

# Hong Kong Olympiad in Informatics 2022/23 Senior Group

## Task Overview

ID	Name	Time Limit	Memory Limit	Subtasks
S231	Ambiguous Undecimal System	1.000 s	256 MB	4 + 7 + 7 + 6 + 25 + 27 + 24
S232	Team Formation	2.000 s	256 MB	11 + 19 + 16 + 8 + 15 + 31
S233	Doctor String in the Coordinates of Madness	1.000 s	256 MB	7 + 21 + 33 + 39
S234	Triangle Tour	2.000 s	256 MB	11 + 12 + 16 + 27 + 34

**Notice:**

Unless otherwise specified, inputs and outputs shall follow the format below:

- One space between a number and another number or character in the same line.
- No space between characters in the same line.
- Each string shall be placed in its own separate line.
- Outputs will be automatically fixed as follows: Trailing spaces in each line will be removed and an end-of-line character will be added to the end of the output if not present. All other format errors will not be fixed.

C++ programmers should be aware that using C++ streams (`cin` / `cout`) may lead to I/O bottlenecks and substantially lower performance.

For some problems 64-bit integers may be required. In Pascal it is `int64`. In C/C++ it is `long long` and its token for `scanf` / `printf` is `%lld`.

All tasks are divided into subtasks. You need to pass all test cases in a subtask to get points.

## S231 - AMBIGUOUS UNDECIMAL SYSTEM

Time Limit: 1.000 s / Memory Limit: 256 MB

"Is  $1011101.1101101_2$  less than, greater than or equal to  $5C.C9_{16}$ ?"

Alice was frustrated when she encountered this question in the HKOI Heat Event. "Why do computer scientists love binary and hexadecimal numbers so much? Can't they give the numbers in base 11?" she murmured.

After knowing that she could not enter the Final Event, she decides to reinvent the **Undecimal System**, which represents numbers in base 11, instead of base 10 in our usual decimal system. In the undecimal system, each digit could range from 0 to 10 inclusive. As the counting is done by elevens, the number  $238_A$  ( $A$  stands for Alice) will represent the value  $2 \times 11^2 + 3 \times 11^1 + 8 \times 11^0 = 283$  in decimal. She uses the two characters  $\boxed{10}$  to represent the digit 10.

However, Bob found out that Alice's system is ambiguous. Take the number  $510_A$  as an example. One can interpret 10 as a single digit, where the value of  $510_A$  would be  $5 \times 11^1 + 10 \times 11^0 = 65$  in decimal. One can also interpret 10 as two separate digits, in which case  $510_A$  represents the value  $5 \times 11^2 + 1 \times 11^1 + 0 \times 11^0 = 616$  in decimal. There are many other cases where a number can represent two or more values indistinguishable from each other.

To tease Alice, Bob writes two different positive integers  $X$  and  $Y$  on the blackboard, expressed in Alice's ambiguous undecimal system. Then he asks: "Can you determine whether  $X$  is less than or greater than  $Y$ ?"

Failing to solve this problem, Alice decides to leave this problem for contestants to solve in the HKOI Final Event (her real identity is an HKOI trainer!).

## INPUT

The first line contains a positive integer  $X$ , expressed in Alice's undecimal number system.

The second line contains a positive integer  $Y$ , also expressed in Alice's undecimal number system.

It is guaranteed that both  $X$  and  $Y$  do not contain leading zeros, and  $X$  and  $Y$  are not identical when viewed as a string.

## OUTPUT

If it must be true that  $X > Y$ , output  $\boxed{>}$ .

If it must be true that  $X < Y$ , output  $\boxed{<}$ .

If it is impossible to infer whether  $X > Y$  or  $X < Y$ , output  $\boxed{?}$ .

## SAMPLE TESTS

	Input	Output
1	<div>81056</div> <div>80823</div>	<div>?</div>

Consider the number  $81056_A$ .

- If 10 represents a single digit, then  $81056_A < 80823_A$ .
- If 10 represents two separate digits, then  $81056_A > 80823_A$ .

Thus, it is impossible to determine whether  $81056_A$  or  $80823_A$  is greater.

2	<div>108</div> <div>1106</div>	<div>&lt;</div>
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$108_A < 1106_A$  is always true no matter what values  $X$  and  $Y$  represent.

3	<table border="1"> <tr> <td>1010</td><td>&gt;</td></tr> <tr> <td>55</td><td></td></tr> </table>	1010	>	55	
1010	>				
55					

Consider the number  $1010_A$ . The possible values it can represent (in decimal) are:

- $10 \times 11^1 + 10 \times 11^0 = 120$
- $1 \times 11^2 + 0 \times 11^1 + 10 \times 11^0 = 131$
- $10 \times 11^2 + 1 \times 11^1 + 0 \times 11^0 = 1221$
- $1 \times 11^3 + 1 \times 11^2 + 1 \times 11^1 + 0 \times 11^0 = 1342$

Meanwhile, the number  $55_A$  must represent  $5 \times 11^1 + 5 \times 11^0 = 60$  in decimal, which must be less than that of  $1010_A$ .

4	<table border="1"> <tr> <td>240</td><td>&gt;</td></tr> <tr> <td>236</td><td></td></tr> </table>	240	>	236	
240	>				
236					

## SUBTASKS

For all cases:

Let  $|S|$  be the number of characters in  $S$ .

$$1 \leq |X|, |Y| \leq 10^6$$

$X$  and  $Y$  are not identical when viewed as a string

$X$  and  $Y$  do not contain leading zeros

	Points	Constraints
1	4	$X$ and $Y$ do not contain $\boxed{10}$ as a substring $1 \leq  X ,  Y  \leq 9$
2	7	$X$ and $Y$ do not contain $\boxed{10}$ as a substring $ X  \neq  Y $ $1 \leq  X ,  Y  \leq 1000$
3	7	$X$ and $Y$ do not contain $\boxed{10}$ as a substring $ X  =  Y $ $1 \leq  X ,  Y  \leq 1000$
4	6	$X$ and $Y$ do not contain $\boxed{10}$ as a substring $1 \leq  X ,  Y  \leq 1000$
5	25	$X$ has exactly one substring $\boxed{10}$ $Y$ does not contain $\boxed{10}$ as a substring $1 \leq  X ,  Y  \leq 1000$
6	27	$Y$ does not contain $\boxed{10}$ as a substring $1 \leq  X ,  Y  \leq 1000$
7	24	No additional constraints

## S232 - TEAM FORMATION

Time Limit: 2.000 s / Memory Limit: 256 MB

Dr Jones wishes to partition  $N$  students into  $K$  teams (numbered  $1, 2, 3, \dots, K$ ) for the upcoming programming contest but he quickly realizes this is not an easy job.

Dr Jones has labelled the students  $1, 2, 3, \dots, N$ . For various reasons,  $M$  pairs of students have become enemies over the years. In an effort to avoid heated arguments during the contest, Dr Jones tries not to place too many pairs of enemies in the same team. Specifically, if the total number of pairs of enemies that are placed in the same team is less than or equal to  $\frac{M}{K}$ , Dr Jones calls such a partition *peaceful*.

Upon further investigation, Dr Jones worked out that it is always possible to find a *peaceful* partition. Therefore, it will not come as a surprise that the problem of finding a *peaceful* partition has become one of the tasks in the programming contest. Can you find such a partition?

## INPUT

The first line consists of three integers  $N$ ,  $M$  and  $K$ .

$M$  lines follow. The  $i$ -th line consists of two integers  $X_i$  and  $Y_i$  meaning students  $X_i$  and  $Y_i$  are enemies. Enemy pairs will not appear more than once.

## OUTPUT

Output a *peaceful* partition in  $K$  lines. The  $i$ -th line should begin with an integer  $S_i$  denoting the number of students in team  $i$ , followed by  $S_i$  integers representing the members of team  $i$ .

Every student must appear in exactly one team. Teams are allowed to be empty.

## SAMPLE TESTS

	Input	Output
1	<div> <div>6 6 2</div> <div>1 4</div> <div>3 6</div> <div>2 1</div> <div>2 3</div> <div>2 6</div> <div>5 4</div> </div>	<div> <div>4 3 1 6 5</div> <div>2 2 4</div> </div>

Team 1 has one pair of enemies:  $(3, 6)$ . Team 2 has no pairs of enemies. The total number of enemy pairs in the same team is 1 which is  $\leq \frac{6}{2} = 3$  so this is a *peaceful* partition.

If Dr Jones assigned students  $1, 2, 3, 4, 5$  to team 1 and student 6 to team 2, it would not be a *peaceful* partition because there are 4 enemy pairs placed in the same team.

2

5 1 4	1 3
2 4	0
	2 5 2
	2 1 4

$\frac{M}{K} = \frac{1}{4}$  so any *peaceful* partitions must have no pairs of enemies in the same team.

3	6 6 2	3 1 2 3
	1 2	3 4 5 6
	1 3	
	1 4	
	1 5	
	1 6	
	5 6	

Team 1 has two pairs of enemies:  $(1, 2)$  and  $(1, 3)$ . Team 2 has one pair of enemies:  $(5, 6)$ . The total number of enemy pairs in the same team is 3 which is equal to  $\frac{M}{K}$ .

## SUBTASKS

For all cases:

$$2 \leq N \leq 50000$$

$$1 \leq M \leq \min\left(500000, \frac{N(N-1)}{2}\right)$$

$$1 \leq K \leq N$$

$$1 \leq X_i, Y_i \leq N \text{ and } X_i \neq Y_i \text{ for all } 1 \leq i \leq M$$

	Points	Constraints
1	11	$K = 2$ $2 \leq N \leq 15$
2	19	$K = 2$ $2 \leq N \leq 2000$
3	16	$K = 2$
4	8	For all $1 \leq i < j \leq N$ , students $i$ and $j$ are enemies
5	15	$2 \leq N \leq 2000$
6	31	No additional constraints

## S233 - DOCTOR STRING IN THE COORDINATES OF MADNESS

Time Limit: 1.000 s / Memory Limit: 256 MB

Doctor String is the most powerful sorcerer in Byteland-616. He is a master of the Mystic Arts, using his magic powers to protect Byteland-616 from various threats. Doctor String also has a Time Stone, one of the six Infinity Stones, where he could foresee all possible futures from every universe.

However, Villian Xormammu from Dark Dimension plans to kill Doctor String. He is terrifying as everyone who touches him will turn into ashes directly. One day, he decides to sneak into Byteland-616 and assassinate Doctor String secretly. Luckily, with the aid of Time Stone, Doctor String is able to know all the evil plans of Xormammu and he is thinking how to save himself from danger.

Byteland-616 can be viewed as an infinite 2-Dimensional coordinate plane and Doctor String stays at  $(0, 0)$  initially. Doctor String will perform  $N$  moves and the movement of Doctor String is controlled by a string  $S$  with length  $N$  consists of  $\text{U}$ ,  $\text{D}$ ,  $\text{L}$ ,  $\text{R}$ . On the  $i^{\text{th}}$  move, the position of Doctor String will change as follows:

- If  $S[i] = \text{U}$ , Doctor String will move from  $(x, y)$  to  $(x, y + 1)$
- If  $S[i] = \text{D}$ , Doctor String will move from  $(x, y)$  to  $(x, y - 1)$
- If  $S[i] = \text{L}$ , Doctor String will move from  $(x, y)$  to  $(x - 1, y)$
- If  $S[i] = \text{R}$ , Doctor String will move from  $(x, y)$  to  $(x + 1, y)$

Doctor String predicts that there are  $Q$  possible locations  $(X_i, Y_i)$  where Xormammu will stay at. If Doctor String moves to  $(X_i, Y_i)$ , he will die immediately as Xormammu activates his ability. For each of the possible locations, he wants to prevent himself from going to  $(X_i, Y_i)$  at all costs by changing string  $S$  with the following 2 operations:

1. Rearrange the characters in string  $S$  in any order.
2. Replace one of the character in string  $S$  with  $\text{U/D/L/R}$ .

Doctor String can apply these 2 operations any number of times. For each of the  $Q$  possible locations, Doctor String wants to know the minimum number of operations he needs to apply to  $S$  in order to avoid going to  $(X_i, Y_i)$ . Please be noted that the  $Q$  possible locations should be considered independently as only 1 location will become reality at last. As a top talent from HKOI, can you help Doctor String to solve the crisis?

## INPUT

The first line contains two integers  $N$  and  $Q$ , the length of  $S$  and the number of possible locations Xormammu will stay at.

The second line contains string  $S$ , the string that controls Doctor String's movement.

The next  $Q$  lines contain two integers  $X_i$  and  $Y_i$ , which is the  $i^{\text{th}}$  possible location  $(X_i, Y_i)$  Xormammu will stay at.

## OUTPUT

Output  $Q$  lines, where the  $i^{\text{th}}$  line should contain 1 number, which is the minimum number of operations that needed to be applied to  $S$  in order to avoid going to  $(X_i, Y_i)$ .

## SAMPLE TESTS

	Input	Output
<b>1</b>	<div> <div>2 3</div> <div>UR</div> <div>0 1</div> <div>1 1</div> <div>1 0</div> </div>	<div> <div>1</div> <div>1</div> <div>0</div> </div>

For (0,1), Doctor String can apply operation (1) once and rearrange  $S$  into  $\boxed{\text{RU}}$  which the movement will be  $(0,0) \rightarrow (1,0) \rightarrow (1,1)$ .

For (1,1), Doctor String can apply operation (2) once and turn  $S$  into  $\boxed{\text{RR}}$  which the movement will be  $(0,0) \rightarrow (1,0) \rightarrow (2,0)$ .

For (1,0), Doctor String doesn't have to apply any operations as he won't transfer to (1,0) with  $S$ .

## SUBTASKS

For all cases:

$$1 \leq N, Q \leq 100000$$

$$-10^9 \leq X_i, Y_i \leq 10^9$$

$$(X_i, Y_i) \neq (0, 0)$$

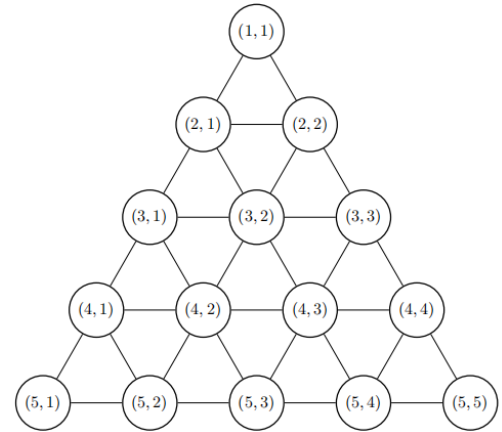
	Points	Constraints
<b>1</b>	7	All of the characters in $S$ are the same
<b>2</b>	21	All occurrences of $\boxed{\text{U}}/\boxed{\text{D}}$ appear before $\boxed{\text{R}}/\boxed{\text{L}}$ in $S$
<b>3</b>	33	$1 \leq N, Q \leq 1000$
<b>4</b>	39	No additional constraints

## S234 - TRIANGLE TOUR

Time Limit: 2.000 s / Memory Limit: 256 MB

Consider a triangular grid with  $N$  layers. The rows are numbered 1 to  $N$  from top to bottom, so the row at the top is row 1, and the row at the bottom is row  $N$ . For all  $1 \leq i \leq N$ , on row  $i$ , there are exactly  $i$  vertices, numbered as 1 to  $i$  from left to right. We denote the vertex  $c$  on row  $r$  as  $(r, c)$ .

Some pairs of vertices in the grid are connected by bidirectional edges. For each vertex  $(r, c)$  that is not located on the last row ( $1 \leq c \leq r < N$ ), it is connected to two vertices  $(r+1, c)$  and  $(r+1, c+1)$  on the next row. Besides, for each vertex  $(r, c)$  that is not the last vertex of the row ( $1 \leq c < r \leq N$ ), it is connected to the vertex  $(r, c+1)$ . The figure on the right shows an example with  $N = 5$ .



(A grid with  $N = 5$ )

$K$  vertices in the grid,  $(X_1, Y_1), (X_2, Y_2), \dots, (X_K, Y_K)$ , have been marked as *traps*. These vertices are dangerous and cannot be visited. Luckily, it is known that for any three consecutive rows, there is at most one trap. Formally, for  $1 \leq r \leq N-2$ , there is at most one  $i$  ( $1 \leq i \leq K$ ) such that  $r \leq X_i \leq r+2$ .

You are curious whether you can start at an arbitrary vertex, and visit each of the other vertices **exactly once** using the edges that connects the vertices, without visiting any traps.

In fact, this is always possible. Can you find such a path?

## INPUT

The first line of input consists of two integers  $N$  and  $K$ , the number of layers in the triangular grid and the number of traps.

$K$  lines follows. The  $i$ -th line consists of two integers  $X_i, Y_i$ , where  $(X_i, Y_i)$  are position of the  $i$ -th trap. The traps are given in increasing order of  $X_i$ .

## OUTPUT

Output an integer  $T$  on the first line, which denotes the number of vertices visited in your path.

Then, output  $T$  more lines. The  $i$ -th line should consist of two integers  $x_i$  and  $y_i$ , where  $(x_i, y_i)$  is the  $i$ -th vertex visited in your path.

Your output should represent a path **in sequence** that visits each vertex (except the traps) exactly once. Each consecutive pair of vertices must be directly connected by an edge.

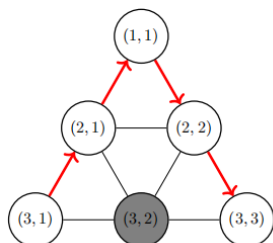
It can be proven that a solution exists under the given constraints. If there are multiple solutions, output any of them.



## SAMPLE TESTS

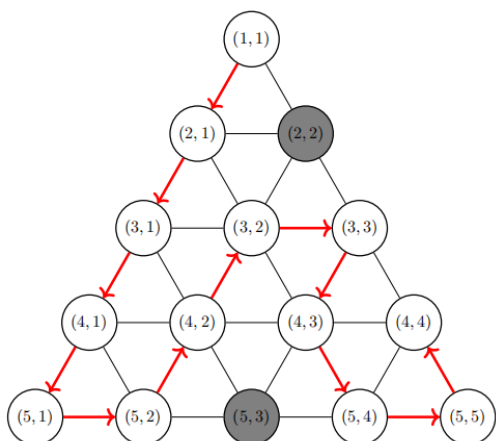
	Input	Output
1	<div> 3 1  3 2 </div>	<div> 5  3 1  2 1  1 1  2 2  3 3 </div>

The following figure denotes the path:



2	<div> 5 2  2 2  5 3 </div>	<div> 13  1 1  2 1  3 1  4 1  5 1  5 2  4 2  3 2  3 3  4 3  5 4  5 5  4 4 </div>
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The following figure denotes the path:



**SUBTASKS**

For all cases:

$$2 \leq N \leq 1000$$

$$0 \leq K \leq \lceil \frac{N}{3} \rceil$$

$$1 \leq Y_i \leq X_i \leq N \text{ for all } 1 \leq i \leq K$$

$$X_i \geq X_{i-1} + 3 \text{ for all } 1 < i \leq K$$

	Points	Constraints
<b>1</b>	11	$K = 0$
<b>2</b>	12	$K = 1$ The trap does not lie on the borders of the triangle (formally, $X_1 \neq 1$ , $X_1 \neq N$ , $Y_1 \neq 1$ and $X_1 \neq Y_1$ )
<b>3</b>	16	$K = 1$
<b>4</b>	27	$N$ is a multiple of 3 $N = 3K$ The row numbers of the traps are $2, 5, 8, \dots, (N - 1)$ (formally, $X_i = 3i - 1$ for all $1 \leq i \leq K$ )
<b>5</b>	34	No additional constraints