

	Format 類型	# Questions 題目數	Total Marks 佔分
Section A 甲部	Multiple Choice 多項選擇題	25	25
Section B 乙部	Fill-in-the-blanks 填充題	8 (A - M)	20
Total 總分			45

- (1) Assume that all variables without declaration shown in the following program segments have already been declared properly as 32-bit signed integers (Pascal: `longint`, C / C++: `int`).

下列程序段中所有未有列出宣告的變量，均假設已經適當地宣告為 32 位元有符號的整數 (Pascal: `longint`, C / C++: `int`)。

- (2) The following code is added to the beginning of all C and C++ programs.

在所有 C 和 C++ 程序的頂部加入以下程式碼:

C

```
#include <stdio.h>
#include <string.h>
#include <math.h>
#include <stdlib.h>
#include <stdbool.h>
```

C++

```
#include <cstdio>
#include <cmath>
#include <cstdlib>
#include <string>
#include <iostream>
using namespace std;
```

For C, `stdbool.h` defines the boolean data type `bool` and values `true` (equivalent to 1) and `false` (equivalent to 0).

對於 C，`stdbool.h` 定義了布爾數據類型 `bool` 及值 `true` (等同 1) 及 `false` (等同 0)。

- (3) Other than questions that mention compilation, assume all programs are compiled properly in Ubuntu 16.04 using the compilers and commands below.

除了有提及編譯的題目之外，假設所有程序都在 Ubuntu 16.04 下使用以下編譯器及指令正確地編譯。

```
Pascal: Free Pascal (fp-compiler 3.0.0)   fpc program.pas
C:      GNU GCC (gcc-4.9 4.9.3)         gcc -std=c99 program.c -o program
C++:   GNU G++ (g++-4.9 4.9.3)       g++ -std=c++98 program.cpp -o program
```

Section A 甲部 (25 marks 分)

For each question, choose the **most appropriate** answer and mark the corresponding box (A, B, C, or D) on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.

請為下列每題各選一個**最適合**的答案，然後把答題紙對應的空格（A、B、C、或D）填滿。
答對得一分，答錯不扣分。

1. What is the output of the following program? 以下程序的輸出是什麼？

Pascal	C	C++
<pre>var a, b, c, d: longint; begin a := -4 mod 5; b := -3 mod -2; c := 6 mod 4; d := a * (b + c); write(-d) end.</pre>	<pre>int a, b, c, d; int main() { a = -4 % 5; b = -3 % -2; c = 6 % 4; d = a * (b + c); printf("%d", -d); return 0; }</pre>	<pre>int a, b, c, d; int main() { a = -4 % 5; b = -3 % -2; c = 6 % 4; d = a * (b + c); cout << -d; return 0; }</pre>

- A. -4
- B. -12
- C. 4
- D. 12

2. Consider the following program: 考慮以下程序：

Pascal	C	C++
<pre>var a: array[0..4] of longint = (0, 9, 7, 3, 5); i, answer: longint; begin read(a[0]); answer := 0; for i := 1 to 4 do if (a[i] < answer) then answer := i; write(answer) end.</pre>	<pre>int a[5] = {0, 9, 7, 3, 5}; int i, answer; int main() { scanf("%d", &a[0]); answer = 0; for (i = 1; i <= 4; i++) if (a[i] < answer) answer = i; printf("%d", answer); return 0; }</pre>	<pre>int a[5] = {0, 9, 7, 3, 5}; int i, answer; int main() { cin >> a[0]; answer = 0; for (i = 1; i <= 4; i++) if (a[i] < answer) answer = i; cout << answer; return 0; }</pre>

How many integer inputs between 0 and 10 (inclusive) are there, so that the program will output 0?

請問有多少個介乎 0 和 10 之間（含）的整數輸入，會使程序輸出 0？

- A. 0
- B. 3
- C. 4
- D. 11

3. What is the output of the following program? 以下程序的輸出是什麼？

Pascal

```
var
  dx: array[0..7] of longint =
    (3, 2, 5, 4, 1, 6, 2, 4);
  dy: array[0..7] of longint =
    (2, 5, 3, 1, 4, 2, 7, 3);
  x, y, xx, yy, i: longint;
begin
  x := 0;
  y := 0;
  for i := 1 to 100 do
  begin
    xx := dx[x];
    yy := dy[y];
    x := (x + xx) mod 8;
    y := (y + yy) mod 8
  end;
  write(x, ' ', y)
end.
```

C

```
int dx[8] =
  {3, 2, 5, 4, 1, 6, 2, 4};
int dy[8] =
  {2, 5, 3, 1, 4, 2, 7, 3};
int x, y, xx, yy, i;
int main() {
  x = 0;
  y = 0;
  for (i = 1; i <= 100; i++) {
    xx = dx[x];
    yy = dy[y];
    x = (x + xx) % 8;
    y = (y + yy) % 8;
  }
  printf("%d %d", x, y);
  return 0;
}
```

C++

```
int dx[8] =
  {3, 2, 5, 4, 1, 6, 2, 4};
int dy[8] =
  {2, 5, 3, 1, 4, 2, 7, 3};
int x, y, xx, yy, i;
int main() {
  x = 0;
  y = 0;
  for (i = 1; i <= 100; i++) {
    xx = dx[x];
    yy = dy[y];
    x = (x + xx) % 8;
    y = (y + yy) % 8;
  }
  cout << x << " " << y;
  return 0;
}
```

- A. 7 2
- B. 3 5
- C. 7 5
- D. 3 7

4. Which of the following expressions is equivalent to (NOT (A AND (NOT B OR C)))?
以下哪一表達式與 (NOT (A AND (NOT B OR C))) 等價？

- A. NOT A OR (B OR NOT C)
- B. NOT A OR (B AND NOT C)
- C. NOT A AND (B OR NOT C)
- D. NOT A AND (B AND NOT C)

5. There will be a party next week.
下星期將會有一場派對。

- i. If Bob goes, Alice will go.
如果波比出席，那麼愛麗絲會出席。
- ii. If Bob does not go, Charlie will not go.
如果波比不出席，那麼查理不會出席。

Which of the following statements must be true?
以下哪一個陳述必為真？

- A. If Alice goes, Charlie must go. 如果愛麗絲出席，那麼查理會出席。
- B. If Charlie goes, Alice must go. 如果查理出席，那麼愛麗絲會出席。
- C. Bob will go. 波比會出席。
- D. Alice will go. 愛麗絲會出席。

6. Consider the following program: 考慮以下程序：

Pascal

```
var
  a: array[0..99] of longint;
  i: longint;
begin
  a[0] := 0;
  a[1] := 1;
  for i := 2 to 99 do
    a[i] := a[i - 1] + a[i - 2]
  end.
```

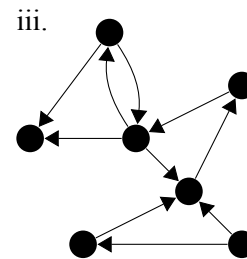
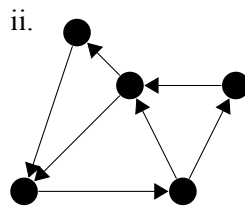
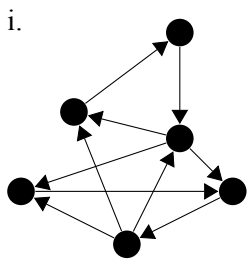
C / C++

```
int a[100];
int i;
int main() {
  a[0] = 0;
  a[1] = 1;
  for (i = 2; i <= 99; i++)
    a[i] = a[i - 1] + a[i - 2];
  return 0;
}
```

When calculating the value of which of the followings, integer overflow occurs the first time?
 當計算以下哪項的值時，首次出現整數溢出？

- A. a[31]
- B. a[47]
- C. a[93]
- D. Integer overflow does not occur. 沒有出現整數溢出。

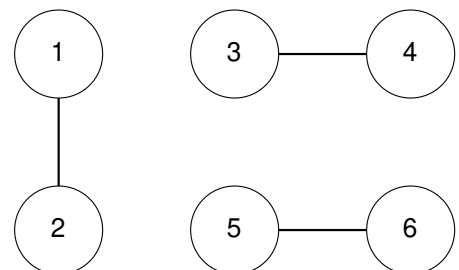
7. Which of the following graph(s) contains at least one path that visits every edge exactly once?
 以下哪個圖存在至少一條路徑，使得每條邊都被訪問剛好一次？



- A. i only 只有 i
- B. ii only 只有 ii
- C. iii only 只有 iii
- D. i, ii only 只有 i 和 ii

8. Consider the undirected graph consisting of 3 connected components, as shown on the right. How many ways to add 2 edges such that the graph becomes one single connected component?

考慮右方所示由 3 個連通分量組成的無向圖。有多少種方法添加 2 條邊，使得此圖變成單一個連通分量？



- A. 16
- B. 24
- C. 36
- D. 48

9. Consider the following function: 考慮以下函數：

Pascal

```
function f(n: longint): longint;  
var result, i: longint;  
begin  
  result := 0;  
  for i := 1 to n do  
    if ((i and 31) = 31) then  
      result := result + i;  
  f := result  
end;
```

C / C++

```
int f(int n) {  
  int result, i;  
  result = 0;  
  for (i = 1; i <= n; i++)  
    if ((i & 31) == 31)  
      result = result + i;  
  return result;  
}
```

What is the return value of $f(125)$?

$f(125)$ 的傳回值是甚麼？

- A. 189
- B. 310
- C. 316
- D. 496

10. Consider the following function: 考慮以下函數：

Pascal

```
function f(n: longint): longint;  
begin  
  if (n = 0) then  
    f := -1  
  else if (n = 1) then  
    f := 2  
  else  
    f := n + f(n - 1) + f(n - 2)  
end;
```

C / C++

```
int f(int n) {  
  if (n == 0)  
    return -1;  
  else if (n == 1)  
    return 2;  
  else  
    return n + f(n - 1) + f(n - 2);  
}
```

What is the return value of $f(2)$?

$f(2)$ 的傳回值是甚麼？

- A. 0
- B. 1
- C. 2
- D. 3

11. Refer to the previous question. What is the return value of $f(7)$?

承上題， $f(7)$ 的傳回值是甚麼？

- A. 28
- B. 36
- C. 49
- D. 84

12. Suppose that a_1, a_2, \dots, a_9 is a permutation of $1, 2, \dots, 9$. i.e. for any $1 \leq x \leq 9$, there exists $1 \leq i \leq 9$ such that $a_i = x$.

設 a_1, a_2, \dots, a_9 為一 $1, 2, \dots, 9$ 的排列。意即：對於所有 $1 \leq x \leq 9$ ，存在 $1 \leq i \leq 9$ 使得 $a_i = x$ 。

What is the number of different permutations satisfying both of the following two conditions?

問有多少不同的排列可同時符合以下兩條件。

- $a_1 < a_2 < a_3 < a_4 < a_5$
- $a_1 < a_6 < a_7 < a_8 < a_4 < a_9$

Two permutations are considered different if and only if there exists $1 \leq i \leq 9$ such that a_i is different in the two permutations.

兩排列會被視為不同當且僅當存在 $1 \leq i \leq 9$ 使得 a_i 在兩排列中不同。

- A. 12
- B. 20
- C. 24
- D. 64

13. There is a rooted tree which each node have zero or at least two children. Assume we already know the tree has 2020 leaf nodes, which of the following numbers can be the size of the tree?

有一棵有根樹，它的每個節點都有零個或至少兩個子節點。假設我們已經知道該樹有 2020 個葉節點，以下哪個數字可能是樹的大小？

- i. 4038
- ii. 4039
- iii. 4040
- iv. 4041

- A. i only 只有 i
- B. i, ii only 只有 i 和 ii
- C. i, ii and iii only 只有 i、ii 和 iii
- D. i, ii, iii and iv i、ii、iii 和 iv

14. Alice is navigating through a maze which can be represented by a grid of 4×4 numbered cells. She can start at any of the cells in the bottom-most row, and would like to end at any of the cells in the top-most row. In each move, she may only move to the cell to the left, to the right or the cell above, and she may only move to a cell that has its number larger than the one on the current cell. How many different paths may Alice possibly take?

16	15	14	13
9	10	11	12
8	7	6	5
1	2	3	4

愛麗絲正嘗試穿過一個 4×4 數字格的迷宮，她可以從最底一行的其中一格開始，並想在最頂端一行的其中一格結束。每一步他她以向左方、右方或上方的空格移動，並且只能移動到數字比目前空格中數字更大的空格。問愛麗絲共有多少種不同的路徑走法？

- A. 64
- B. 246
- C. 340
- D. 964

15. Consider the following function: 考慮以下函數：

Pascal

```
function f(x: longint): longint;  
begin  
  if (x = 0) then  
    f := 0  
  else if ((x mod 2) = 1) then  
    f := f(x div 2) + 1  
  else  
    f := f(x div 2) + 2  
end;
```

C / C++

```
int f(int x) {  
  if (x == 0)  
    return 0;  
  else if ((x % 2) == 1)  
    return f(x / 2) + 1;  
  else  
    return f(x / 2) + 2;  
}
```

What is the return value of $f(88)$?

$f(88)$ 的傳回值是甚麼？

- A. 11
- B. 12
- C. 13
- D. 14

16. In this question, we only consider simple graphs, i.e. undirected graphs with no self-loops and repeated edges. We call a graph bipartite if its vertices can be divided into two disjoint sets A and B , such that each edge connects one vertex in A with one vertex in B .

在這題，我們只考慮簡單圖，即沒有自環和重邊的無向圖。如果一個圖的點可以劃分成兩個不相交的集合 A 和 B ，使得每條邊都是將一個 A 的點和一個 B 的點連結，則我們稱之為二分圖。

What is the largest possible number of edges of a **disconnected** bipartite graph with 8 vertices?

請問一個有 8 個點的非連通二分圖，最多可有多少條邊？

- A. 4
- B. 8
- C. 12
- D. 16

17. Alice and Bob are having an archery duel, where they take turn alternatively to shoot a target. In every independent round, the player shoots an arrow and it would have a $\frac{5}{8}$ chance of hitting the target. The player who hits the target first wins the duel. If Alice shoots first, what is the probability that Alice wins the game?

愛麗絲和鮑伯正在進行箭術比賽，他們輪流對一個目標進行射擊。在每一個獨立回合中，射擊者會射出一枝箭，而該箭有 $\frac{5}{8}$ 機率命中目標。先命中目標的參賽者勝出。如果愛麗絲先進行射擊，愛麗絲勝出比賽的機率是多少呢？

- A. $\frac{9}{88}$
- B. $\frac{5}{8}$
- C. $\frac{365}{512}$
- D. $\frac{8}{11}$

18. What is the output of the following program? 以下程序的輸出是什麼？

Pascal	C	C++
<pre> var q: array[0..9] of longint; head, tail, sum, i, v: longint; procedure push(x: longint); begin q[tail] := x; tail := (tail + 1) mod 10 end; function pop(): longint; var v: longint; begin if (head = tail) then pop := -1 else begin v := q[head]; head := (head + 1) mod 10; pop := v end end; begin head := 0; tail := 0; sum := 0; v := 0; for i := 1 to 15 do push(i); while (v <> -1) do begin sum := sum + v; v := pop() end; write(sum) end.</pre>	<pre> int q[10]; int head, tail, sum, i, v; void push(int x) { q[tail] = x; tail = (tail + 1) % 10; } int pop() { int v; if (head == tail) return -1; else { v = q[head]; head = (head + 1) % 10; return v; } } int main() { head = 0; tail = 0; sum = 0; v = 0; for (i = 1; i <= 15; i++) push(i); while (v != -1) { sum = sum + v; v = pop(); } printf("%d", sum); }</pre>	<pre> int q[10]; int head, tail, sum, i, v; void push(int x) { q[tail] = x; tail = (tail + 1) % 10; } int pop() { int v; if (head == tail) return -1; else { v = q[head]; head = (head + 1) % 10; return v; } } int main() { head = 0; tail = 0; sum = 0; v = 0; for (i = 1; i <= 15; i++) push(i); while (v != -1) { sum = sum + v; v = pop(); } cout << sum; }</pre>

- A. 0
- B. 65
- C. 105
- D. 120

19. Recall that English vowels are A, E, I, O, and U. How many substrings of the string EXHAUSTION contains exactly three vowels? A substring of a string is a continuous segment of characters appearing in the given string.

英文的母音字母為 A、E、I、O 和 U。請問字串 EXHAUSTION 有多少個子字串恰好包含三個母音字母？我們稱一個字串的任何一段連續的字為它的子字串。

- A. 8
- B. 9
- C. 10
- D. 11

20. What is the output of the following program? 以下程序的輸出是什麼？

Pascal

```
var
  a, b: longint;
function f(x: longint): longint;
begin
  if (x <= 1) then
    f := 1
  else
    begin
      a := f(x - 1);
      b := f(x - 2);
      f := a + b
    end
end;
begin
  write(f(5))
end.
```

C

```
int a, b;
int f(int x) {
  if (x <= 1)
    return 1;
  else {
    a = f(x - 1);
    b = f(x - 2);
    return a + b;
  }
}

int main() {
  printf("%d", f(5));
  return 0;
}
```

C++

```
int a, b;
int f(int x) {
  if (x <= 1)
    return 1;
  else {
    a = f(x - 1);
    b = f(x - 2);
    return a + b;
  }
}

int main() {
  cout << f(5);
  return 0;
}
```

- A. 3
- B. 5
- C. 8
- D. 13

21. What is the output of the following program? 以下程序的輸出是什麼？

Pascal

```
var
  a, b, c, i: longint;
begin
  a := 56;
  b := 0;
  for i := 0 to 4 do
    begin
      c := a and (1 shl i);
      b := b + (c shr i)
    end;
  write(b)
end.
```

C

```
int a, b, c, i;
int main() {
  a = 56;
  b = 0;
  for (i = 0; i <= 4; i++) {
    c = a & (1 << i);
    b = b + (c >> i);
  }
  printf("%d", b);
  return 0;
}
```

C++

```
int a, b, c, i;
int main() {
  a = 56;
  b = 0;
  for (i = 0; i <= 4; i++) {
    c = a & (1 << i);
    b = b + (c >> i);
  }
  cout << b;
  return 0;
}
```

- A. 1
- B. 2
- C. 3
- D. 4

22. Alice has n coins and Bob has $(n + 1)$ coins. Every coin is fair and independent. When they flip all the coins at the same time, what is the probability of Bob getting more heads than Alice?

愛麗絲有 n 枚硬幣，鮑伯則有 $(n + 1)$ 枚硬幣。每枚硬幣均為公正和獨立。當他們同時投擲所有硬幣，鮑伯擲到「公」的數目比愛麗絲多的概率是什麼？

- A. $\frac{1}{2}$
- B. $\frac{2}{3}$
- C. $\frac{n+1}{n+3}$
- D. None of the other options 其他選項皆非

23. Consider the following program: 考慮以下程序：

Pascal	C	C++
<pre> var a, b: array[0..3] of longint; x, y, i: longint; begin for i := 0 to 3 do read(a[i]); for i := 0 to 3 do read(b[i]); x := 0; y := 0; for i := 0 to 7 do if ((x < 4) and ((y = 4) or (a[x] <= b[y]))) then begin write(a[x], ' '); inc(x) end else begin write(b[y], ' '); inc(y) end end end. </pre>	<pre> int a[4], b[4]; int x, y, i; int main() { for (i = 0; i <= 3; i++) scanf("%d", &a[i]); for (i = 0; i <= 3; i++) scanf("%d", &b[i]); x = 0; y = 0; for (i = 0; i <= 7; i++) { if (x != 4 && (y == 4 a[x] <= b[y])) { printf("%d ", a[x]); x++; } else { printf("%d ", b[y]); y++; } } return 0; } </pre>	<pre> int a[4], b[4]; int x, y, i; int main() { for (i = 0; i <= 3; i++) cin >> a[i]; for (i = 0; i <= 3; i++) cin >> b[i]; x = 0; y = 0; for (i = 0; i <= 7; i++) { if (x != 4 && (y == 4 a[x] <= b[y])) { cout << a[x] << " "; x++; } else { cout << b[y] << " "; y++; } } return 0; } </pre>

Which of the following inputs give the same output?

以下哪些輸入擁有相同輸出？

- i. 3 1 5 7 2 4 8 6
 - ii. 3 5 8 7 2 4 1 6
 - iii. 3 1 8 6 2 4 5 7
- A. i and ii only 只有 i 和 ii
 - B. i and iii only 只有 i 和 iii
 - C. ii and iii only 只有 ii 和 iii
 - D. i, ii and iii i、ii 和 iii

24. How many ways are there to delete 1 to 5 elements from the array $[1, 4, 2, 8, 5, 7]$, such that the remaining element(s) are in ascending order? The elements' relative order are preserved.

有多少種方法從陣列 $[1, 4, 2, 8, 5, 7]$ 中刪除 1 至 5 個元素，使得剩下的元素是由小至大排列的？(保留元素的相對次序)

- A. 27
- B. 28
- C. 29
- D. 30

25. Some blocks are placed in a $3 \times 3 \times 3$ cube, neglecting the gravity.

某些方塊在忽視重力的情況下被放置在一個 $3 \times 3 \times 3$ 的立體中。

The following is an example for you to familiarize with the setting of the question:

以下是一個協助你熟悉題目設定的例子：

The top view and the front view of this cube are shown:

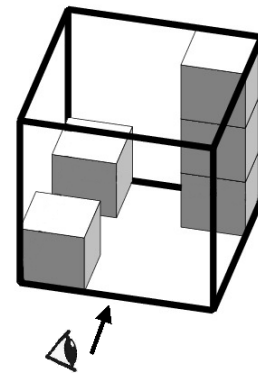
下圖顯示該立體的俯視圖和正視圖：

Top view 俯視圖

B	.	B
.	.	.
B	.	.

Front view 正視圖

.	.	B
.	.	B
B	.	B



where "B" means there is at least 1 block and "." means there is no block.
而 "B" 表示那裏有至少一個方塊，"." 則表示那裏沒有方塊。

The following are the top view and the front view of another cube:

以下是另一個立體的俯視圖和正視圖：

Top view 俯視圖

B	.	B
.	.	B
B	B	.

Front view 正視圖

B	.	.
.	B	.
B	.	B

Let x be the number of blocks in this cube. What is the difference between the minimum possible x and the maximum possible x ?

設 x 為這立體中方塊的數量。 x 的最小值和 x 的最大值之差是多少？

- A. 0
- B. 1
- C. 2
- D. 3

END OF SECTION A 甲部完

Section B 乙部 (20 marks 分)

The blanks are labeled from A to M. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 A 至 M，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Select exactly one programming language on the Answer Sheet. Answers must be in that language.
您必須在答題紙上選擇剛好一種編程語言，並只使用該語言作答。
- (2) For C and C++, you must not use the ?: operator.
對於 C 及 C++，答案不可以包括 ?: 運算元。
- (3) You must not use any library function unless the appropriate library has been included. (See Page 1)
除非適當的函數庫已被引用 (見頁一)，否則答案不可以包括任何函數庫內的函數。
- (4) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (5) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (6) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。
- (7) If blank X is divided into N parts X1, X2, ..., XN, it means that marks will only be given when X1, X2, ..., XN are all correct.
如果空格 X 分為 N 部份 X1、X2、...、XN，那麼 X1、X2、...、XN 皆為正確才會給分。

1. In a room escape game, some clues have dependencies on other clues. That means the player needs to find out some particular clues before some other particular clues could be possible for discovery. There are 7 clues A, B, C, D, E, F and G, their dependencies are listed below:

在一個密室逃脫遊戲裏，有些線索依賴著其他線索。這指玩家需要找出某些特定的線索，某些其他特定的線索才能被發現。現有 7 個線索 A, B, C, D, E, F 和 G，以下列出這些線索的依賴：

- Clues A and C together unlock the access to clues E and F.
線索 A 和 C 結合解鎖線索 E 和 F。
- Clue B unlocks the access to clue D.
線索 B 解鎖線索 D。
- Clue D and E together unlocks the access to clue G.
線索 D 和 E 結合解鎖線索 G。

Which of the clues can possibly be the 6th clue the player finds in the room? Write down the letters represented by the clues and separate them with commas. (Example: W, X, Z)

哪些是玩家在密室裏第 6 個可能找到的線索？寫下代表線索的字符，並以逗號分隔。(例: W, X, Z)

Answer 答案: A (2 marks 分)

2. Complete the following program such that it calculates the value of $f(x)$ correctly for all $1 \leq x \leq 100$.
完成以下程序使得程序能夠對於所有 $1 \leq x \leq 100$ 正確地計算 $f(x)$ 。

$$f(x) = 1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + x \cdot (x + 1) \cdot (x + 2)$$

Pascal

```
function f(x: longint): longint;
var sum, i: longint;
begin
  sum := 0;
  for i := 0 to   B1   do
    sum := sum +       B2      ;
  f := sum
end;
```

C / C++

```
int f(int x) {
  int sum, i;
  sum = 0;
  for (i = 0; i <=   B1  ; i++)
    sum = sum +       B2      ;
  return sum;
}
```

Answer 答案: B1 B2 (2 marks 分)

3. Complete the following program, so that it outputs the number of integers between 1 and n (inclusive) divisible by 6 or 8, but not both. It is guaranteed that the input n is a positive integer less than or equal to 10^9 .
完成以下程序，使其輸出在 1 和 n 之間（含），被 6 或者 8 整除，但不同時被兩數整除的數的個數。保證輸入 n 為小於或等於 10^9 的正整數。

Pascal

```
var
  i, n, answer: longint;
begin
  read(n);
  answer := 0;
  for i := 1 to n do
    if (      C      ) then
      inc(answer);
  write(answer)
end.
```

C

```
int i, n, answer;
int main() {
  scanf("%d", &n);
  answer = 0;
  for (i = 1; i <= n; i++)
    if (      C      )
      answer++;
  printf("%d", answer);
  return 0;
}
```

C++

```
int i, n, answer;
int main() {
  cin >> n;
  answer = 0;
  for (i = 1; i <= n; i++)
    if (      C      )
      answer++;
  cout << answer;
  return 0;
}
```

Answer 答案: C (1.5 marks 分)

Now, complete the following program, so that it also outputs the number of integers between 1 and n (inclusive) divisible by 6 or 8, but not both. It is guaranteed that the input n is a positive integer less than or equal to 10^9 .
現在完成以下程序，使其同樣輸出在 1 和 n 之間（含），被 6 或者 8 整除，但不同時被兩數整除的數的個數。保證輸入 n 為小於或等於 10^9 的正整數。

Pascal

```
var
  n, answer: longint;
begin
  read(n);
  answer :=       D      ;
  write(answer)
end.
```

C

```
int n, answer;
int main() {
  scanf("%d", &n);
  answer =       D      ;
  printf("%d", answer);
  return 0;
}
```

C++

```
int n, answer;
int main() {
  cin >> n;
  answer =       D      ;
  cout << answer;
  return 0;
}
```

Answer 答案: D (1 mark 分)

4. Complete the following program such that its output is hko*i*.
完成以下程序使得其輸出 hko*i*。

Pascal

```
var
  v, m, i: longint;
begin
  v := ord('___E1___');
  m := 1;
  for i := 2 to 6 do
    if (___E2___) then
      m := -1
    else
      begin
        v := v + m * i;
        write(chr(v))
      end
  end.
end.
```

C

```
int v, m, i;
int main() {
  v = (int)'___E1___';
  m = 1;
  for (i = 2; i <= 6; i++)
    if (___E2___)
      m = -1;
    else {
      v = v + m * i;
      printf("%c", (char)v);
    }
  return 0;
}
```

C++

```
int v, m, i;
int main() {
  v = (int)'___E1___';
  m = 1;
  for (i = 2; i <= 6; i++)
    if (___E2___)
      m = -1;
    else {
      v = v + m * i;
      cout << (char)v;
    }
  return 0;
}
```

Answer 答案: ___E1___ ___E2___ (2 marks 分)

5. Consider the following functions: 考慮以下函數：

Pascal

```
function f(x: longint): longint;
begin
  if (x = 0) then
    f := 0
  else
    f := f(x div 2) xor x
end;
function g(): longint;
var
  y, i: longint;
begin
  y := 0;
  for i := 1 to 64 do
    y := y + f(i);
  g := y
end;
```

C / C++

```
int f(int x) {
  if (x == 0)
    return 0;
  else
    return f(x / 2) ^ x;
}
int g() {
  int y, i;
  y = 0;
  for (i = 1; i <= 64; i++)
    y = y + f(i);
  return y;
}
```

Write down the return value of f(25).

寫下 f(25) 的傳回值。

Answer 答案: ___F___ (1 mark 分)

Write down an integer x such that f(x) returns 25.

寫下一個整數 x 使得 f(x) 傳回 25。

Answer 答案: ___G___ (1.5 marks 分)

Write down the return value of g().

寫下 g() 的傳回值。

Answer 答案: ___H___ (1.5 marks 分)

6. Consider a function f which, for integer input x , returns the remainder of $(ax^2 + bx + c)$ divided by 3. a, b, c are equal to 0, 1, or 2. We denote this return value by $f(x)$.
考慮一函數 f ，當輸入整數 x 時， f 會返回 $(ax^2 + bx + c)$ 除 3 的餘數。 a, b, c 等於 0、1 或 2。我們以 $f(x)$ 表示這一返回值。

Here is an example. Suppose $a = 0, b = 2, c = 0$. Then, when the input x is -1 , $f(x)$ equals 1, because:
以下有一例子。假設 $a = 0, b = 2, c = 0$ 。那麼，當輸入 x 是 -1 時， $f(x)$ 等於 1，因為：

$$ax^2 + bx + c = 0 \cdot (-1)^2 + 2 \cdot (-1) + 0 = -2,$$

and the remainder of -2 divided by 3 is 1.
而 -2 除 3 的餘數是 1。

The values of a, b , and c are unknown to you, and you want to find them out by making queries. Each time you make a query, you can pick an integer x as input, and you will then receive the value of $f(x)$.

你不知道 a, b, c 的值，而你想透過詢問將它們找出。每次詢問，你可以選擇一個整數 x 作輸入，而你將會得知 $f(x)$ 的值。

In the following questions, you need to determine the minimum number of queries needed to find out the value of one unknown (a, b , or c). Note that you do not need to find out the values of the other two. If it is impossible to do so, write X as your answer.

在以下數題，你需要判定找出某一個未知值 (a, b 或 c) 的最少詢問次數。留意，你不需要同時找出其餘兩個未知值。如果不可能做到，請填 X 作答案。

What is the minimum number of queries needed to find out the value of a ?

要找出 a 的值，至少要詢問多少次？

Answer 答案: I (1 mark 分)

What is the minimum number of queries needed to find out the value of b ?

要找出 b 的值，至少要詢問多少次？

Answer 答案: J (1 mark 分)

What is the minimum number of queries needed to find out the value of c ?

要找出 c 的值，至少要詢問多少次？

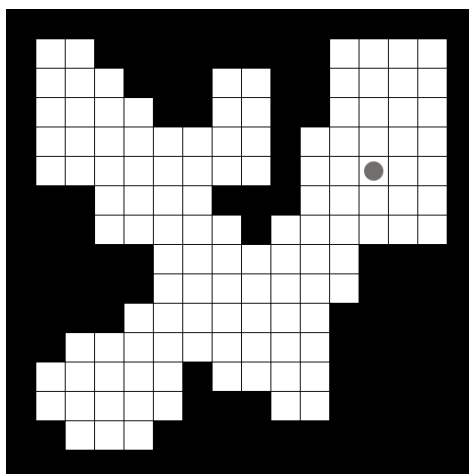
Answer 答案: K (1 mark 分)

7. The input to the following program is 801 integers between 1 and 100 inclusive. Complete the following program so that it outputs the inputted numbers in ascending order.
以下程序的輸入是 801 個 1 及 100 之間(含)的整數。完成以下程序使得它將輸入的數字由小至大輸出。

Pascal	C	C++
<pre> var a, b: array[0..800] of longint; c: array[0..100] of longint; i, j: longint; begin for i := 0 to 100 do c[i] := 0; for i := 0 to 800 do begin read(a[i]); inc(c[a[i]]) end; for i := 1 to 100 do c[i] := c[i] + c[i - 1]; for i := 1 to 100 do for j := <u> L1 </u> to <u> L2 </u> do <u> L3 </u>; for i := 0 to 800 do write(b[i], ' '); end. </pre>	<pre> int a[801], b[801]; int c[101]; int i, j; int main() { for (i = 0; i <= 100; i++) c[i] = 0; for (i = 0; i <= 800; i++) { scanf("%d", &a[i]); c[a[i]]++; } for (i = 1; i <= 100; i++) c[i] = c[i] + c[i - 1]; for (i = 1; i <= 100; i++) for (j = <u> L1 </u>; j <= <u> L2 </u>; j++) <u> L3 </u>; for (i = 0; i <= 800; i++) printf("%d ", b[i]); return 0; } </pre>	<pre> int a[801], b[801]; int c[101]; int i, j; int main() { for (i = 0; i <= 100; i++) c[i] = 0; for (i = 0; i <= 800; i++) { cin >> a[i]; c[a[i]]++; } for (i = 1; i <= 100; i++) c[i] = c[i] + c[i - 1]; for (i = 1; i <= 100; i++) for (j = <u> L1 </u>; j <= <u> L2 </u>; j++) <u> L3 </u>; for (i = 0; i <= 800; i++) cout << b[i] << " "; return 0; } </pre>

Answer 答案: L1 L2 L3 (2 marks 分)

8. Consider the following 16×16 grid. Black cells represent obstacles and the circle represents the robot.
考慮以下 16×16 網格。黑色的格代表障礙物，圓形則代表機器人。



Whenever the robot arrives at a new position (including the initial position), information of the adjacent 8 cells will be given and stored in array *s* (Pascal: *s*[0..7] of boolean, C / C++: *bool s*[8]), where *s*[*i*] equals true if and only if that cell is an obstacle.

每當機器人到達新的位置（包括初始的位置），鄰近 8 格的資訊會被給定並儲存在陣列 *s* (Pascal: *s*[0..7] of boolean, C / C++: *bool s*[8])，*s*[*i*] 等於 true 當且僅當該格為障礙物。

s[0]	s[1]	s[2]
s[7]	●	s[3]
s[6]	s[5]	s[4]

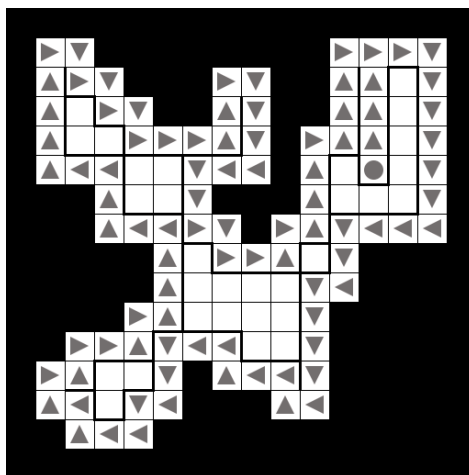
Given array s , the robot can take four possible actions: UP, DOWN, LEFT, RIGHT.

給予陣列 s ，機器人可以作出四種行動：上 (UP)、下 (DOWN)、左 (LEFT)、右 (RIGHT)。

Complete the following function $f(s)$ such that by repeated calling $f(s)$, the robot can:

完成以下函數 $f(s)$ 使得透過重複呼叫 $f(s)$ ，機器人能：

1. Move UP until it reaches the boundary, and then
向上移動直至到達邊界，然後
2. Follow the boundary in CLOCKWISE direction:
沿著邊界、順時針方向移動：



Pascal

```

type
  Dir = (UP, DOWN, LEFT, RIGHT);
function f(s: array of boolean): Dir;
begin
  if (_____ M1 _____) then
    f := RIGHT
  else if (_____ M2 _____) then
    f := DOWN
  else if (_____ M3 _____) then
    f := LEFT
  else
    f := UP // default
end;
```

C / C++

```

enum Dir { UP, DOWN, LEFT, RIGHT };
enum Dir f(bool s[]){
  if (_____ M1 _____)
    return RIGHT;
  else if (_____ M2 _____)
    return DOWN;
  else if (_____ M3 _____)
    return LEFT;
  else
    return UP; // default
}
```

Answer 答案: _____ M1 _____ M2 _____ M3 _____ (2.5 marks 分)

END OF PAPER 全卷完