++ programs.
在所有 C++ 程序的頂部加入以下程式碼:

C++

```
#include <algorithm>
#include <array>
#include <cmath>
#include <cstdlib>
#include <deque>
#include <forward_list>
#include <iostream>
#include <list>
#include <queue>
#include <stack>
#include <string>
#include <utility>
#include <vector>
using namespace std;
```

- (3) Assume all programs shall be compiled and executed in Debian 12 using the compilers and commands below.

假設所有程序都將在 Debian 12 下使用以下編譯器及指令進行編譯，然後執行。

GNU G++ (g++-12 12.2.0)

```
g++ -std=c++20 program.cpp -o program
```

```
./program
```

Section A 甲部 (5 marks 分)

For each question, choose the **most appropriate** answer and mark the corresponding box (A, B, C, or D) on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.

請為下列每題各選一個**最適合**的答案，然後把答題紙對應的空格（A、B、C、或 D）填滿。
答對得一分，答錯不扣分。

1. What is the output of the following program? 以下程序的輸出是什麼？

C++

```
int main() {  
    int a[5] = {1, 2, 3, 4, 5};  
    for (int i = 1; i < 5; ++i)  
        a[i] += a[i - 1];  
    for (int i = 0; i < 5; ++i)  
        cout << a[i] << " ";  
}
```

- A. 1 3 5 7 9
B. 1 3 6 10 15
C. 1 2 3 4 5
D. 2 4 6 8 10
2. Which of the following program segments correctly exchanges the values of a and b? It is known that a and b are integer variables, and $-10^9 \leq a, b \leq 10^9$.

以下哪些程序段正確地把 a 和 b 的值對調？已知 a 和 b 是整數變量，且 $-10^9 \leq a, b \leq 10^9$ 。

i. C++

```
int c = a;  
b = c;  
a = b;
```

ii. C++

```
a ^= b;  
b ^= a;  
a ^= b;
```

iii. C++

```
a = a + b;  
b = a - b;  
a = a - b;
```

- A. i and ii only 只有 i 和 ii
B. i and iii only 只有 i 和 iii
C. ii and iii only 只有 ii 和 iii
D. i, ii and iii i、ii 和 iii

3. What is the output of the following program? 以下程序的輸出是什麼？

C++

```
int main() {  
    string s1 = "1";  
    char s2 = '1';  
    int s3 = '1';  
    s1 = s1 + '1';  
    s2 = s2 + '1';  
    s3 = s3 + '1';  
    cout << s1 << " " << s2 << " " << s3;  
}
```

- A. b b 98
- B. 11 b 98
- C. b 11 98
- D. 11 11 2

4. What is the output of the following program? 以下程序的輸出是什麼？

C++

```
int main() {  
    double a = 1.7, b = 1.7, c = 1.7;  
    int d = (int) a + b + c;  
    cout << d;  
}
```

- A. 3
- B. 4
- C. 5
- D. 5.1

5. What is the output of the following program? 以下程序的輸出是什麼？

C++

```
int main() {  
    int x[5] = {5, 18, 31, 14, 16};  
    int y = 0;  
    int z = 0;  
    for (int i = 0; i < 5; ++i) {  
        y ^= x[i];  
        if ((y & (1 << i)) != 0) {  
            z = z + 1;  
        }  
    }  
    cout << z;  
}
```

- A. 1
- B. 2
- C. 3
- D. 4

END OF SECTION A 甲部完

Section B 乙部 (9 marks 分)

The blanks are labeled from L to Q. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 L 至 Q，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Answers must be in C++. Completed programs shall be compiled and executed according to the procedure specified on Page 10.
您必須使用 C++ 作答。完成的程序將以第 10 頁之步驟進行編譯及執行。
- (2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。

1. What is the output of the following program? 以下程序的輸出是什麼？

C++

```
int main() {  
    int a = 2, b = 3, c = 5;  
    b *= c;  
    a += b;  
    b -= c;  
    a /= c;  
    cout << a + b + c;  
}
```

Answer 答案: _____ L _____ (1.5 marks 分)

2. What is the output of the following program? 以下程序的輸出是什麼？

C++

```
int main() {  
    int ans = 0;  
    for (int i = 2; i < 16; ++i) {  
        for (int j = i * i; j < 16; j += i) ans += i;  
    }  
    cout << ans;  
}
```

Answer 答案: _____ M _____ (1.5 marks 分)

3. Consider the following program: 考慮以下程序：

C++

```
int main() {  
    int n;  
    cin >> n;  
    n &= n >> 1;  
    n &= n >> 1;  
    n &= n >> 1;  
    cout << n;  
}
```

Assume the input is an integer between 0 and 63 (inclusive). Across all inputs, how many distinct integers can the program output?

假設輸入為一個 0 至 63（含）之整數。對於所有輸入，程式總共能輸出多少個不同的整數？

Answer 答案: N (1.5 marks 分)

4. Complete the function `IsNotEqual(x, y)` such that it returns `true` if and only if x is not equal to y for all integers $-10000 \leq x, y \leq 10000$.

完成函數 `IsNotEqual(x, y)` 使其對於所有整數 $-10000 \leq x, y \leq 10000$ ，傳回 `true` 當且僅當 x 不等於 y 。

C++

```
bool IsNotEqual(int x, int y) {  
    return   0  ;  
}
```

Answer 答案: 0 (1.5 marks 分)

5. Let a be an array of integers of length n , where $n \geq 1$. For all $0 \leq i < n$, it is guaranteed that $0 \leq a[i] \leq 1$. Complete the following function `AreAllOnes(a)`, such that it returns `true` if and only if all elements in the array are equal to 1.

設 a 為一長度為 n 的整數陣列，其中 $n \geq 1$ 。對於所有 $0 \leq i < n$ ，保證 $0 \leq a[i] \leq 1$ 。完成以下函數 `AreAllOnes(a)`，使其當且僅當陣列中所有元素皆為 1 時傳回 `true`。

C++

```
bool AreAllOnes(vector<int> a) {  
    bool ok = true;  
    int n = a.size();  
    for (int i = 0; i < n; ++i) {  
        ok =   P  ;  
    }  
    return ok;  
}
```

Answer 答案: P (1.5 marks 分)

6. This is the 30th edition of HKOI. If you look up HKOI's website, you will find that the 1st HKOI was "HKOI 1997". Let e be the edition number, the naming format, "HKOI {1997 + e - 1}", remained in use until the 18th HKOI ("HKOI 2014"). Since the 19th HKOI ("HKOI 2014/15"), the naming format has changed to "HKOI {1997 + e - 2}/{the two least significant digits of (1997 + e - 1)}".

本屆是第 30 屆香港電腦奧林匹克競賽 (HKOI)。如果你查看 HKOI 的網站，你會發現第 1 屆 HKOI 是 "HKOI 1997"。設 e 為屆數，直至第 18 屆 HKOI ("HKOI 2014")，命名格式是 "HKOI {1997 + e - 1}"。而從第 19 屆 HKOI ("HKOI 2014/15") 開始，命名格式則改為 "HKOI {1997 + e - 2}/{(1997 + e - 1) 的尾兩數位}"。

Assuming that the naming format will not change again in the future, complete the function $f(e)$ below, such that for all $1 \leq e \leq 100$, it outputs the correct e^{th} HKOI's name following the naming formats above.

假設未來不會再轉用其他命名格式，完成以下函數 $f(e)$ ，使其對於所有 $1 \leq e \leq 100$ ，按照以上的命名格式輸出第 e 屆 HKOI 的正確名稱。

C++

```
void f(int e) {  
    if (e <= 18) {  
        cout << "HKOI " << 1997 + e - 1;  
    }  
    else {  
        cout << "HKOI " << Q;  
    }  
}
```

Answer 答案: Q (1.5 marks 分)

END OF SECTION B 乙部完

Section C 丙部 (6 marks 分)

The blanks are labeled from R to V. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 R 至 V，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Answers must be in C++. Completed programs shall be compiled and executed according to the procedure specified on Page 10.
您必須使用 C++ 作答。完成的程序將以第 10 頁之步驟進行編譯及執行。
- (2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。
- (5) If blank X is divided into N parts $X_1, X_2, \dots, X_N$, it means that marks will only be given when $X_1, X_2, \dots, X_N$ are all correct.
如果空格 X 分為 N 部份 $X_1, X_2, \dots, X_N$ ，那麼 $X_1, X_2, \dots, X_N$ 皆為正確才會給分。

1. Let n and m be integers. We call the pair (n, m) a good pair if and only if both n and m are even. For example, $(8, 4)$, $(2, 6)$ and $(10, 10)$ are good pairs, but $(5, 2)$ and $(7, 7)$ are not.

設 n 和 m 為整數。當且僅當 n 和 m 都是偶數時，我們稱數對 (n, m) 為一個好數對。例如， $(8, 4)$ 、 $(2, 6)$ 和 $(10, 10)$ 都是好數對，但 $(5, 2)$ 和 $(7, 7)$ 不是。

(a) Complete the function `IsGoodPair( $n, m$ )` such that it returns `true` if and only if (n, m) is a good pair, where $1 \leq n, m \leq 10^9$.

完成函數 `IsGoodPair( $n, m$ )`，使其當且僅當 (n, m) 是一個好數對時傳回 `true`，其中 $1 \leq n, m \leq 10^9$ 。

C++

```
bool IsGoodPair(int n, int m) {  
    return ____R____;  
}
```

Answer 答案: ____R____ (1 mark 分)

(b) Assume `IsGoodPair( $n, m$ )` is implemented correctly, complete the function `CountGoodPairs1( $k$ )` such that it returns the number of unique good pairs where $1 \leq n, m \leq k$ for all $1 \leq k \leq 1000$. The pairs are unordered, i.e. (n, m) and (m, n) are considered the same pair.

假設 `IsGoodPair( $n, m$ )` 已被正確實現，完成函數 `CountGoodPairs1( $k$ )`，使其對於 $1 \leq k \leq 1000$ ，傳回滿足 $1 \leq n, m \leq k$ 的不同好數對的數量。數對沒有順序，即 (n, m) 和 (m, n) 算作同一個數對。

C++

```
int CountGoodPairs1(int k) {  
    int count = 0;  
    for (int n = 1; n <= k; ++n) {  
        for (int ____S____) {  
            if (IsGoodPair(n, m))  
                count++;  
        }  
    }  
    return count;  
}
```

Answer 答案: ____S____ (1 mark 分)

(c) Complete the function `CountGoodPairs2( $k$ )` such that it returns the same value as `CountGoodPairs1( $k$ )` for all $1 \leq k \leq 1000$.

完成函數 `CountGoodPairs2( $k$ )` 使其對於 $1 \leq k \leq 1000$ ，傳回值與 `CountGoodPairs1( $k$ )` 相同。

C++

```
int CountGoodPairs2(int k) {  
    int count = 0;  
    for (int n = 2; n <= k; n += 2) {  
        count += ____T____;  
    }  
    return count;  
}
```

Answer 答案: ____T____ (1 mark 分)

2. Given a string s only consisting of digits from 0 to 9 inclusive. s possibly containing leading zeros. A contiguous substring $s[i..j]$ (where $i \leq j$) is called *good* if and only if it, when being seen as a number, is a multiple of 3.

給定一個只包含數位 0 至 9 的字串 s 。 s 可能含有前導零。一個連續的子串 $s[i..j]$ (其中 $i \leq j$) 被稱為「良好」，當且僅當它被當成數字時是 3 的倍數。

(a) Complete the function $f(s)$ such that it returns the number of good substrings of a string s , where s is a string of length at most 1000.

完成函數 $f(s)$ ，使得它傳回 s 中良好子串的數量，其中 s 是一個長度不大於 1000 的字串。

C++

```
int f(string s) {
    int n = s.length();
    int a = 0;
    for (int i = 0; i < n; ++i) {
        int sum = 0;
        for (int j = i; j < n; ++j) {
            sum += s[j] - '0';
            if (____ U1 ____ ) {
                ____ U2 ____;
            }
        }
    }
    return a;
}
```

Answer 答案: _____ U1 _____ U2 _____ (1.5 marks 分)

(b) Complete the function $g(s)$ so that for all string s with length at most 1000, the return value of $g(s)$ is the same as the return value of $f(s)$.

完成函數 $g(s)$ ，使其對所有長度不大於 1000 的字串 s ， $g(s)$ 與 $f(s)$ 的傳回值相等。

C++

```
int g(string s) {
    int n = s.length();
    int a = 0;
    int p[3] = {1, 0, 0};
    int sum = 0;
    for (int i = 0; i < n; ++i) {
        sum += s[i] - '0';
        ____ V ____;
        ++p[sum % 3];
    }
    return a;
}
```

Answer 答案: _____ V _____ (1.5 marks 分)

END OF PAPER 2 卷二 (C++) 完