

Assume that all variables without declaration shown in the following program segments have already been declared properly. Integers in problem statements are 32-bit signed variables (**Pascal**: longint, **C**: int). Assume all the programs are compiled properly without using any compiler flag (except the "-o" option in C).

	<b>Format</b>	<b># Questions</b>	<b>Total Marks</b>
<b>Section A</b>	Multiple Choice	25	25
<b>Section B</b>	Fill-in-the-blanks	7 (A-J)	20
<b>Total</b>			<b>45</b>

### Section A (25 marks)

For each question, choose the most appropriate answer and write the letter (**A**, **B**, **C** or **D**) in the corresponding space on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.

1. Given 5 equalities:

$$a + d = 10$$

$$c + e = -3$$

$$b + e = 1$$

$$d + c = 5$$

$$b + f = 2$$

Which of the following expression(s) can be evaluated?

i.  $a + f$

ii.  $b + d$

iii.  $d + e$

A. iii only

B. i and ii only

C. ii and iii only

D. All of the above

2. What is the output of the following program?

#### Pascal Version

```
var
  sum1, sum2: double;
  i: longint;
begin
  sum1 := 0.0;
  sum2 := 0.0;
  for i := 1 to 6 do
  begin
    sum1 := sum1 + 1.0 / i;
    sum2 := sum2 - 1.0 / i;
  end;
  if (sum1 = 2.45) then write('A');
  if (sum1 + sum2 = 0.0) then
  write('B');
end.
```

#### C Version

```
#include <stdio.h>
double sum1, sum2;
int i;
int main() {
  sum1 = 0.0;
  sum2 = 0.0;
  for (i = 1; i <= 6; i++) {
    sum1 = sum1 + 1.0f / i;
    sum2 = sum2 - 1.0f / i;
  }
  if (sum1 == 2.45) printf("A");
  if (sum1 + sum2 == 0.0) printf("B");
  return 0;
}
```

- A. A
- B. B
- C. AB
- D. Empty output

3. 689 is such a beautiful number, in a way that when it is rotated 180 degrees, it is still 689. 101 is another beautiful number with length 3, while 2015 is not a beautiful number with length 4. Note that beautiful numbers do not contain leading zeroes. How many beautiful numbers with length 2015 are there? What is the units digit of the answer?

- A. 2
- B. 0
- C. 1
- D. 5

For questions 4 to 5, consider the following program segment:

#### Pascal Version

```
var
  max, i: longint;
  a: array[0..4] of longint;
begin
  max := 0;
  for i := 0 to 4 do
    read(a[i]);
  for i := 0 to 3 do
    if (a[i] + a[i+1] > max) then
      max := a[i] + a[i+1];
  write(max);
end.
```

#### C Version

```
#include <stdio.h>
int max, n, i, a[5];
int main() {
  max = 0;
  for (i = 0; i < 5; i++)
    scanf("%d", &a[i]);
  for (i = 0; i < 4; i++)
    if (a[i] + a[i+1] > max)
      max = a[i] + a[i+1];
  printf("%d", max);
  return 0;
}
```

4. What is the output of the program when the input is 5 -6 2 -5 3?

- A. -2
- B. -1
- C. 0
- D. 8

5. Which of the following inputs will give the numerically largest output?

- A. -2100000000 2000000000 999999999 -1500000000 1234567890
- B. 5 1000 -300 0 5
- C. -500 50000 -2000000000 -1000000000 800
- D. -800000 50000 -99999999 100000 5000

6. Palindromes are strings that read the same forward or reversed. For example, "abcba" is a palindrome while "accda" isn't.

Substring of a string is any continuous segment of the string, For example, Both "a", "b", "c", "ab", "bc" and "abc" are substrings of "abc", while "ac" isn't.

How many strings of length exactly 4 does not consist of any substring that is palindrome and with length larger than 1? Consider characters from "a" to "z" only.

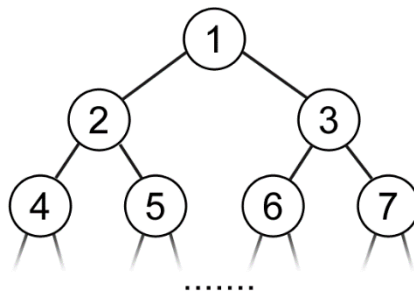
For example, "abcd" is one of the strings that match the condition, while "aabc" is not. (Since "aa" is a palindrome).

- A. 374400
- B. 390000
- C. 405600
- D. 422500

7. Alice has just learnt to use file compressing tools in today's ICT lesson. She compressed an image flower.jpg (2MB) to flower.zip (1MB). Now, she is compressing flower.zip to flower.zip.zip. What is the most appropriate prediction of the size of the double compressed file?

- A. 0MB
- B. 0.5MB
- C. 1MB
- D. 2MB

8. In this problem, we use a binary tree, the root of which has a value of 1. Every node in this tree with value N has a left child with value  $N \times 2$  and a right child with value  $N \times 2 + 1$ . The first few levels of the binary tree look like this:



We define the ancestors of a node include itself and the ancestors of its parent. For example, 1 and 3 are the ancestors of 7.

The common ancestors of two nodes v and w are ancestors of both v and w. For example, 'the common ancestors of 6 and 7 are 1 and 3. The lowest (i.e. deepest) one is called the lowest common ancestor (LCA). Therefore, the LCA of 6 and 7 is 3.

What is the LCA of 2338 and 2015?

- A. 1
- B. 2
- C. 1024
- D. 2048

9. We have a 4 x 4 chessboard. A white or black chess must be placed in each cell. Two cells which share a common edge are considered as neighbors.

Every chess must have an odd number of neighboring chess of the same color. For example, the following is a valid configuration.

```
oxxx
ooxo
oxoo
xxxxo
```

In how many valid ways can you fill the following chessboard? ("x", "o", "." represent black chess, white chess and a cell to be filled respectively)

```
x.o.o
....
....
....
```

- A. 1
- B. 2
- C. 8
- D. 16

10. What is the output of the following program?

**Pascal Version**

```
var
  a: array[0..3] of longint;
  i, t: longint;
begin
  a[0] := 3; a[1] := 2; a[2] := 9; a[3] :=
5;
  t := a[0];
  for i := 1 to 3 do
  begin
    t := ((t * 2) xor (a[i]));
  end;
  write(t);
end.
```

**C Version**

```
#include <stdio.h>
int a[4], i, t;
int main() {
  a[0] = 3; a[1] = 2; a[2] = 9; a[3] =
5;
  t = a[0];
  for (i = 1; i <= 3; i++){
    t = ((t * 2) ^ (a[i]));
  }
  printf("%d", t);
  return 0;
}
```

- A. 2
- B. 3
- C. 5
- D. 7

11. What is the output of the following program?

**Pascal Version**

```
var
  ans, i, j: longint;
begin
  ans := 0;
  for i := 1 to 2003 do
  begin
    j := i;
    while j > 0 do
    begin
      ans := ans + j mod 10;
      j := j div 10;
    end;
  end;
  write(ans);
end.
```

**C Version**

```
#include <stdio.h>
int ans, i, j;
int main() {
  ans = 0;
  for(i = 1; i <= 2003; i++){
    j = i;
    while(j > 0){
      ans = ans + j % 10;
      j = j / 10;
    }
  }
  printf("%d", ans);
  return 0;
}
```

- A. 13510
- B. 14510
- C. 27014
- D. 28014

For questions 12 to 13, consider the following situation:

Alice and Bob is playing a two-player game called X, in which player takes turn to make a move until one player makes the winning move. They have just discovered that the first player (the player who makes the first move) could guarantee winning no matter how the second player reacts. They get bored so they add the following rules to the game.

- The game will be repeated for N rounds. (Each round is identical to the original version of X)
- Alice will be first player in round 1.
- The loser of the  $k^{\text{th}}$  round ( $1 \leq k < N$ ) will be the first player in the  $(k+1)^{\text{th}}$  round.
- The winner of the  $N^{\text{th}}$  round will be the winner of the whole game.

12. Given that both players do not want to lose. In which of the following situation(s), Alice will win?

N =

- i. 5
  - ii. 10
  - iii. 13
- A. ii only
  - B. i and iii only
  - C. Cannot be determined
  - D. All of the above

13. After they thoroughly studied the game, they figured out that in each round, the first player could guarantee losing no matter how the second player reacts, too.

Given that both players do not want to lose. In which of the following situation(s), Alice will win?

N =

- i. 5
- ii. 10
- iii. 13

- A. ii only
- B. i and iii only
- C. Cannot be determined
- D. All of the above

14. What is the output of the following program?

**Pascal Version**

```
var
  i, x: longint;
begin
  x := 0;
  for i := 53 to 102 do
    x := x xor i;
  write(x)
end.
```

**C Version**

```
#include <stdio.h>
int i, x;
int main() {
  x = 0;
  for(i = 53; i <= 102; i++)
    x = x ^ i;
  printf("%d", x);
  return 0;
}
```

- A. 83
- B. 0
- C. 1
- D. 52

15. The goal of the following program is to factorize a positive integer n into product of prime numbers.

**Pascal Version**

```
var
  n, k: longint;
begin
  read(n);
  k := 2;
  while (k <= n) do
  begin
    if (n mod k = 0) then
    begin
      n := n div k;
      if (n > 1) then
        write(k, ' * ');
      else
        write(k)
    end;
    k := k+1;
  end
end.
```

**C Version**

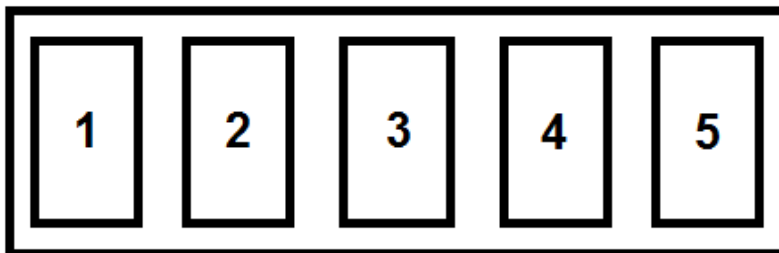
```
#include <stdio.h>
int k, n;
int main(){
  scanf("%d", &n);
  k = 2;
  while (k <= n) {
    if (n%k == 0) {
      n = n/k;
      if (n > 1)
        printf("%d * ", k);
      else
        printf("%d", k);
    }
    k = k+1;
  }
}
```

However, the program has a logic error. Which of the following input  $n$  will cause the program fail to output the expected result?

- i. 15
  - ii. 16
  - iii. 17
  - iv. 18
- A. iii only
  - B. iv only
  - C. ii and iv only
  - D. i, ii and iv only

16. There is a machine  $X$  in the office, which is invented to store  $N$  paper document securely. Each document is stored in an independent cell inside the machine. We will denote the leftmost cell as  $1^{\text{st}}$  cell and the rightmost cell as  $N^{\text{th}}$  cell.

Following is the internal view of machine  $X$  when there are 5 documents for storing, in this case, the  $p^{\text{th}}$  document is stored in the  $p^{\text{th}}$  cell. When a staff request access for the document inside  $k^{\text{th}}$  cell (not necessarily the  $k^{\text{th}}$  document),  $X$  will first move the document to the nearest end of the machine for access and return it to the original cell after it is done. After that,  $\min(N - k + 1, k)$  unit energy is consumed. ( $\min(a, b)$  will return the minimum value among  $a$  and  $b$ )



Using the picture above as an example, when  $k = 2$ ,  $X$  will use 2 unit energy to move the document to the left end and return it. When  $k = 5$ ,  $X$  will use 1 unit energy to move the document to the right end and return it.

The inventor of  $X$  has recorded the frequency of usage of each document per day

Document	1	2	3	4	5
Frequency	4	2	5	1	6

Assuming the above frequency is constant every day, if he could rearrange the document order inside  $X$ , what is the minimum energy unit usage per day?

- A. 26
- B. 27
- C. 28
- D. 29

17. There are 20 people in a class, which all of them have a distinct seat in a 4 x 5 grid system. The teacher is separating the class into three for a group project. As the teacher is sitting in front of all students, he has four ways of grouping. He will start count 1, 2, 3, 1, 2, 3... from the rightmost/leftmost student in front of him in two possible directions (As illustrated in the graph), until everyone has been assigned a group number. (1, 2 or 3)

Obviously, group 3 is the group with least number of workforce, how many seat are/is there, such that whoever sit on it would stay in group 3 no matter which grouping method the teacher used?

```

12312  12...  ...21  21321
3....  23...  ...32  ...13
.....  3....  ....3  .....
.....  1....  ....1  .....
    
```

- A. 0
- B. 2
- C. 4
- D. 6

18. What is the output of the following program?

**Pascal Version**

```

var
  tmp, cnt, i: longint;
begin
  tmp := 0;
  cnt := 0;
  i := 1;
  while (i < 100) do
  begin
    tmp := tmp * i;
    tmp := tmp + i;
    tmp := tmp mod 1000;
    if (tmp mod 5 = 0) then
      cnt := cnt + 1;
    i := i + 1;
  end;
  write(cnt);
end.
    
```

**C Version**

```

#include <stdio.h>
int tmp, cnt, i;
int main() {
  tmp = 0;
  cnt = 0;
  i = 1;
  while (i < 100) {
    tmp = tmp * i;
    tmp = tmp + i;
    tmp = tmp % 1000;
    if (tmp % 5 == 0)
      cnt = cnt + 1;
    i = i + 1;
  }
  printf("%d", cnt);
  return 0;
}
    
```

- A. 20
- B. 29
- C. 39
- D. 40



19. What is the output of the following program?

#### Pascal Version

```
var
  k, x, y, tx, ty:longint;
  a: array[0..2, 0..4] of longint =
  ((11, 3, 7, 15, 1), (16, 62, 53, 44, 37),
  (10, 12, 11, 31, 22)) ;
begin
  k := 127;
  x := 1;
  tx := 1;
  y := 0;
  ty := 0;
  while k>0 do
  begin
    dec(k);
    x := (a[tx][ty]+tx) mod 3;
    y := (a[tx][ty]+ty) mod 5;
    tx := x;
    ty := y;
  end;
  write(x, ' ', y);
end.
```

#### C Version

```
#include <stdio.h>
int k, x, y, tx, ty, a[3][5] = {{11, 3,
7, 15, 1}, {16, 62, 53, 44, 37}, {10,
12, 11, 31, 22}};
int main(){
  k = 127;
  x = 1;
  tx = 1;
  y = 0;
  ty = 0;
  while(k>0){
    k--;
    x = (a[tx][ty] + tx) % 3;
    y = (a[tx][ty] + ty) % 5;
    tx = x;
    ty = y;
  }
  printf("%d %d", x, y);
  return 0;
}
```

- A. 0 4
- B. 0 3
- C. 1 3
- D. 2 3

20. Assume we keep flipping a fair coin until we have two heads in a row. What is the average number of flips before we stop?

- A. 4.00
- B. 4.50
- C. 5.25
- D. 6.00

21. Variables  $a$  and  $b$  are integers. Which of the following expression(s) is/are equivalent to  $a > b$ ?

- i.  $a - b > 0$
- ii.  $a + 1 > b + 1$
- iii.  $a * 1 > b * 1$
- iv.  $a * -1 < b * -1$

- A. iii only
- B. i and ii only
- C. iii and iv only
- D. All of the above

For questions 22 to 23, consider the following situation:

In the game League of Legends, a team consists of 5 players. Lee Sin, Fizz, Vayne, Thresh and Ezreal have formed a team.

There are 4 roles in a team which are the jungler, the top laner, the middle laner and the bottom laner.

A player should take exactly one role.

22. The team requires 1 jungler, 1 top laner, 1 middle laner and 2 bottom laners.

How many different assignments of roles are there?

- A. 60
- B. 120
- C. 240
- D. 600

23. As some new tactics have been developed, now the team requires at least 1 player and at most 2 players for each of the 4 roles.

How many different assignments of roles are there?

- A. 60
- B. 120
- C. 240
- D. 600

24. What is the output of the following program?

**Pascal Version**

```
var
  n, count: longint;
begin
  n := 75290635;
  count := 0;
  while (n > 0) do
  begin
    while (n mod 10 <> 0) do
    begin
      n := n - 1;
      if (n mod 10 = 4) then
        continue;
      count := count + n mod 10;
    end;
    n := n div 10;
  end;
  write(count);
end.
```

**C Version**

```
#include <stdio.h>
int n, count;
int main() {
  n = 75290635;
  count = 0;
  while (n > 0) {
    while (n % 10 != 0) {
      n = n - 1;
      if (n % 10 == 4)
        continue;
      count = count + n % 10;
    }
    n = n / 10;
  }
  printf("%d", count);
  return 0;
}
```

- A. 32
- B. 37
- C. 76
- D. 113

25. What is the output of the following program?

**Pascal Version**

```
var
  a:array[0..4] of longint;
  i, j, temp: longint;
begin
  a[0] := 1; a[1] := 2; a[2] := 4; a[3] :=
3; a[4] := 5;
  for i := 1 to 5102 do
  begin
    temp := a[0];
    for j := 0 to 3 do
      a[j] := a[j+1];
    a[4] := temp;
  end;
  for i := 0 to 4 do
    write(a[i], ' ');
  end.
```

**C Version**

```
#include <stdio.h>
int a[5];
int i, j, temp;
int main() {
  a[0] = 1; a[1] = 2; a[2] = 4; a[3] =
3; a[4] = 5;
  for (i = 1; i <= 5102; i++) {
    temp = a[0];
    for (j = 0; j <= 3; j++)
      a[j] = a[j+1];
    a[4] = temp;
  }
  for (i = 0; i <= 4; i++)
    printf("%d ", a[i]);
  return 0;
}
```

- A. 1 2 4 3 5
- B. 2 1 5 4 3
- C. 4 3 5 1 2
- D. 5 1 2 4 3

**END OF SECTION A**

**Section B (20 marks)**

The blanks are labeled from A to J. Please fill in the blanks on the answer sheet.

Except otherwise specified, two marks for each correct blank. No marks will be deducted for wrong answers.

**Note:**

- (1) You must not use the ? : operator in C.
  - (2) You must not use any library function unless the appropriate library(s) is/are included.
  - (3) You can write only one character in each box on the answer sheet.
  - (4) No answer with length greater than the designated number of boxes will be accepted.
1. The NAND(not-and) function has many applications. The truth table of NAND, with two inputs, is as follows

A	B	A NAND B
T	T	F
T	F	T
F	T	T
F	F	T

Let's use Q to represent the NAND function. Therefore the NAND of X and Y is (XQY), and this result is also a boolean expression. Also A and B themselves are boolean expressions. For example, the NOT function of A can be expressed as (AQA).

Please note that brackets in the above example are compulsory, no mark will be given for answers that miss brackets in the following questions.

Given two inputs A and B, represent the following truth table using boolean expressions with variables A, B, brackets and Q (NAND) only.

A	B	Result
T	T	T
T	F	F
F	T	F
F	F	F

Answer: \_\_\_\_\_ A \_\_\_\_\_ (2 marks)

Given two inputs A and B, represent the following truth table using boolean expressions with variables A, B, brackets and Q (NAND) only.

A	B	Result
T	T	T
T	F	F
F	T	F
F	F	T

Answer: \_\_\_\_\_ B \_\_\_\_\_ (2 marks)

2. The following program reads an integer  $n$  and outputs a  $(2n+1) \times (2n+1)$  grid.

**Pascal Version**

```

var
  n, i, j: longint;
begin
  read(n);
  for i := 0 to 2*n do
  begin
    for j := 0 to 2*n do
      if (_____ C, D _____) then
        write('*')
      else
        write(' ');
    writeln();
  end;
end.

```

**C Version**

```

#include <stdio.h>
int main() {
  int n, i, j;
  scanf("%d", &n);
  for (i = 0; i <= 2*n; i++) {
    for (j = 0; j <= 2*n; j++) {
      if (_____ C, D _____)
        printf("*");
      else
        printf(" ");
    }
    printf("\n");
  }
  return 0;
}

```

If the input  $n$  is 4, complete the program such that the output is

```

*       *
*       *
*       *
* *
*
* *
* *
* *
*       *
*       *

```

Answer: \_\_\_\_\_ C \_\_\_\_\_ (2 marks)

If the input  $n$  is 5, complete the program such that the output is

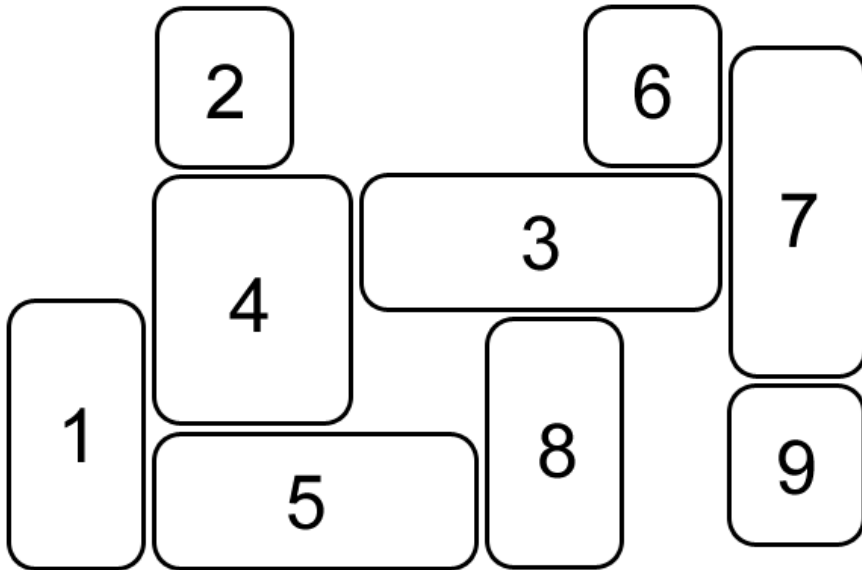
```

*****
*       *
*       * *
*       *
*       *
* * * * *
*       *
*       *
*       *
*       *
*****

```

Answer: \_\_\_\_\_ D \_\_\_\_\_ (2 marks)

3. There are some ice cubes floating. The rectangles represent the cubes and the narrow gaps between rectangles mean that you can walk from between the two ice cubes. Alice is on one of the ice cubes. As ice cubes keep on melting, she needs to move from one ice cube to another ice cube. Once she leaves an ice cube, the ice cube melts and Alice can no longer stand on that ice cube. Eventually she will be trapped on one of the ice cube having no way to go. Bob, who is the enemy of Alice, would like to know all the possibilities of Alice's position when she will be trapped. Please write down the index(es) of that/those ice cube(s).



Answer:       E       (2 marks)

4. In International Chess, "Bishop" can attack in 4 diagonal directions. Given that there are only two bishops on a 8 x 8 chessboard, the first bishop is on row r1, column c1, the second bishop is on row r2, column c2.

When the following function is given the coordinates of two bishops, it will return 0 (**Pascal** : false) if there exists a cell in the board that is attacked by both bishops, return 1 (**Pascal** : true) if otherwise. Please complete the following function.

The function `check` satisfies:  
 $1 \leq r1, r2, c1, c2 \leq 8$

**Pascal Version**

```
function check(r1, c1, r2, c2:
longint) : boolean;
begin
  check :=       F      ;
end
```

**C Version**

```
int check(int r1, int c1, int r2, int
c2){
  return       F      ;
}
```

Answer:       F       (2 marks)

5. Consider the following program:

**Pascal version**

```

var
  a: array[0..9] of longint =
    (2 + 1, 0 + 5, 6 + 0, 0, 0, 1 + 9,
 8 + 2, 0, 9 + 6, 5 + 8);
procedure f(x, y: longint);
var
  i, tmp: longint;
begin
  for i := 1 to 7 do
  begin
    write(x);
    tmp := y;
    y := a[y] - x;
    x := tmp;
  end;
end;
procedure G;
begin
  _____ G _____
end;
procedure H;
begin
  _____ H _____
end;
end;

```

**C version**

```

#include <stdio.h>

int a[10] = {2 + 1, 0 + 5, 6 + 0, 0, 0,
1 + 9, 8 + 2, 0, 9 + 6, 5 + 8};
void f(int x, int y) {
  int i, tmp;
  for (i = 0; i < 7; i++) {
    printf("%d", x);
    tmp = y;
    y = a[y] - x;
    x = tmp;
  }
}

void G() {
  _____ G _____
}

void H() {
  _____ H _____
}

```

Complete the program by filling in     G     so that calling G () will output 2015986. (2 marks)

Complete the program by filling in     H     so that calling H () will output 5102689. (2 marks)

6. Consider the following program:

**Pascal version**

```

var
  i: longint;
begin
  for i := 0 to 15 do
  begin
    write(chr(_____ I _____));
    if (i mod 4 = 3) then
      writeln()
  end;
end.

```

**C version**

```

#include <stdio.h>
int main() {
  int i;
  for (i = 0; i <= 15; i++) {
    printf("%c", _____ I _____);
    if (i % 4 == 3)
      printf("\n");
  }
  return 0;
}

```

Complete the program so that the output will be

dhlp  
cgko  
bfjn  
aeim

Answer:     I     (2 marks)

Please note that chr () is provided for Pascal version.

7. Variable `d` from the original problem is renamed to variable `x` to avoid name conflict in Pascal version. Scores were also given to those who wrote down “compile error” as their answers.

What is the output of the following program?

#### Pascal Version

```
var
  n, x, A, B, C, D : longint;
begin
  n := 2015;
  x := 3;
  A := (n div x) * x + n mod x;
  B := (n div (-x)) * (-x) + n mod (-x);
  C := ((-n) div x) * x + (-n) mod x;
  D := ((-n) div (-x)) * (-x) + (-n) mod
(-x);
  write(A + B + C + D)
end.
```

#### C Version

```
#include <stdio.h>
int n, x, A, B, C, D;
int main() {
  n = 2015;
  x = 3;
  A = (n / x) * x + n % x;
  B = (n / (-x)) * (-x) + n % (-x);
  C = ((-n) / x) * x + (-n) % x;
  D = ((-n) / (-x)) * (-x) + (-n) % (-x);
  printf("%d", A + B + C + D);
  return 0;
}
```

Answer:       J       (2 marks)

**END OF PAPER**