Paper 1 (Compulsory Part) 卷一（必答部分）

	Format 類型	# Questions 題目數	Total Marks 佔分
Section A 甲部	Multiple Choice 多項選擇題	10	10
Section B 乙部	Short Questions 短答題	6 (A - F)	9
Section C 丙部	Long Questions 長填充題	2 (G - J)	6
Total 總分			**25**

(1) The term "arrays" in Paper 1 generally refers to lists in Python.
卷一提及的「陣列」泛指 Python 語言的串列。

(2) The following code is added to the beginning of all C++ programs.
在所有 C++ 程序的頂部加入以下程式碼:

C++

```cpp
#include <algorithm>
#include <array>
#include <cmath>
#include <cstdlib>
#include <deque>
#include <forward_list>
#include <iostream>
#include <list>
#include <queue>
#include <stack>
#include <string>
#include <utility>
#include <vector>
using namespace std;
```

(3) Assume all programs shall be compiled and executed in Debian 12 using the compilers and commands below.
假設所有程序都將在 Debian 12 下使用以下編譯器及指令進行編譯，然後執行。

GNU G++ (g++-12 12.2.0)
```
g++ -std=c++20 program.cpp -o program
./program
```

Python 3 (python3.11 3.11.2)
```
python3.11 -S -m py_compile program.py
python3.11 -O -S program.py
```

Section A 甲部 (10 marks 分)

For each question, choose the **most appropriate** answer and mark the corresponding box (A, B, C, or D) on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.
請為下列每題各選一個**最適合**的答案，然後把答題紙對應的空格（A、B、C、或 D）填滿。
答對得一分，答錯不扣分。

1. The size of the Discord app for MacOS is approximately 400 MB. Which of the following is closest to the size of the app in bytes?
    MacOS 版本的 Discord 應用程式大小約為 400 MB。以下哪一個是最接近該應用程式大小的位元組數值？

    A. $\dfrac{400}{1024 \times 1024}$
    B. $\dfrac{400}{1024}$
    C. 400×1024
    D. $400 \times 1024 \times 1024$

2. What is the output of the following program? 以下程序的輸出是什麼？

    Python
    ```python
    a = [1, 2, 3, 4, 5]

    for i in range(5):
        a[i] = a[(i * 3) % 5]

    for i in range(5):
        print(a[i], end=" ")
    ```

    C++
    ```cpp
    int main() {
      int a[5] = {1, 2, 3, 4, 5};
      for (int i = 0; i < 5; ++i)
        a[i] = a[(i * 3) % 5];

      for (int i = 0; i < 5; ++i)
        cout << a[i] << " ";
    }
    ```

    A. 1 4 4 5 4
    B. 1 4 4 5 5
    C. 1 4 5 4 4
    D. 1 5 4 4 5

3. Which of the following boolean expressions are equivalent to A XOR B?
    以下哪些布爾表達式等同於 A XOR B？

    i.   NOT(((NOT A) OR B) AND (A OR (NOT B)))
    ii. NOT((A AND B) OR ((NOT A) AND (NOT B)))

    A. i only   只有 i
    B. ii only   只有 ii
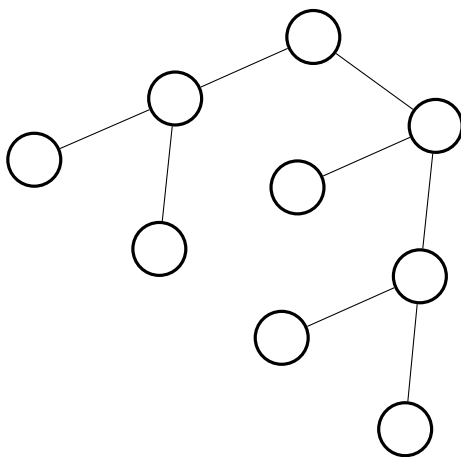   C. i and ii   i 和 ii
    D. None of them   無

4. Which of the following statements about sorting an array are true?
    以下哪些關於排序陣列的陳述為真？

    i.   Merge Sort involves merging arrays that are already sorted.
         合併排序涉及合併已經排序的陣列。

    ii. Quicksort involves quickly selecting the smallest value in the array during each iteration.
         快速排序涉及在每次迭代中快速選擇陣列中的最小值。

    iii. Insertion Sort involves inserting the next unsorted element into its correct position in the sorted portion.
         插入排序涉及將下一個未排序的元素插入到已排序部分的正確位置。

    A.   i and ii only   只有 i 和 ii
    B.   i and iii only   只有 i 和 iii
    C.   ii and iii only   只有 ii 和 iii
    D.   i, ii and iii   i、ii 和 iii

5. Consider the following graph: 考慮下圖：

    What is the minimum number of vertices needed to be chosen such that for every edge, **at least** one of its endpoints is chosen?
    至少需要選擇多少個節點，才能使得對於圖中所有邊，**至少**其中一個端點被選擇？

    A.   2
    B.   3
    C.   4
    D.   5

6. You have an string S containing the characters A and B only. You can perform the following operations any number of times in any order on the string:
    你有一個只包含 A 和 B 字符的字串 S。你可以對該字串進行以下操作，次數及順序不限：

    1. Select three (not necessarily consecutive) characters that read A, B, A from left to right and erase them from S.
        選擇三個（不一定相鄰）從左到右為 A, B, A 的字符，並將其從 S 中刪除。
    2. Select three (not necessarily consecutive) characters that read A, A, B from left to right and erase them from S.
        選擇三個（不一定相鄰）從左到右為 A, A, B 的字符，並將其從 S 中刪除。

    Which of the following strings can be reduced to an empty string by performing the above operations zero or more times?
    以下哪一個字串可以通過執行上述操作零次或多次，最終變為空字串？

    i.   AAAABBBAA
    ii. ABAABBBAABAAAAA
    iii. AABABBAAB

    A. i only   只有 i
    B. ii only   只有 ii
    C. i and iii only 只有 i 和 iii
    D. ii and iii only 只有 ii 和 iii

7. Five players are playing a game of Among Us, where some of them are impostors and the rest are crewmates. Assuming that imposters always lie and crewmates always tell the truth, consider the following statements by the players:
    五名玩家正在玩《Among Us》，其中有些是內鬼，剩下的是船員。假設內鬼總是撒謊，而船員總是說實話，考慮以下玩家的陳述：

    1. Player 1: Player 2 is an impostor.   玩家 1：玩家 2 是內鬼。
    2. Player 2: Player 3 is an impostor.   玩家 2：玩家 3 是內鬼。
    3. Player 3: Player 4 is an impostor.   玩家 3：玩家 4 是內鬼。
    4. Player 4: There is an even number of crewmates.   玩家 4：我們之中有偶數個船員。
    5. Player 5: There is an odd number of crewmates.   玩家 5：我們之中有奇數個船員。

    What is the **minimum** possible number of crewmates?
    最少可能的船員數是多少？

    A. 0
    B. 1
    C. 2
    D. 3

8. It is known that data structures A and B are either a stack or a queue. Initially, both data structures are empty. The following operations are then executed in order on A and B:
   已知數據結構 A 和 B 皆為堆疊（棧）或隊列。最初，兩個數據結構都是空的。接下來，在 A 和 B 上按順序執行以下操作：

   1. Push 5 into A.　　　　　　　　　　　　　　　將 5 推入 A。

   2. Push 8 into A.　　　　　　　　　　　　　　　將 8 推入 A。

   3. Push 7 into B.　　　　　　　　　　　　　　　將 7 推入 B。

   4. Pop an element from A and push this element into B.　　從 A 彈出一個元素並將該元素推入 B。

   5. Pop an element from B and push this element into A.　　從 B 彈出一個元素並將該元素推入 A。

   6. Pop an element from A.　　　　　　　　　　從 A 彈出一個元素。

It is known that the value popped in the last operation is 7. Which of the following options are the data structures A and B?
已知最後一次操作中彈出的值是 7。以下哪一個選項是數據結構 A 和 B？

	A	B
A.	Queue 隊列	Queue 隊列
B.	Queue 隊列	Stack 堆疊
C.	Stack 堆疊	Queue 隊列
D.	Stack 堆疊	Stack 堆疊

9. There is a tournament with $2^7 = 128$ players. The players are assigned a distinct seeding from 1st to 128th, with the 1st seed being the highest and the 128th seed being the lowest. The tournament has 7 rounds. Each round, the remaining players are randomly paired against each other (each pairing is equally likely), and they play a match. The player with the higher seeding always wins. The winner of each match advances to the next round, and the loser is eliminated.
在一個有 $2^7 = 128$ 名選手的錦標賽中，選手們被分配了從 1 到 128 的獨特種子排名，其中 1 為最高，128 為最低。該錦標賽共進行 7 輪。每輪比賽中，剩餘的選手會隨機配對進行比賽（每種配對的概率都是一樣），種子排名高的選手總是獲勝。每場比賽的勝者將晉級下一輪，敗者則被淘汰。

What is the probability that the final round is the first seed versus the second seed? Choose the closest answer.
最後一輪是第一種子對第二種子的概率是多少？選擇最接近的答案。

   A. 0.25

   B. 0.5

   C. 0.75

   D. 1

10. Given a 4-by-4 grid, where each cell is initially coloured black or white.
    給定一個四乘四的網格，其中每個方格在一開始被塗上了黑色或白色。

    You start at the top-left corner cell. Every moment, you may move one cell up, down, left or right, and turn the colour of the new cell from black to white and white to black. At the end, you must go back to the top-left corner cell.
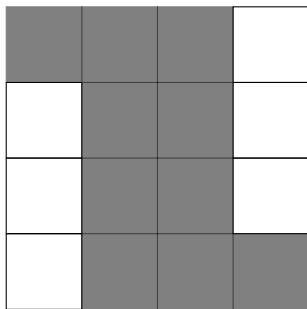   你從左上角方格開始。每一刻，你可以向上、下、左或右移動一格，並將新方格的顏色由黑變白或由白變黑。最後，你必須回到左上角方格。

    Note that at the beginning, the colour of the top-left corner would not be changed.
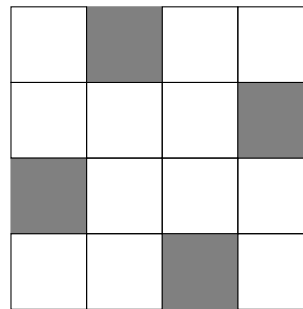   留意，一開始的時候，左上角方格的顏色並不會被改變。

    Which of the following starting configurations can you turn every cell to white?
    對於下列哪個起始局面，你能夠將所有方格變成白色？

    i.

    ii.

    A.   i only   只有 i
    B.   ii only   只有 ii
    C.   i and ii   i 和 ii
    D.   None of them   無

END OF SECTION A 甲部完

Section B 乙部 (9 marks 分)

The blanks are labeled from A to F. Please fill in the blanks on the answer sheet.
下列各空格分別命名為 A 至 F，請在答題紙上對應的地方填上答案。

Note 注意：

    (1) You can write only one character in each box on the answer sheet.
        答題紙上每個小格只可填上一個字符。

    (2) Answers must not exceed the designated number of boxes.
        答案長度不得多於該題提供的小格數目。

    (3) Write legibly. Unrecognizable answers will be regarded as incorrect.
        字體須端正清楚，無法辨別之答案當錯誤論。

1. What is the output of the following program? 以下程序的輸出是什麼？

Python
```python
a, b, c = 3, 2, 1
a = a - b + c
b = a - b + c
c = a - b + c

print(a, b, c, sep=',')
```

C++
```cpp
int main() {
  int a = 3, b = 2, c = 1;
  a = a - b + c;
  b = a - b + c;
  c = a - b + c;
  cout << a << "," << b << "," << c;
}
```

Answer 答案: _____A_____ (1.5 marks 分)

2. Alice distributes candies evenly among 24 people until no further equal distribution is possible. The number of remaining candies is 17. If Alice distributes the same number of candies among 6 people until no further equal distribution is possible, what is the number of remaining candies?
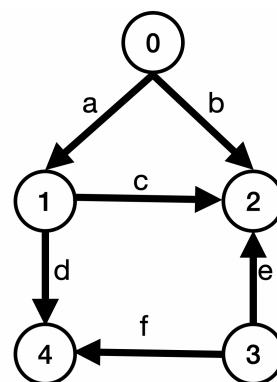   愛麗絲將糖果平均分配給 24 個人，直到無法進一步平均分配為止。剩餘糖果的數量為 17 顆。如果愛麗絲將相同數量的糖果分配給 6 個人，直到無法進一步平均分配為止，那麼剩餘糖果的數量是多少？

   Answer 答案: _____B_____ (1.5 marks 分)

3. A graph is strongly connected if and only if for all pair of vertices (u, v) in the graph, one can reach v from u by traversing some edges and can also reach u from v by traversing some edges. Find a set of edges in the graph shown on the right such that the number of edges in the set is minimized and when all the edges in the set is reversed, the graph is strongly connected. List all the edges in the set by separating them with commas.
   一個圖被稱為強連通當且僅當對於圖中所有頂點對 (u, v)，可以從 u 遍歷一些邊到達 v，且可以從 v 遍歷一些邊到達 u。在右圖找出一個邊的集合使得集合中邊的數目最少，且當集合中所有邊被反轉時，圖會強連通。列出所有集合中的邊並用逗號分隔。

   Answer 答案: _____C_____ (1.5 marks 分)

4. Complete the program such that it outputs 14.
    完成以下程序，使其輸出 14。

Python

```
s = '___D___'
a = 0
b = 0

for i in range(8):
    if s[i] == 'a':
        a = a + 1
    elif s[i] == 'b':
        b = b + a

print(b)
```

C++

```
int main() {
  string s = "___D___";
  int a = 0;
  int b = 0;
  for (int i = 0; i < 8; ++i) {
    if (s[i] == 'a')
      a = a + 1;
    else if (s[i] == 'b')
      b = b + a;
  }
  cout << b;
}
```

Answer 答案: _____D_____ (1.5 marks 分)

5. Suppose there are 3 students and 6 different candies. How many ways are there to distribute the candies to the students such that each student has at least one candy?
    假設有 3 名學生和 6 顆不同的糖果。有多少種方法可以將糖果分給學生，使每位學生至少有一顆糖果？

Answer 答案: _____E_____ (1.5 marks 分)

6. What is the output of the following program? 以下程序的輸出是什麼？

Python

```
a = [4, 5, 7, 2]
b = [1, 7, 3, 4, 1, 9, 0]
c = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

for i in range(10):
    for j in range(i + 1):
        if (j < 4) and (i - j < 7):
            c[i] ^= (a[j] ^ b[i - j])

ans = 0

for i in range(10):
    ans ^= c[i]

print(ans)
```

C++

```
int main() {
  int a[4] = {4, 5, 7, 2};
  int b[7] = {1, 7, 3, 4, 1, 9, 0};
  int c[10] = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0};
  for (int i = 0; i < 10; ++i) {
    for (int j = 0; j < i + 1; ++j) {
      if (j < 4 && i - j < 7)
        c[i] ^= (a[j] ^ b[i - j]);
    }
  }
  int ans = 0;
  for (int i = 0; i < 10; ++i) {
    ans ^= c[i];
  }
  cout << ans;
}
```

Answer 答案: _____F_____ (1.5 marks 分)

END OF SECTION B 乙部完

Section C 丙部 (6 marks 分)

The blanks are labeled from G to J. Please fill in the blanks on the answer sheet.
下列各空格分別命名為 G 至 J，請在答題紙上對應的地方填上答案。

Note 注意：

(1) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。

(2) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。

(3) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。

(4) If blank X is divided into N parts X1, X2, ..., XN, it means that marks will only be given when X1, X2, ..., XN are all correct.
如果空格 X 分為 N 部份 X1、X2、...、XN，那麼 X1、X2、...、XN 皆為正確才會給分。

1. S is an infinite string defined as follows:
S 是一個無限長的字串，定義如下：

   1. S initially is a string A.
   S 最初是一個字串 A。

   2. Repeat the following action indefinitely: take a copy S' of the entire current string S, replace every letter in S' by shifting it one position forward in the alphabet, so that A becomes B, B becomes C, ..., and Z wraps around to become A. Then place S' at the end of S.
   重複以下動作無限次：將目前字串 S 複製至 S'，將 S' 中的每個字母在字母表中向前移動一個位置來取代，如此 A 變成 B，B 變成 C，...，Z 繞一圈變成 A。然後將 S' 置於 S 的末端。

For illustration, this is how the string S is constructed in the first few steps:
為了說明，在最初的幾個步驟中，字串 S 是這樣構成的：

A → AB → ABBC → ABBCBCCD → ...

(a) What is the 65th letter of S?
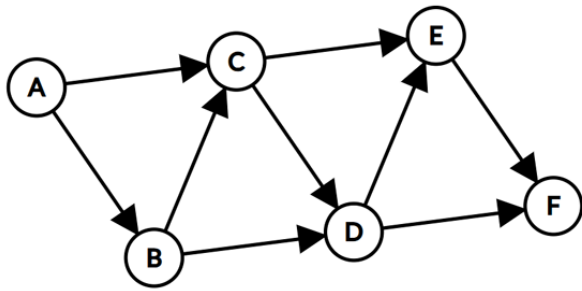S 的第 65 個字母是什麼？

Answer 答案: _____G_____ (1 mark 分)

(b) What is the 160th letter of S?
S 的第 160 個字母是什麼？

Answer 答案: _____H_____ (2 marks 分)
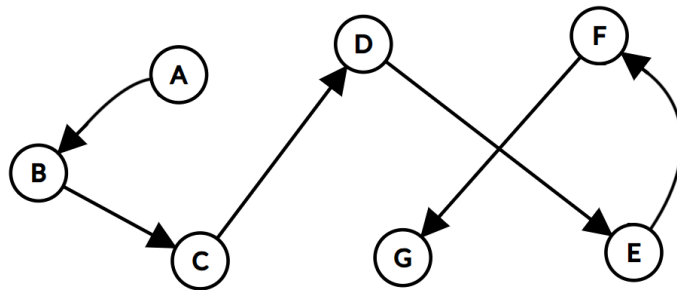
2. (a) Consider the following directed graph. How many different paths are there between vertex A and vertex F?
考慮以下有向圖，請問節點 A 及節點 F 之間有多少條不同的路徑？

Answer 答案: _____I_____ (1 mark 分)

(b) Consider the following directed graph. Currently, it has only 1 path between vertex A and vertex G.
考慮以下有向圖，現時節點 A 及節點 G 之間只有 1 條路徑。

Add exactly 3 directed edges to the graph such that there are 8 different paths between vertex A and vertex G. Draw the additional edges inside the rectangular box on the answer sheet using arrows. Multiple edges that go from the same vertex to the same vertex are not allowed. You must not create cycles in the graph.
請在圖加入剛好 3 條有向邊，使得節點 A 及節點 G 之間有 8 條不同的路徑。請在答題紙上的方框內用箭咀畫上加入的邊。不可以有重覆邊由一節點去另外一節點，並且不可以使圖變得有環。

Answer 答案: _____J_____ (2 marks 分)

END OF PAPER 1 卷一完

Paper 2 卷二

Important Instructions 重要指示

(1) Choose ONE programming language: Python OR C++.
請從 Python 或 C++ 中選擇一款編程語言。

(2) Indicate your choice by marking the corresponding box on the answer sheet.
將答題紙對應的空格填滿，以表示你的選擇。

(3) If you choose the Python version, attempt all questions from this page to page 8, and put your answers in the Python sections of the answer sheet.
如選擇 Python 版本，應作答此頁起至第 8 頁之所有題目，並將你的答案填寫在答題紙的 Python 部分。

(4) If you choose the C++ version, attempt all questions from page 9 to page 16, and put your answers in the C++ sections of the answer sheet.
如選擇 C++ 版本，應作答第 9 至 16 頁之所有題目，並將你的答案填寫在答題紙的 C++ 部分。

Paper 2 卷二 (Python)

	Format 類型	# Questions 題目數	Total Marks 佔分
Section A 甲部	Multiple Choice 多項選擇題	5	5
Section B 乙部	Short Questions 短答題	6 (A - F)	9
Section C 丙部	Long Questions 長填充題	2 (G - J)	6
Total 總分			**20**

Assume all programs shall be compiled and executed in Debian 12 using the compilers and commands below.
假設所有程序都將在 Debian 12 下使用以下編譯器及指令進行編譯，然後執行。

Python 3 (python3.11 3.11.2)
```
python3.11 -S -m py_compile program.py
python3.11 -O -S program.py
```

Section A 甲部 (5 marks 分)

For each question, choose the **most appropriate** answer and mark the corresponding box (A, B, C, or D) on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.
請為下列每題各選一個**最適合**的答案,然後把答題紙對應的空格(A、B、C、或 D)填滿。
答對得一分,答錯不扣分。

1. What is the output of the following program? 以下程序的輸出是什麼?

Python

```python
x = 4
if 2 ** x == 16:
    x = 5
    print(x, end=' ')
print(x)
```

    A.   4

    B.   4 4

    C.   5 4

    D.   5 5

2. The following operations are performed on a list `L` of $100,000$ numbers. Order the operations from the fastest to the slowest according to the most reasonable approximation of the order of time needed to perform them.
對一個大小為 $100,000$ 的串列 `L` 執行以下操作。根據執行時間的最合理估算,將以下操作從最快到最慢排序。

    i. `L.pop(0)`

    ii. `L.pop(-1)`

    iii. `L.sort()`

    A.   i, ii, iii

    B.   i, iii, ii

    C.   ii, i, iii

    D.   ii, iii, i

3. What is the output of the following program? 以下程序的輸出是什麼?

Python

```python
a = ['H', 'K', 'O', 'I']
print('+'.join(a), sep='-')
```

    A.   HKOI

    B.   ['H', 'K', 'O', 'I']

    C.   H-K-O-I

    D.   H+K+O+I

4. Assuming the input a is a 32-bit signed integer $(-2^{31} \le a \le 2^{31} - 1)$, what is the number of input a that does not crash the program?

   假設輸入 a 是一個 32 位元有符號整數 $(-2^{31} \le a \le 2^{31} - 1)$，不使程式崩潰的輸入 a 的數量是多少？

   Python

```python
a = int(input())
b = 1 // a
c = 1 // b
print(c)
```

   Remark: We regret that there were no correct option in the original paper.
   注：我們很抱歉原試卷中沒有正確選項。

5. Which of the input value will make the following program output `HkOi`?
   以下哪一個輸入會使以下程序輸出 `HkOi`？

   Python

```python
x = int(input())
alphabets = 'ABCDEFGHIJKLMNOPQRSTUVwXYZabcdefghijklmnopqrstuvwxyz'
result = ''
while (x > 0):
    residue = x % 100
    if residue < 52:
        result += alphabets[residue]
    x = x // 100
print(result)
```

   A.   7361434
   B.   8141007
   C.   34143607
   D.   34361407

END OF SECTION A 甲部完

Section B 乙部 (9 marks 分)

The blanks are labeled from A to F. Please fill in the blanks on the answer sheet.
下列各空格分別命名為 A 至 F，請在答題紙上對應的地方填上答案。

Note 注意：

(1) Answers must be in Python. Completed programs shall be compiled and executed according to the procedure specified on Page 1.
您必須使用 Python 作答。完成的程序將以第 1 頁之步驟進行編繹及執行。

(2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。

(3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。

(4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。

1. What is the output of the following program? 以下程序的輸出是什麼？

Python

```python
def magic(a):
    a[0] = a[2]
    a[2] = a[0]

b = [
    [1, 2, 3],
    [4, 5, 6],
    [7, 8, 9]
]
magic(b)
magic(b[0])
b[0][2] = 0
print(b[0][0] + b[1][1] + b[2][2])
```

Answer 答案: _____A_____ (1.5 marks 分)

2. What is the output of the following program? 以下程序的輸出是什麼？

Python

```python
a = [1, 3, 6, 0, 5, 2, 1, 9, 6, 8]
x = 0
for i in range(2025):
    if x % 2 == 0:
        x += a[x]
    else:
        x += a[-x]
    x %= 10
print(x)
```

Answer 答案: _____B_____ (1.5 marks 分)

W - 4

3. What is the output of the following program? 以下程序的輸出是什麼？

Python
```python
def f(x: int) -> int:
    if x == 0:
        return 3
    return f(x // 4) + g(x // 3)

def g(x: int) -> int:
    if x == 0:
        return 2
    return f(x // 3) + g(x // 4)

print(g(10))
```

Answer 答案: _____C_____ (1.5 marks 分)

4. Complete the program below, such that for all the given inputs, the output of the program matches the given output:
   完成以下程序，使得對於所有給定之輸入，程序之輸出與給定之輸出相符：

Input 輸入	Output 輸出
1	0
2	0
3	3
4	7
5	12
6	12
7	12

Python
```python
n = int(input())
total = 0
for i in range(1, n + 1):
    if _____D_____:
        total = total + i
print(total)
```

Answer 答案: _____D_____ (1.5 marks 分)

5. Let `a` be a list of integers. Complete the function `f` so that `f(a)` returns `a` sorted in ascending order.
    設 a 為一個整數串列。完成函數 f，使得 f(a) 以升序排序返回 a。

    Python

```
def f(a: list[int]) -> list[int]:
    n = len(a)
    if (n == 0):
        return []
    idx = 0
    for i in range(1, n):
        if a[idx] > a[i]:
            idx = i
    return [a[idx]] + f(_____E_____)
```

    Answer 答案: _____E_____ (1.5 marks 分)

6. You are provided with a function `rnd()` which uniformly and independently returns a random floating point value between 0 and 1 inclusive.
    你被給予一個函數 rnd()，函數會均勻且獨立地隨機回傳一個數值為 0 和 1 之間（含）的浮點數。

    Complete the following function `rnd_point_in_triangle()` such that `rnd_point_in_triangle()` uniformly and independently returns a random coordinate (x, y) where $0 \leq x, y \leq 1$ and $0 \leq x + y \leq 1$.
    完成以下函數 rnd_point_in_triangle() 使得 rnd_point_in_triangle() 會均勻且獨立地隨機回傳一個坐標 (x, y)，且 $0 \leq x, y \leq 1$ 和 $0 \leq x + y \leq 1$.

    Python

```
def rnd_point_in_triangle() -> tuple[float, float]:
    x = rnd()
    y = rnd()
    if x + y > 1:
        return (___F___)
    return (x, y)
```

    Answer 答案: _____F_____ (1.5 marks 分)

END OF SECTION B 乙部完

Section C 丙部 (6 marks 分)

The blanks are labeled from G to J. Please fill in the blanks on the answer sheet.
下列各空格分別命名為 G 至 J，請在答題紙上對應的地方填上答案。

Note 注意：

(1) Answers must be in Python. Completed programs shall be compiled and executed according to the procedure specified on Page 1.
您必須使用 Python 作答。完成的程序將以第 1 頁之步驟進行編繹及執行。

(2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。

(3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。

(4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。

(5) If blank X is divided into N parts X1, X2, ..., XN, it means that marks will only be given when X1, X2, ..., XN are all correct.
如果空格 X 分為 N 部份 X1、X2、...、XN，那麼 X1、X2、...、XN 皆為正確才會給分。

1. The following table shows the number of days in each month of 2025. 下表顯示 2025 年中每個月份的天數。

Month 月份	1	2	3	4	5	6	7	8	9	10	11	12
Number of days 天數	31	28	31	30	31	30	31	31	30	31	30	31

(a) Complete the function `has_thirty_days` so that the function `has_thirty_days(m)` returns whether there are exactly 30 days in the m-th month of 2025. It is guaranteed that $1 \leq m \leq 12$.
完成函數 `has_thirty_days` 使得函數 `has_thirty_days(m)` 返回 2025 年第 m 個月份是否恰好有 30 天。保證 $1 \leq m \leq 12$。

Python
```
def has_thirty_days(m: int) -> bool:
    return bool(___G___)
```

Answer 答案: _____G_____ (1.5 marks 分)

(b) Complete the function `has_thirty_one_days` so that the function `has_thirty_one_days(m)` returns whether there are exactly 31 days in the m-th month of 2025. It is guaranteed that $1 \leq m \leq 12$.
完成函數 `has_thirty_one_days` 使得函數 `has_thirty_one_days(m)` 返回 2025 年第 m 個月份是否恰好有 31 天。保證 $1 \leq m \leq 12$。

Python
```
def has_thirty_one_days(m: int) -> bool:
    return bool(___H___)
```

Answer 答案: _____H_____ (1.5 marks 分)

W - 7

2. Let a be a list of integers of length n, where $n = 2^k$ for some integer $1 \leq k \leq 10$. We define its *interleaved list* as another list b such that b[0], b[2], ... are the elements a[0], a[1], ..., a[$n/2-1$] in order, and b[1], b[3], ... are the elements a[$n/2$], a[$n/2+1$], ..., a[$n-1$] in order.
設 a 為一長度為 n 的整數串列，而對於某整數 $1 \leq k \leq 10$ 有 $n = 2^k$。我們定義串列的交錯串列為另一條串列 b 使得 b[0], b[2], ... 依序為元素 a[0], a[1], ..., a[$n/2-1$]，而 b[1], b[3], ... 依序為元素 a[$n/2$], a[$n/2+1$], ..., a[$n-1$]。

For example, the interleaved list of [0, 1, 2, 3, 4, 5, 6, 7] is [0, 4, 1, 5, 2, 6, 3, 7].
例如，[0, 1, 2, 3, 4, 5, 6, 7] 的交錯串列為 [0, 4, 1, 5, 2, 6, 3, 7]。

(a) Complete the function find_interleave so that the function find_interleave(a) returns the interleaved list of a.
完成函數 find_interleave 使得函數 find_interleave(a) 返回 a 的交錯串列。

Python

```
def find_interleave(a: list[int]) -> list[int]:
    n = len(a)
    b = [0 for _ in range(n)]
    l = 0
    r = n // 2
    for i in range(0, n // 2):
        b[___I1___] = a[l]
        b[___I2___] = a[r]
        l += 1
        r += 1
    return b
```

Answer 答案: _____ I1 _____ _____ I2 _____ (1 mark 分)

(b) Complete the functions helper and to_interleave so that the function to_interleave(a) **modifies** the list a to its interleaved list.
完成函數 helper 和 to_interleave 使得函數 to_interleave(a) **修改**串列 a 為它的交錯串列。

Python

```
def helper(a: list[int], l: int, r: int) -> None:
    n = r - l
    if n <= 2:
        return
    for i in range(0, ___J1___):
        x = ___J2___
        y = ___J3___
        a[x], a[y] = a[y], a[x]
    mid = (l + r) // 2
    helper(a, l, mid)
    helper(a, mid, r)

def to_interleave(a: list[int]) -> None:
    n = len(a)
    helper(a, 0, n)
```

Answer 答案: _____ J1 _____ _____ J2 _____ _____ J3 _____ (2 marks 分)

END OF PAPER 2 卷二 (Python) 完

Paper 2 卷二 (C++)

	Format 類型	# Questions 題目數	Total Marks 佔分
Section A 甲部	Multiple Choice 多項選擇題	5	5
Section B 乙部	Short Questions 短答題	6 (K - P)	9
Section C 丙部	Long Questions 長填充題	2 (Q - T)	6
Total 總分			**20**

(1) Assume that the size of the `int` data type is 32 bits.
假設 `int` 資料類型的長度為 32 位元。

(2) The following code is added to the beginning of all C++ programs.
在所有 C++ 程序的頂部加入以下程式碼:

C++

```cpp
#include <algorithm>
#include <array>
#include <cmath>
#include <cstdlib>
#include <deque>
#include <forward_list>
#include <iostream>
#include <list>
#include <queue>
#include <stack>
#include <string>
#include <utility>
#include <vector>
using namespace std;
```

(3) Assume all programs shall be compiled and executed in Debian 12 using the compilers and commands below.
假設所有程序都將在 Debian 12 下使用以下編譯器及指令進行編譯，然後執行。

GNU G++ (`g++-12` 12.2.0)
`g++ -std=c++20 program.cpp -o program`
`./program`

Section A 甲部 (5 marks 分)

For each question, choose the **most appropriate** answer and mark the corresponding box (A, B, C, or D) on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.
請為下列每題各選一個**最適合**的答案，然後把答題紙對應的空格（A、B、C、或 D）填滿。
答對得一分，答錯不扣分。

1. Given an integer variable x, where $0 \le x \le 30$, which of the following expressions can be used to calculate 2^x?
   給定一個整數變量 x，而 $0 \le x \le 30$，以下哪項表達式能用來計算 2^x？

   A.　2 ^ x
   B.　2 ** x
   C.　1 << x
   D.　x ^ 2

2. What is the output of the following program? 以下程序的輸出是什麼？

   C++
   ```
   int main() {
     int a[5] = {3, 2, 7, 6, 8};
     int sum = 0;
     for (int i = 1; i < 3; ++i) {
       sum = sum + a[i];
     }
     cout << sum;
   }
   ```

   A.　5
   B.　9
   C.　12
   D.　15

3. a is an array of type vector<int> with $100,000$ integers. The following operations are performed on a. Order the operations from the fastest to the slowest according to the most reasonable approximation of the order of time needed to perform them.
   a 是一個類型為 vector<int>，包含 $100,000$ 個整數的陣列。對 a 執行以下操作。根據執行時間的最合理估算，將以下操作從最快到最慢排序。

   　i. reverse(a.begin(), a.end());
   　ii. sort(a.begin(), a.end());
   　iii. a.pop_back();

   A.　i, ii, iii
   B.　i, iii, ii
   C.　iii, i, ii
   D.　iii, ii, i

4. What is the output of the following program? 以下程序的輸出是什麼？

C++

```cpp
int main() {
  int x = 20;
  int y = 24;
  if (x + y == 2024) {
    x = 24;
    cout << x << " ";
  } else {
    int x = 24;
  }
  cout << x;
}
```

    A.   20

    B.   24

    C.   20 24

    D.   24 24

5. What is the output of the following program? 以下程序的輸出是什麼？

C++

```cpp
void f(int x) {
  cout << x;
}
int g() {
  cout << 3;
  return 2;
}

int main() {
  f(g());
}
```

    A.   2

    B.   3

    C.   23

    D.   32

END OF SECTION A 甲部完

Section B 乙部 (9 marks 分)

The blanks are labeled from K to P. Please fill in the blanks on the answer sheet.
下列各空格分別命名為 K 至 P，請在答題紙上對應的地方填上答案。

Note 注意：

(1) Answers must be in C++. Completed programs shall be compiled and executed according to the procedure specified on Page 9.
您必須使用 C++ 作答。完成的程序將以第 9 頁之步驟進行編繹及執行。

(2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。

(3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。

(4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。

1. What is the output of the following program? 以下程序的輸出是什麼？

C++
```cpp
int a, b, c;

void foo(int &a, int b) {
  c = a + b;
  a = b + c;
  b = a + c;
}

int main() {
  a = 3; b = 5; c = 7;
  foo(b, c);
  cout << a << "," << b << "," << c;
}
```

Answer 答案: _____K_____ (1.5 marks 分)

2. What is the output of the following program? 以下程序的輸出是什麼？

C++
```cpp
int main() {
  int a[2][2] = {
    {34, 14},
    {11, 21}
  };
  int x = 0;
  for (int i = 0; i < 2024; ++i) {
    x += a[(x & 2) >> 1][x & 1];
  }
  cout << x % 4;
}
```

Answer 答案: _____L_____ (1.5 marks 分)

W - 12

3. What is the output of the following program? 以下程序的輸出是什麼？

C++

```cpp
int f(int x);
int g(int x);

int f(int x) {
  if (x == 0)
    return 2;
  return f(x / 2) + g(x / 3);
}

int g(int x) {
  if (x == 0)
    return 1;
  return f(x / 3) + g(x / 2);
}

int main() {
  cout << f(11);
}
```

Answer 答案: _____M_____ (1.5 marks 分)

4. Complete the program below, such that for all the given inputs, the output of the program matches the given output:
完成以下程序，使得對於所有給定之輸入，程序之輸出與給定之輸出相符：

Input 輸入	Output 輸出
1	1
2	1
3	4
4	4
5	9
6	9
7	16

C++

```cpp
int main() {
  int total = 0;
  int n;
  cin >> n;
  for (int i = 1; i <= n; ++i) {
    if (___N___)
      total = total + i;
  }
  cout << total;
}
```

Answer 答案: _____N_____ (1.5 marks 分)

5. Complete the program such that it outputs `knrl hdvb mre`.
   完成以下程序，使其輸出 `knrl hdvb mre`。.

   C++

```
bool t(char x) {
  return x >= 'a' && x <= 'z';
}

char s(char x) {
  return ___O___;
}

int main() {
  string str1 = "hkoi easy job";
  string str2 = "";
  for (char x : str1) {
    if (t(x)) {
      str2 += s(x);
    } else {
      str2 += x;
    }
  }
  cout << str2;
}
```

   Answer 答案: _____O_____ (1.5 marks 分)

6. Complete the program such that it outputs `10 9 8 7 6 5 4 3 3 2 2 2 1 0`.
   完成以下程序，使其輸出 `10 9 8 7 6 5 4 3 3 2 2 2 1 0`。

   C++

```
int a[14] = {3, 2, 2, 3, 6, 0, 5, 7, 10, 2, 9, 1, 8, 4};
int b[11];
int main() {
  for (int i = 0; i < 11; ++i) {
    b[i] = 0;
  }
  for (int i = 0; i < 14; ++i) {
    _____P_____;
  }
  for (int i = 10; i >= 0; --i) {
    for (int j = 0; j < b[i]; ++j) {
      cout << i;
      if (i > 0 || j + 1 < b[i]) {
        cout << ' ';
      }
    }
  }
}
```

   Answer 答案: _____P_____ (1.5 marks 分)

END OF SECTION B 乙部完

Section C 丙部 (6 marks 分)

The blanks are labeled from Q to T. Please fill in the blanks on the answer sheet.
下列各空格分別命名為 Q 至 T，請在答題紙上對應的地方填上答案。

Note 注意：

(1) Answers must be in C++. Completed programs shall be compiled and executed according to the procedure specified on Page 9.
您必須使用 C++ 作答。完成的程序將以第 9 頁之步驟進行編繹及執行。

(2) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。

(3) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。

(4) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。

(5) If blank X is divided into N parts X1, X2, ..., XN, it means that marks will only be given when X1, X2, ..., XN are all correct.
如果空格 X 分為 N 部份 X1、X2、...、XN，那麼 X1、X2、...、XN 皆為正確才會給分。

1. Complete the program below, such that it converts a given duration t in minutes into a string in the format of H:MM ($0 \leq t < 600$), where H is a digit representing hours, MM are two digits representing minutes. For example, f(123) should return the string 2:03 as 123 minutes can be converted to 2 hours and 3 minutes.
完成以下函數，使其將給定的時長 t (分鐘) 轉換成格式為 H:MM 的字串 ($0 \leq t < 600$)，其中 H 是代表小時的一個數位，MM 是代表分鐘的兩個數位。例如，f(123) 應該返回字符串 2:03，因為 123 分鐘可以轉換為 2 小時 3 分鐘。

C++

```cpp
string f(int t) {
  string s = "";
  s += '0' +    Q1   ;
  s += ':';
  s += '0' +    Q2   ;
  s += '0' +    Q3   ;
  return s;
}
```

Answer 答案: _____Q1_____ _____Q2_____ _____Q3_____ (2 marks 分)

2. Consider the following function: 考慮以下函數：

C++

```cpp
int f(int x) {
  int cnt = 0;
  for (int i = x; i >= 1; (--i) &= x) {
    ++cnt;
  }
  return cnt;
}
```

(a) What is the return value of `f(26)`?

`f(26)` 的傳回值是什麼？

Answer 答案: _____ R _____ (1 mark 分)

You **CANNOT** call other functions in your answers to parts (b) and (c), including `f`, `g` and `h`.

你 **不能** 在 (b) 和 (c) 部分的答案中呼叫其他函數，包括 `f`，`g` 和 `h`。

(b) Complete the function below, such that for all $1 \le x < 2^{10}$, the return value of `g(x)` is same as the return value of `f(x)`:

完成以下函數，使得對於所有 $1 \le x < 2^{10}$，`g(x)` 的傳回值與 `f(x)` 的傳回值相等：

C++

```cpp
int g(int x) {
  int cnt = 0;
  for (int i = 1; i <= x; ++i) {
    if ((___S1___) == (___S2___)) {
      ++cnt;
    }
  }
  return cnt;
}
```

Answer 答案: _____ S1 _____ _____ S2 _____ (1.5 marks 分)

(c) Complete the function below, such that for all $1 \le x < 2^{10}$, the return value of `h(x)` is same as the return value of `f(x)`:

完成以下函數，使得對於所有 $1 \le x < 2^{10}$，`h(x)` 的傳回值與 `f(x)` 的傳回值相等：

C++

```cpp
int h(int x) {
  int cnt = 0;
  for (int i = 0; i < 10; ++i) {
    if (___T1___) {
      ++cnt;
    }
  }
  return ___T2___;
}
```

Answer 答案: _____ T1 _____ _____ T2 _____ (1.5 marks 分)

END OF PAPER 2 卷二 (C++) 完