Hong Kong Olympiad in Informatics 2018/19
Heat Event (Senior Group)
Official Solution

Statistics (N=220)
Full mark = 45. Maximum = 42. Median = 14. Advance to Final = 16 marks of above.

Section A

Q	A	**Explanation**
1	D	f(g(2)) = f(-1) = 2 g(f(2)) = g(8) = 17
2	D	The function calculates the value of x in "base y". Therefore, f(513, 4, 1) = $5 * 4^2 + 1 * 4^1 + 3 * 4^0 = 87$.
3	D	The range of marks is $0 \leq x \leq 100$, so there are at most 101 possible marks. Let i and j be the number of 5 and 3 marks Alice gets respectively, where i + j <= 20. We can assume the remaining marks are 0. 1, 2, 4, 7 can not be obtained by any linear combinations of 5 and 3. 97 = 5 * 17 + 3 * 4 which is impossible. 99 = 5 * 18 + 3 * 3 which is also impossible. Hence, the number of possible marks is 101 - 6 = 95.
4	C	`(x << 31) & 1` is 0 as the last bit of `(x << 31)` is 0. Therefore, `x` will not change and there is an infinite loop except when `f(0)` is called.
5	D	Boolean expressions with two variables can be represented by a 2x2 truth table. As there are only 4 cells in a 2x2 truth table, the number of inequivalent Boolean expressions = 2^4 = 16.
6	D	Note that logically A → B (If A, then B) does not imply B → A (If B, then A). From the information given in (1), (2), (3) and (4), no information can be inferred regarding whether it rains outside, whether Charlie fails his exam, whether Charlie feels sad, whether it rains outside and whether Charlie rides a bike. Hence, all three statements in (i), (ii) and (iii) need not be true.
7	D	The program finds the sum of all positive odd numbers, subtracted by other numbers. Hence the sum = (7 + 9 + 11) - [(-3) + 2 + 6 + (-5) + (-10)] = 37.
8	D	Assigning a string to an integer variable is a syntax error that will be detected at compile time.
9	D	The function assigns a to result and multiplies a to result e times. Therefore, it returns the value a^{e+1}. Hence the value of my_pow(2, 10) = $2^{10+1} = 2^{11}$ = 2048.
10	D	f(1) + f(2) + f(3) + … + f(29) + f(30) = [f(0) + f(1) + f(2) + … + f(26)] + [f(27) + f(28) + f(29) + f(30)] Note that 0 = 000_3, 26 = 222_3. Hence f(0) + f(1) + f(2) + … + f(26) = 27 * 3 * (0 + 2) / 2 = 81. By 27 = 1000_3, 28 = 1001_3, 29 = 1002_3, 30 = 1010_3, we have f(1) + f(2) + f(3) + … + f(29) + f(30) = 81 + 1 + 1 + 1 + 1 + 2 + 1 + 1 = 89.
11	C	For a sum of some numbers to be odd, the number of odd numbers among the numbers must be odd. Inserting a plus sign after an even number does not change the count of odd numbers, while inserting a plus sign after an odd number increases the count of odd numbers by one. Among the first 9 digits, there are 6 even numbers and 3 odd numbers. Hence, the

		number of different expressions = $2^6 * ({}^3C_1 + {}^3C_3) = 64 * 4 = 256$.
12	A	For a product to be an odd number, all the numbers multiplied must be odd numbers. Among the first 9 digits, the 2nd, 4th, 7th digit are odd numbers. The multiple signs can only be inserted after the 2nd, 4th and 7th digit. Therefore, there are $2^3 = 8$ possibilities.
13	D	The for-loop finds the largest pair (i, j) such that i < j and a[i] = a[j], which is a[5] and a[7].
14	B	For (i), $11_6 = 7_{10}$, which is a prime number. Hence (i) is false. For (ii), consider the digits of a n-digit number. Note that $8 \equiv -1 \pmod 9$ and $8^2 \equiv 1 \pmod 9$. Let the 0^{th} digit be the unit digit. As the i^{th} digit must be same as the $(n - 1 - i)^{th}$ digit, and i and (n - 1 - i) have different parity, the remainder when divided by 9 must be 0. Hence, all even-length palindrome in base 8 must be a factor of 9.
15	B	A single cycle of bubble sort is executed on a, with the original result stored in b. For (i), consider b = {1, 0, 2, 3, …}. After the loop, a = {0, 1, 2, …}, where a[1] > b[1]. Hence, (i) need not be true. For (ii), consider b = {0, 1, 3, 4, 2, 5, 6, …}. After the loop, a = {0, 1, 3, 2, 4, 5, …} with a[2] > a[3]. Hence, (ii) need not be true. For (iii), as the last iteration of the loop swaps a[8] and a[9] if a[8] > a[9], a[8] <= a[9] must be true. Hence, (iii) must be true.
16	C	The combination that results in the minimum amount is hiring Bob, Daisy and Emily.
17	C	A pair of boolean expression is logically equivalent if they have the same truth table. The truth tables are as follows:

`x || (!x && y)`			`(x && !y) || y`		
	x = 0	x = 1		x = 0	x = 1
y = 0	0	1	y = 0	0	1
y = 1	1	1	y = 1	1	1
`(x || !y) ^ (!x || y)`			`(x && !y) || (!x && y)`		
	x = 0	x = 1		x = 0	x = 1
y = 0	0	1	y = 0	0	1
y = 1	1	0	y = 1	1	0

		From the truth tables, both pairs are logically equivalent.
18	A	The program repeatedly applies a permutation to an array that is initialized with a[i] = i. The permutation contains two cycle: one of them is $(2 \rightarrow 0 \rightarrow 6 \rightarrow 1 \rightarrow 3)$ and the other is $(7 \rightarrow 4 \rightarrow 5)$. For a[2], as 2 is in the first cycle and $2018 \equiv 3 \pmod 5$, the result should be 1. For a[7], as 7 is in the second cycle and $2018 \equiv 2 \pmod 3$, the result should be 5.
19	D	f(x) = x AND x = x. g(x) = x XOR x = 0. For (i), f(g(x)) = f(0) = 0. g(f(x)) = g(x) = 0 = f(g(x)). Therefore, (i) is true. For (ii), f(f(x)) = f(x) = x = f(x). Therefore, (ii) is true. For (iii), g(g(x)) = g(0) = 0 = g(x). Therefore, (iii) is true.
20	D	As Alice should not sit next to Bob, and Charlie must not sit next to Dave, Alice and Bob must sit in either the 1st and the 3rd seat or the 2nd and the 4th seat. With either Alice sitting before Bob or not and Charlie sitting before Dave or not, the number of combinations = 2 * 2 * 2 = 8.
21	C	An element in a stack can only be popped out when all elements pushed after it is popped. Therefore, a stack is ‘first in, last out’. An element in a queue can only be popped out when all elements pushed before it is popped. Therefore, a queue in ‘first in, first out’.

22	B	Let the top left cell of the board be (0, 0). The ideal arrangement is to put a king in each cell (i, j) where both i and j are even. Hence there are (98/2 - 0 + 1)^2 = 50^2 = 2500 kings at most.
23	C	For each trainer, there are 3 possibilities: either he is assigned to only the competition committee, he is assigned to only the training committee or he is assigned to both committees. Excluding the 2 invalid cases that all 6 of the trainers are assigned to the same committee, the number of different assignments = $3^6 - 2 = 727$.
24	D	The question is equivalent to finding out the number of trees with 4 labeled vertices. Cayley's formula states that for every positive integer n, the number of trees on n labeled vertices is n^{n-2}. Thus, the answer is $4^{4-2} = 4^2 = 16$.
25	B	Binary search and linear search are algorithms for searching an element in an array. Monte Carlo tree search is a search algorithm for searching a game state tree. Breadth-first search is the most suitable algorithm for finding shortest path in an unweighted, undirected graph.

Section B

Answer and Explanation			
	Pascal	C	C++
A	`10`		
	The array `a[]` is initialized to all 0s. For any number `j`, when a factor of `j` is found, `a[j]` is flipped. As only square numbers have an odd number of factors, only `a[i]` for square numbers `i` is 1, while all other `a[i]` are 0. There are 10 square numbers less than or equal to 100, which is the answer.		
B	`ans:=ans xor x`	`ans=ans^x` `ans^=x`	`ans=ans^x` `ans^=x`
	As `x XOR x = 0`, all the numbers inputted twice will not have an effect on ans. The only number inputted once is stored in ans.		
C	`(x and (x-1))=0` `x and -x=0`	`!(x&(x-1))` `(x&-x)==x` `1073741824%x==0`	`!(x&(x-1))` `(x&-x)==x` `1073741824%x==0`
	For power of two only, `x&(x-1)` will be 0. For other numbers, `x&(x-1)` will remove the lowest bit from the number.		
D1, D2	`16, 25, 36, 49, 64, 81 (any two)`		
	When `n` is a square number, √`n` is double counted.		
E1	`i*i=n` `sqrt(n)=i` `n/i=i`	`i*i==n` `sqrt(n)==i`	`i*i==n` `sqrt(n)==i`
E2	`sum-i`	`sum-i`	`sum-i`
	To prevent double counting √`n`, subtract `i` from the sum.		
F	`sum_factors(n)`	`sum_factors(n)`	`sum_factors(n)`
G1	`sum>2*n`	`sum>2*n`	`sum>2*n`
G2	`sum=2*n`	`sum==2*n`	`sum==2*n`
H	`45`		
I	`21`		
J	`10 9 8 7 6 5 4 1 2 3`		
	This program finds the number of inversions in the array, the number of inversions is calculated by finding the number of pair `(i, j)` where `i < j` and `a[i] > a[j]`.		

	To construct an array with 42 inversions, we start with an array in descending order, and we flip any 3 pairs of numbers which the pair is an inversion.		
K	`50`		
	Note that the value of the first two digits determines the last two digits in the password, there are at most 9 * 9 = 81 possible combinations. Let i and j be the first two digits. Both of the digits cannot be 0, so i * j must not be a factor of 10 and i * j must be larger than 10. By subtracting the number of pairs of (i, j) that contain 0, we can find the answer.		
L1	`a[m]>=target`	`a[m]>=target`	`a[m]>=target`
L2	`l:=m+1`	`l=m+1`	`l=m+1`
L3	`m-1`	`m-1`	`m-1`
	The program is a binary search. If the index is not found, m will be equal to 0 at the end of the while loop. If the index is found, `m` will be (the required index + 1). Therefore `(m - 1)` is the required index or `-1` if it is not found.		
M	`2`		
	As the two if statements do not have brackets, the else statement follows the last if statement. Hence, any even number that is not a multiple of 3 will cause the program to output 'ODD'.		
N1	`17 // 18`	`46 // 47`	`76 // 77`
N2	`else // [space]else`	`else; // ;else`	`else; // ;else`
	It ends the inner if statement and adds an else statement to the outer if statement.		