

Format 類型	# Questions 題目數	Total Marks 佔分
Section A 甲部	Multiple Choice 多項選擇題	25
Section B 乙部	Fill-in-the-blanks 填充題	8 (A - N)
Total 總分		45

(1) Assume that all variables without declaration shown in the following program segments have already been declared properly as 32-bit signed integers (Pascal: `longint`, C / C++: `int`).

下列程序段中所有未有列出宣告的變量，均假設已經適當地宣告為 32 位元有符號的整數 (Pascal: `longint`, C / C++: `int`)。

(2) The following code is added to the beginning of all C and C++ programs.

在所有 C 和 C++ 程序的頂部加入以下程式碼:

C

```
#include <stdio.h>
#include <string.h>
#include <math.h>
#include <stdlib.h>
#include <stdbool.h>
```

C++

```
#include <cstdio>
#include <cmath>
#include <cstdlib>
#include <string>
#include <iostream>
using namespace std;
```

For C, `stdbool.h` defines the boolean data type `bool` and values `true` (equivalent to 1) and `false` (equivalent to 0).

對於 C，`stdbool.h` 定義了布爾數據類型 `bool` 及值 `true` (等同 1) 及 `false` (等同 0)。

(3) Other than questions that mention compilation, assume all programs are compiled properly in Ubuntu 16.04 using the compilers and commands below.

除了有提及編譯的題目之外，假設所有程序都在 Ubuntu 16.04 下使用以下編譯器及指令正確地編譯。

Pascal: Free Pascal (<code>fpc-compiler 3.0.0</code>)	<code>fpc program.pas</code>
C: GNU GCC (<code>gcc-4.9 4.9.3</code>)	<code>gcc -std=c99 program.c -o program</code>
C++: GNU G++ (<code>g++-4.9 4.9.3</code>)	<code>g++ -std=c++98 program.cpp -o program</code>

Section A 甲部 (25 marks 分)

For each question, choose the **most appropriate** answer and mark the corresponding box (A, B, C, or D) on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.

請為下列每題各選一個**最適合**的答案，然後把答題紙對應的空格（A、B、C、或D）填滿。
答對得一分，答錯不扣分。

1. What is the output of the following program? 以下程序的輸出是什麼？

Pascal

```
function f(n: longint): longint;
begin
  f := 2 * n * n;
end;
function g(n: longint): longint;
begin
  g := 3 * n - 7;
end;
begin
  write(f(g(2)), ' ', g(f(2)))
end.
```

C

```
int f(int n) {
  return 2 * n * n;
}
int g(int n) {
  return 3 * n - 7;
}
int main() {
  printf("%d %d", f(g(2)),
         g(f(2)));
}
```

C++

```
int f(int n) {
  return 2 * n * n;
}
int g(int n) {
  return 3 * n - 7;
}
int main() {
  cout << f(g(2)) << ' '
      << g(f(2));
}
```

- A. -2 11
- B. 2 5
- C. 2 11
- D. 2 17

2. Consider the following program: 考慮以下程序：

Pascal

```
function f(x, y, z: longint): longint;
begin
  if (x = 0) then
    f := 0
  else
    f := x mod 10 * z + f(x div 10, y, z * y)
end;
```

C / C++

```
int f(int x, int y, int z) {
  if (x == 0)
    return 0;
  else
    return x % 10 * z + f(x / 10, y, z * y);
}
```

What is the return value of $f(513, 4, 1)$?

$f(513, 4, 1)$ 的傳回值是甚麼？

- A. 27
- B. 77
- C. 84
- D. 87

3. Alice is taking a test consisting of 20 questions. She may get 5, 3 or 0 marks for each question. How many different possible total marks can she get?

愛麗絲正在接受一個由 20 條問題組成的測試。每條問題她可能會得到 5 分、3 分或 0 分。她能獲取多少種可能的總分？

- A. 92
- B. 93
- C. 94
- D. 95

4. Consider the following function: 考慮以下函數：

Pascal

```
function f(x: longint): longint;
begin
  while (x <> 0) do
    x := x + ((x shl 31) and 1);
  f := x
end;
```

C / C++

```
int f(int x) {
  while (x != 0)
    x = x + ((x << 31) & 1);
  return x;
}
```

Which of the following function calls will result in an infinite loop?

以下哪個函數呼叫會導致無限循環？

- i. f(-1)
 - ii. f(1)
-
- A. i only 只有 i
 - B. ii only 只有 ii
 - C. i and ii i 和 ii
 - D. None of them 無

5. Given two Boolean variables A and B. Using zero or more A, B, brackets, NOT, and OR, how many pairwise inequivalent Boolean expressions can be formed?

給予兩個布爾變數 A 和 B。利用零個或多個 A、B、括號、NOT(非) 和 OR(或)，可以組成多少互不等價的布爾表達式？

- A. 2
- B. 4
- C. 8
- D. 16

6. Suppose that the following statements are true: 假設以下陳述為真：

1. If it rains outside or Charlie fails his exam, he feels sad.
如果外面下雨或查理考試不合格，他會感到難過。
2. If Charlie feels sad and it rains outside, he practices programming at home.
如果查理感到難過並且外面下雨，他會在家裡練習編程。
3. If Charlie feels sad and it does not rain outside, he rides a bike.
如果查理感到難過並且外面沒有下雨，他就會騎單車。
4. Charlie practices programming at home.
查理在家裡練習編程。

Which of the following statements must then be true?

那麼以下哪一項陳述必須為真？

- i. Charlie rides a bike
查理騎單車
 - ii. It does not rain outside
外面沒有下雨
 - iii. Charlie does not feel sad
查理沒有感到難過
- A. i only 只有 i
 B. ii only 只有 ii
 C. iii only 只有 iii
 D. None of them 無

7. What is the output of the following program? 以下程序的輸出是什麼？

Pascal

```
var
arr: array[0..7] of longint =
(-3, 2, 6, 7, 9, -5, -10, 11);
ans: longint = 0;
i: longint;
begin
for i := 0 to 7 do
begin
if (arr[i] mod 2 = 1) then
ans := ans + arr[i]
else
ans := ans - arr[i]
end;
writeln(ans)
end.
```

C

```
int arr[8] = {-3, 2, 6, 7, 9,
-5, -10, 11};
int ans = 0;
int i;
int main() {
for (i = 0; i <= 7; i++) {
if (arr[i] % 2 == 1)
ans = ans + arr[i];
else
ans = ans - arr[i];
}
printf("%d", ans);
}
```

C++

```
int arr[8] = {-3, 2, 6, 7, 9,
-5, -10, 11};
int ans = 0;
int i;
int main() {
for (i = 0; i <= 7; i++) {
if (arr[i] % 2 == 1)
ans = ans + arr[i];
else
ans = ans - arr[i];
}
cout << ans;
}
```

- A. 21
 B. 27
 C. 29
 D. 37

8. x is an integer variable. What kind of error is present in the following code?
 x 是個整數變量，以下代碼有何種錯誤？

Pascal

```
x := 'pi';
```

C / C++

```
x = "pi";
```

- A. Logic Error 邏輯錯誤
- B. Precision Error 精度誤差
- C. Runtime Error 執行錯誤
- D. Syntax Error 語法錯誤

9. Consider the following program segment: 考慮以下程序段：

Pascal

```
function my_pow(a, e: longint): longint;
var i, result: longint;
begin
  result := a;
  for i := 1 to e do
    result := result * a;
  my_pow := result
end;
```

C / C++

```
int my_pow(int a, int e) {
  int i, result;
  result = a;
  for (i = 1; i <= e; i++)
    result = result * a;
  return result;
}
```

What is the return value of $\text{my_pow}(2, 10)$?

$\text{my_pow}(2, 10)$ 的傳回值是甚麼？

- A. 20
- B. 22
- C. 1024
- D. 2048

10. For any non-negative integer n , let $f(n)$ be the sum of digits of n , when n is written in base 3.

For example, $f(0) = f(0_3) = 0$, $f(11) = f(102_3) = 1 + 0 + 2 = 3$.

Calculate the sum $f(1) + f(2) + f(3) + \dots + f(29) + f(30)$.

對於任何非負整數 n ，設 $f(n)$ 為 n 寫成 3 進制時的數字之和。

例如 $f(0) = f(0_3) = 0$ ， $f(11) = f(102_3) = 1 + 0 + 2 = 3$ 。

$f(1) + f(2) + f(3) + \dots + f(29) + f(30)$ 的總和是甚麼？

- A. 86
- B. 87
- C. 88
- D. 89

11. Consider the number 2147483647. It has 10 digits. There are $2^{10-1} = 512$ ways to insert (possibly zero) plus signs (+) between the digits to obtain an arithmetic expression. For example, 2147483647, 2+1+4+7+4+8+3+6+4+7, and 214+74+8+3647 are three ways. How many of these arithmetic expressions evaluates to an odd number?
考慮數字 2147483647。它有 10 位數字。有 $2^{10-1} = 512$ 種方式在數字之間插入（可能為零個）加號（+）以獲得算術表達式。例如，2147483647，2+1+4+7+4+8+3+6+4+7 和 214+74+8+3647 是三種方式。有多少種算術表達式算出的值為奇數？
- A. 64
B. 128
C. 256
D. 512
12. Consider the number 2147483647. It has 10 digits. There are $2^{10-1} = 512$ ways to insert (possibly zero) multiplication signs (*) between the digits to obtain an arithmetic expression. For example, 2147483647, 2*1*4*7*4*8*3*6*4*7, and 214*74*8*3647 are three ways. How many of these arithmetic expressions evaluates to an odd number?
考慮數字 2147483647。它有 10 位數字。有 $2^{10-1} = 512$ 種方式在數字之間插入（可能為零個）乘號（*）以獲得算術表達式。例如，2147483647，2*1*4*7*4*8*3*6*4*7 和 214*74*8*3647 是三種方式。有多少種算術表達式算出的值為奇數？
- A. 8
B. 16
C. 32
D. 64
13. What is the output of the following program? 以下程序的輸出是什麼？

Pascal

```
var
  a: array[0..8] of longint =
    (1, 2, 1, 2, 3, 3, 2, 3, 1);
  x, y, i, j: longint;
begin
  for i := 0 to 8 do
    for j := i + 1 to 8 do
      if (a[i] = a[j]) then
        begin
          x := i;
          y := j;
          break;
          break;
        end;
    write(x, ' ', y)
end.
```

C

```
int a[9] = {
  1, 2, 1, 2, 3, 3, 2, 3, 1
};
int x, y, i, j;
int main() {
  for (i = 0; i <= 8; i++)
    for (j = i + 1; j <= 8; j++)
      if (a[i] == a[j]) {
        x = i;
        y = j;
        break;
        break;
      }
  printf("%d %d", x, y);
  return 0;
}
```

C++

```
int a[9] = {
  1, 2, 1, 2, 3, 3, 2, 3, 1
};
int x, y, i, j;
int main() {
  for (i = 0; i <= 8; i++)
    for (j = i + 1; j <= 8; j++)
      if (a[i] == a[j]) {
        x = i;
        y = j;
        break;
        break;
      }
  cout << x << " " << y;
  return 0;
}
```

- A. 0 0
B. 0 2
C. 0 8
D. 5 7

14. A palindromic number is a number that remains the same when its digits are reversed. For example, in base 10, 12321 is a palindromic number and 12322 is not.

迴文數是正讀反讀都一樣的數字。舉例說，在十進制，12321 是迴文數，而 12322 則不是。

Considering positive integers only, which of the following statements is true?

只考慮正整數，以下哪項陳述正確？

- i. In base 6, every palindromic number with an even number of digits is a composite number.
在六進制，所有偶數位數的迴文數都是合成數。
- ii. In base 8, every palindromic number with an even number of digits is a composite number.
在八進制，所有偶數位數的迴文數都是合成數。

- | | |
|-----------------|--------|
| A. i only | 只有 i |
| B. ii only | 只有 ii |
| C. i and ii | i 和 ii |
| D. None of them | 無 |

15. Consider the program below:

考慮以下程序：

Pascal

```
var
  a, b: array[0..9] of longint;
  i, tmp: longint;
begin
  for i := 0 to 9 do
    read(a[i]);
  for i := 0 to 9 do
    b[i] := a[i];
  for i:= 1 to 9 do
    if (a[i] < a[i - 1]) then
    begin
      tmp := a[i];
      a[i] := a[i - 1];
      a[i - 1] := tmp;
    end
end.
```

C

```
int a[10], b[10];
int i, tmp;
int main() {
  for (i = 0; i <= 9; i++)
    scanf("%d", &a[i]);
  for (i = 0; i <= 9; i++)
    b[i] = a[i];
  for (i = 1; i <= 9; i++)
    if (a[i] < a[i - 1]) {
      tmp = a[i];
      a[i] = a[i - 1];
      a[i - 1] = tmp;
    }
  return 0;
}
```

C++

```
int a[10], b[10];
int i, tmp;
int main() {
  for (i = 0; i <= 9; i++)
    cin >> a[i];
  for (i = 0; i <= 9; i++)
    b[i] = a[i];
  for (i = 1; i <= 9; i++)
    if (a[i] < a[i - 1]) {
      tmp = a[i];
      a[i] = a[i - 1];
      a[i - 1] = tmp;
    }
  return 0;
}
```

Which of the followings must be true?

以下哪項必為正確？

- i. $a[1] \leq b[1]$
 - ii. $a[2] \leq a[3]$
 - iii. $a[8] \leq a[9]$
- | | |
|--------------------|-------------|
| A. i only | 只有 i |
| B. iii only | 只有 iii |
| C. i and iii only | 只有 i 和 iii |
| D. ii and iii only | 只有 ii 和 iii |

16. Percy needs to prepare an exam paper with 40 questions. He wants to hire question writers to finish the job. Here is a list of five question writers, how many questions they can prepare, and how much they charge.
珀西需要準備一份包含 40 條問題的試卷。他想聘請出題人來完成這項工作。
以下列出了五位出題人，他們可以準備多少問題以及他們收取多少費用。

Writer 出題人	Number of Questions 問題數量	Charge (in dollars) 費用（以元計）
Alice 愛麗絲	25	1000
Bob 鮑伯	13	450
Charlie 查理	16	590
Daisy 黛西	20	770
Emily 艾米莉	9	330

What is the minimum amount (in dollars) that Percy needs to pay in total, so that he will have at least 40 exam questions?

Note that Percy **cannot** hire the same writer more than once.

珀西最少需要支付多少金額（以元計）令他至少會有 40 條問題？

請注意，珀西 **不能** 僱用同一個出題人多於一次。

- A. 1450
- B. 1470
- C. 1550
- D. 1590

17. Which of the following pair(s) of boolean expressions are logically equivalent?

以下哪對布爾表達式在邏輯上等價？

Pascal	C / C++
i. $x \text{ OR } (\text{NOT}(x) \text{ AND } y)$ $(x \text{ AND } \text{NOT}(y)) \text{ OR } y$	$x \mid\mid (\text{!}x \&\& y)$ $(x \&\& \text{!}y) \mid\mid y$
ii. $(x \text{ OR } \text{NOT}(y)) \text{ XOR } (\text{NOT}(x) \text{ OR } y)$ $(x \text{ AND } \text{NOT}(y)) \text{ OR } (\text{NOT}(x) \text{ AND } y)$	$(x \mid\mid \text{!}y) \wedge (\text{!}x \mid\mid y)$ $(x \&\& \text{!}y) \mid\mid (\text{!}x \&\& y)$
A. i only 只有 i B. ii only 只有 ii C. i and ii i 和 ii D. None of them 無	

18. What is the output of the following program? 以下程序的輸出是什麼？

Pascal

```
var
  p: array[0..7] of longint =
    (6, 3, 0, 2, 5, 7, 1, 4);
  a, tmp: array[0..7] of longint;
  i, j: longint;
begin
  for i := 0 to 7 do
    a[i] := i;
  for i := 1 to 2018 do
  begin
    for j := 0 to 7 do
      tmp[j] := a[p[j]];
    for j := 0 to 7 do
      a[j] := tmp[j];
  end;
  write(a[2], ' ', a[7])
end.
```

C

```
int p[8] = {
  6, 3, 0, 2, 5, 7, 1, 4
};
int a[8], tmp[8];
int i, j;
int main() {
  for (i = 0; i <= 7; i++)
    a[i] = i;
  for (i = 1; i <= 2018; i++) {
    for (j = 0; j <= 7; j++)
      tmp[j] = a[p[j]];
    for (j = 0; j <= 7; j++)
      a[j] = tmp[j];
  }
  printf("%d %d", a[2], a[7]);
  return 0;
}
```

C++

```
int p[8] = {
  6, 3, 0, 2, 5, 7, 1, 4
};
int a[8], tmp[8];
int i, j;
int main() {
  for (i = 0; i <= 7; i++)
    a[i] = i;
  for (i = 1; i <= 2018; i++) {
    for (j = 0; j <= 7; j++)
      tmp[j] = a[p[j]];
    for (j = 0; j <= 7; j++)
      a[j] = tmp[j];
  }
  cout << a[2] << " " << a[7];
  return 0;
}
```

- A. 1 5
- B. 3 7
- C. 6 4
- D. 6 5

19. Define two unary functions 定義兩個一元函數

$$f(x) = x \text{ AND } x$$

$$g(x) = x \text{ XOR } x$$

Which of the following must be true?

以下哪項必為正確？

- | | |
|------------------------|--|
| i. $f(g(x)) = g(f(x))$ | |
| ii. $f(f(x)) = f(x)$ | |
| iii. $g(g(x)) = g(x)$ | |
-
- | | |
|--------------------|-------------|
| A. i only | 只有 i |
| B. ii only | 只有 ii |
| C. ii and iii only | 只有 ii 和 iii |
| D. i, ii and iii | i、ii 和 iii |

20. There are four seats arranged in a row. Alice, Bob, Charlie, and Dave are each going to choose a seat. Alice should not sit next to Bob, and Charlie should not sit next to Dave. How many seating arrangements are there?
有四個連續排列的座位。愛麗絲、鮑伯、查理和戴夫都會選擇一個座位。愛麗絲不應該坐在鮑伯旁邊，查理不應該坐在戴夫身邊。有多少種座位安排方法？

- A. 2
- B. 4
- C. 6
- D. 8

21. Which of the following best describes the behaviors of stack and queue?

以下哪項最能描述棧和隊列的特性？

Stack 棠 Queue 隊列

- | | |
|-----------------------------|--------------------------|
| A. First in, first out 先進先出 | First in, first out 先進先出 |
| B. First in, first out 先進先出 | First in, last out 先進後出 |
| C. First in, last out 先進後出 | First in, first out 先進先出 |
| D. First in, last out 先進後出 | First in, last out 先進後出 |

22. In chess, a king can attack all squares which share an edge or a corner with the square he currently occupies.

At most how many kings can be placed on a 99×99 chessboard, so that no two kings attack each other?

在國際象棋中，國王可以攻擊與他目前佔據的方格共享邊緣或角落的所有方格。

最多可以在 99×99 的棋盤上放置多少國王而沒有兩個國王互相攻擊？

- A. 2401
- B. 2500
- C. 4900
- D. 4901

23. 6 HKOI trainers are assigned to competition committee and training committee. Each trainer should be assigned to at least one committee, and each committee should have at least one trainer.

What is the number of different assignments of trainers?

6 名 HKOI 訓練員被分配到比賽小組和訓練小組。每個訓練員應被分配到至少一個小組，每個小組至少應有一名訓練員。訓練員的不同分配方案數量是多少？

- A. 62
- B. 64
- C. 727
- D. 729

24. There are 4 districts in Byteland. How many ways are there to build exactly three bi-directional roads between some districts such that it is possible to reach from any districts to one another?

位元組國有 4 個區域。有多少種方法可以在區域之間建造正好 3 條雙向的道路，令到任何區域都可到達其他區域？

Two ways are different if there exists one road which is built in one plan but not in another plan.

如果存在一條建在一種方法中但不在另一種方法中的道路，兩種方法則視為不同。

- A. 4
- B. 8
- C. 12
- D. 16

25. Given an unweighted, undirected graph.

Which of the following algorithms is the most suitable for finding a shortest path from one node A to another node B?

給定一個無向無權圖。

以下哪種算法最適合尋找從一個節點 A 到另一個節點 B 的最短路徑？

- A. Binary Search 二分搜索
- B. Breadth-First Search 廣度優先搜索
- C. Linear Search 線性搜索
- D. Monte Carlo Tree Search 蒙地卡羅樹搜尋

END OF SECTION A 甲部完

Section B 乙部 (20 marks 分)

The blanks are labeled from A to N. Please fill in the blanks on the answer sheet.

下列各空格分別命名為 A 至 N，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Select exactly one programming language on the Answer Sheet. Answers must be in that language.
您必須在答題紙上選擇剛好一種編程語言，並只使用該語言作答。
- (2) For C and C++, you must not use the ?: operator.
對於 C 及 C++，答案不可以包括 ?: 運算元。
- (3) You must not use any library function unless the appropriate library has been included. (See Page 1)
除非適當的函數庫已被引用 (見頁一)，否則答案不可以包括任何函數庫內的函數。
- (4) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (5) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (6) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。
- (7) If blank X is divided into N parts X1, X2, ..., XN, it means that marks will only be given when X1, X2, ..., XN are all correct.
如果空格 X 分為 N 部份 X1、X2、...、XN，那麼 X1、X2、...、XN 皆為正確才會給分。

1. What is the output of the following program? 以下程序的輸出是什麼？

Pascal

```
var
  a: array[0..100] of longint;
  i, j, sum: longint;
begin
  for i := 1 to 100 do
    a[i] := 0;
  for i := 1 to 100 do
  begin
    j := i;
    while (j <= 100) do
    begin
      a[j] := 1 - a[j];
      j := j + i
    end
  end;
  sum := 0;
  for i := 1 to 100 do
    sum := sum + a[i];
  write(sum)
end.
```

C

```
int a[101];
int i, j, sum;
int main() {
  for (i = 1; i <= 100; i++)
    a[i] = 0;
  for (i = 1; i <= 100; i++) {
    j = i;
    while (j <= 100) {
      a[j] = 1 - a[j];
      j = j + i;
    }
    sum = 0;
    for (i = 1; i <= 100; i++)
      sum = sum + a[i];
    printf("%d", sum);
    return 0;
}
```

C++

```
int a[101];
int i, j, sum;
int main() {
  for (i = 1; i <= 100; i++)
    a[i] = 0;
  for (i = 1; i <= 100; i++) {
    j = i;
    while (j <= 100) {
      a[j] = 1 - a[j];
      j = j + i;
    }
    sum = 0;
    for (i = 1; i <= 100; i++)
      sum = sum + a[i];
    cout << sum;
    return 0;
}
```

Answer 答案: _____ A _____ (2 marks 分)

2. The following program inputs $2n + 1$ integers. Each integer between 1 and $n + 1$ (inclusive), other than one that is only inputted once, is inputted twice. Complete the program, so that it outputs the number that is inputted only once.

以下程序輸入 $2n + 1$ 個整數。在 1 和 $n + 1$ 之間（含）的整數，除了一個只被輸入一次之外，其他均被輸入兩次。完成程序，使其輸出那個只被輸入一次的數字。

Pascal

```
var
  n, x, ans, i: longint;
begin
  read(n);
  ans := 0;
  for i := 0 to 2 * n do
  begin
    read(x);
    _____ B _____
  end;
  write(ans)
end.
```

C

```
int n, x, ans, i;
int main() {
  scanf("%d", &n);
  ans = 0;
  for (i = 0; i <= 2 * n; i++) {
    scanf("%d", &x);
    _____ B _____;
  }
  printf("%d", ans);
  return 0;
}
```

C++

```
int n, x, ans, i;
int main() {
  cin >> n;
  ans = 0;
  for (i = 0; i <= 2 * n; i++) {
    cin >> x;
    _____ B _____;
  }
  cout << ans;
  return 0;
}
```

Answer 答案: _____ B _____ (2 marks 分)

3. Complete the function below, so that the function returns `true` if and only if x is a power of two.
完成以下函數，使得函數傳回 `true` 當且僅當 x 是二的次方。

Pascal

```
function is_power_of_two(x: longint): boolean;
begin
  if (x <= 0) then
    is_power_of_two := false
  else if (_____ C _____) then
    is_power_of_two := true
  else
    is_power_of_two := false
end;
```

C / C++

```
bool is_power_of_two(int x) {
  if (x <= 0)
    return false;
  else if (_____ C _____)
    return true;
  else
    return false;
}
```

Answer 答案: _____ C _____ (1.5 marks 分)

4. Given a positive integer n , we say n is:

給定一正整數 n ，稱 n 為：

- Abundant, if the sum of factors of n is larger than $2n$;
盈數，如果 n 的因數之和大於 $2n$ ；
- Deficient, if the sum of factors of n is smaller than $2n$;
虧數，如果 n 的因數之和小於 $2n$ ；
- Perfect, if the sum of factors of n is equal to $2n$.
完全數，如果 n 的因數之和等於 $2n$ 。

For example, the sum of factors of 4 is $1 + 2 + 4 = 7$, which is smaller than $2 \times 4 = 8$, so 4 is deficient. The sum of factors of 28 is $1 + 2 + 4 + 7 + 14 + 28 = 56$, which is equal to $2 \times 28 = 56$, so 28 is perfect.
例如，4 的因數之和是 $1 + 2 + 4 = 7$ ，小於 $2 \times 4 = 8$ ，所以 4 是虧數。28 的因數之和是 $1 + 2 + 4 + 7 + 14 + 28 = 56$ ，等於 $2 \times 28 = 56$ ，所以 28 是完全數。

You need to write a program which, given input n , determines if n is abundant, deficient, or perfect.

你需要編寫程式，對於輸入 n ，判斷 n 是盈數、虧數、還是完全數。

It is guaranteed that n is a positive integer in the range $[1, 10^7]$.

保證 n 是在 $[1, 10^7]$ 以內的正整數。

(a) First, you need write a function `sum_factors()`, which, given parameter n , returns the sum of factors of n .
首先，你需要編寫函數 `sum_factors()`，給定參數 n ，傳回 n 的因數之和。

Your friend John provides you his implementation of the function. You are advised to test it on several small inputs to understand how it works.

你的朋友約翰為你提供了他的函數實現。建議你在幾個小輸入上進行測試，理解它如何運作。

Pascal

```
function sum_factors(n: longint): longint;
var
  sum: longint = 0;
  i: longint = 1;
begin
  while (i * i <= n) do
  begin
    if (n mod i = 0) then
      sum := sum + i + n div i;
    inc(i);
  end;
  sum_factors := sum
end;
```

C / C++

```
int sum_factors(int n) {
  int sum = 0;
  int i = 1;
  while (i * i <= n) {
    if (n % i == 0)
      sum = sum + i + n / i;
    i++;
  }
  return sum;
}
```

Find two values of n between 11 and 99 (inclusive), for which John's function fails to return the correct answer.
找兩個在 11 和 99 之間（含）的 n 值，使得約翰的函數不能傳回正確答案。

Answer 答案: _____ D1 _____ D2 _____ (1 mark 分)

- (b)** Complete the modified implementation below so that it returns the correct answer for all n.
完成以下修改後的實現，使得它對於所有 n 均能傳回正確答案。

Pascal

```
function sum_factors(n: longint): longint;
var
  sum: longint = 0;
  i: longint = 1;
begin
  while (i * i <= n) do
  begin
    if (n mod i = 0) then
      sum := sum + i + n div i;
    if (_____ E1 _____) then
      sum := _____ E2 _____;
    inc(i)
  end;
  sum_factors := sum
end;
```

C / C++

```
int sum_factors(int n) {
  int sum = 0;
  int i = 1;
  while (i * i <= n) {
    if (n % i == 0)
      sum = sum + i + n / i;
    if (_____ E1 _____)
      sum = _____ E2 _____;
    i++;
  }
  return sum;
}
```

Answer 答案: _____ E1 _____ E2 _____ (1.5 marks 分)

- (c)** Now, you have a correct and efficient implementation for the function `sum_factors()`. Complete the program below, calling the function when appropriate, so that the program correctly determines whether the input number n is abundant, deficient, or perfect.

現在，對於函數 `sum_factors()`，你有一個正確且高效的實現。完成下面的程序，在適當的時候呼叫該函數，使得程序正確地判斷輸入數字 n 是盈數、虧數、還是完全數。

Pascal

```
function sum_factors
  (n: longint): longint;
  ... // omitted 省略
end;
var
  n, sum: longint;
begin
  read(n);
  sum := _____ F _____;
  if (_____ G1 _____) then
    write('Abundant') // 盈數
  else if (_____ G2 _____) then
    write('Perfect') // 完全數
  else
    write('Deficient') // 虧數
end.
```

C

```
int sum_factors(int n) {
  ... // omitted 省略
}
int n, sum;
int main() {
  scanf("%d", &n);
  sum = _____ F _____;
  if (_____ G1 _____)
    printf("Abundant"); // 盈數
  else if (_____ G2 _____)
    printf("Perfect"); // 完全數
  else
    printf("Deficient"); // 虧數
  return 0;
}
```

C++

```
int sum_factors(int n) {
  ... // omitted 省略
}
int n, sum;
int main() {
  cin >> n;
  sum = _____ F _____;
  if (_____ G1 _____)
    cout << "Abundant"; // 盈數
  else if (_____ G2 _____)
    cout << "Perfect"; // 完全數
  else
    cout << "Deficient"; // 虧數
  return 0;
}
```

Answer 答案: _____ F _____ (0.5 marks 分)

Answer 答案: _____ G1 _____ G2 _____ (1 mark 分)

5. Consider the following program: 考慮以下程序：

Pascal

```
var
  a: array[0..9] of longint;
  i, j, t, ans: longint;
begin
  for i := 0 to 9 do
    read(a[i]);
  ans := 0;
  for i := 8 downto 0 do
    for j := 0 to i do
      if (a[j] > a[j + 1]) then
        begin
          t := a[j];
          a[j] := a[j + 1];
          a[j + 1] := t;
          inc(ans)
        end;
    write(ans)
  end.
```

C

```
int a[10];
int i, j, t, ans;
int main() {
  for (i = 0; i <= 9; i++)
    scanf("%d", &a[i]);
  ans = 0;
  for (i = 8; i >= 0; i--)
    for (j = 0; j <= i; j++)
      if (a[j] > a[j + 1]) {
        t = a[j];
        a[j] = a[j + 1];
        a[j + 1] = t;
        ans++;
      }
  printf("%d", ans);
  return 0;
}
```

C++

```
int a[10];
int i, j, t, ans;
int main() {
  for (i = 0; i <= 9; i++)
    cin >> a[i];
  ans = 0;
  for (i = 8; i >= 0; i--)
    for (j = 0; j <= i; j++)
      if (a[j] > a[j + 1]) {
        t = a[j];
        a[j] = a[j + 1];
        a[j + 1] = t;
        ans++;
      }
  cout << ans;
  return 0;
}
```

If the input is 10 9 8 7 6 5 4 3 2 1, what is the output?

若輸入 10 9 8 7 6 5 4 3 2 1，那麼輸出是什麼？

Answer 答案: _____ H _____ (1 mark 分)

If the input is 4 4 10 10 10 3 3 3 6 6, what is the output?

若輸入 4 4 10 10 10 3 3 3 6 6，那麼輸出是什麼？

Answer 答案: _____ I _____ (1 mark 分)

Give an input such that the program outputs 42. 紿一個輸入使得程序輸出 42。

Answer 答案: _____ J _____ (1.5 marks 分)

6. On the entrance gate of a building, there is a numpad with nine buttons, for the digits from 1 to 9. The numpad is used to input gate passcode. The gate passcode is a four digit number that can be input using the numpad. In addition, the product of the first digit and the second digit equals the number formed by the last two digits. For example, 7535 is a possible gate passcode because $7 \times 5 = 35$. How many possible gate passcodes are there?
在一幢大廈的入口大門上，有一個數字鍵盤，鍵盤上有由 1 至 9 的九個數字鍵，用以輸入大門密碼。大門密碼是個可以用數字鍵盤輸入的 4 位數，而且其中第一位及第二位數字的積等於後兩位數組成的數字。例如 7535 是可能的大門密碼，因為 $7 \times 5 = 35$ 。總共有多少個可能的大門密碼？

Answer 答案: _____ K _____ (2 marks 分)

7. Given an array $a[0..99]$ which consists of 100 integers. Its elements are sorted **in descending order**. Complete the function $f(target)$, so that, among all indices i with $a[i]$ greater than or equal to $target$, it returns the largest one. If no such index exists, it should return -1 instead.

給定由 100 個整數組成的數組 $a[0..99]$ 。其元素由大至小排列。完成函數 $f(target)$ ，使得在所有符合 $a[i]$ 大於或等於 $target$ 的索引 i 當中，它傳回最大的一個。如果這樣的索引不存在，那麼傳回 -1 。

It is guaranteed that the array elements and $target$ are integers between -1000 and 1000 inclusive.

保證數組元素及 $target$ 是 -1000 和 1000 之間（含）的整數。

Note that the same integer may appear more than once in the array.

注意，同一整數可以在數組出現多於一次。

Pascal

```
var
  a: array[0..99] of longint;
function f(target: longint): longint;
var
  l: longint = 0;
  r: longint = 100;
  m: longint;
begin
  while (l < r) do
  begin
    m := (l + r) div 2;
    if (_____ L1 _____) then
      _____ L2 _____
    else
      r := m
  end;
  f := _____ L3 _____
end;
```

C / C++

```
int a[100];
int f(int target) {
  int l = 0;
  int r = 100;
  int m;
  while (l < r) {
    m = (l + r) / 2;
    if (_____ L1 _____)
      _____ L2 _____;
    else
      r = m;
  }
  return _____ L3 _____;
}
```

Answer 答案: _____ L1 _____ _____ L2 _____ _____ L3 _____ (2 marks 分)

8. The input to the following program is an integer between 1 and 30 inclusive.

以下程序的輸入為一個 1 至 30 之間（含）的整數。

Pascal

```
11 var
12   x : longint;
13 begin
14   read(x);
15   if (x mod 2 = 0) then
16     if (x mod 3 = 0) then
17       write('six')
18     else
19       write('ODD')
20 end.
```

C

```
41 int x;
42 int main() {
43   scanf("%d", &x);
44   if (x % 2 == 0)
45     if (x % 3 == 0)
46       printf("six");
47   else
48     printf("ODD");
49   return 0;
50 }
```

C++

```
71 int x;
72 int main() {
73   cin >> x;
74   if (x % 2 == 0)
75     if (x % 3 == 0)
76       cout << "six";
77   else
78     cout << "ODD";
79   return 0;
80 }
```

The program tries to output according to the followings: 此程序嘗試根據下列規則輸出：

1. If the input is a multiple of 6, output the message **six**
若輸入是一個 6 的倍數，輸出訊息 **six**
2. If the input is an odd number, output the message **ODD**
若輸入是一個奇數，輸出訊息 **ODD**
3. Nothing should be outputted if none of the above conditions are met
若上述兩個條件皆不成立，不應輸出任何訊息

However, there is a bug in his program, please suggest a valid input such that the program outputs incorrectly.
但是程序中有一錯誤，請提出一個有效的輸入使得這個程序錯誤地輸出。

Answer 答案: _____ M (1 mark 分)

The bug can be fixed by adding some characters at the end of one line. Find the line and correct it. Please assume that there are no whitespace characters at the end of each line in the original program.

那一錯誤能透過在其中一行的末端增加一些字符來修正。請找出並將其改正。請假設在原本的程序中，每行行末均不存在空白字元。

Line number 行數: _____ N1

Characters to add 增加的字符: _____ N2 (2 marks 分)

END OF PAPER 全卷完