Hong Kong Olympiad in Informatics 2014/15

Heat Event (Senior Group)

Official Solution

Statistics (N = 184)

Full mark = 45. Maximum = 41. Median = 13. Advance to Final = 15 marks or above.

Section A

Q	A	Explanation
1	B	Assume a is a free variable, b, c, d, e and f can all be expressed in terms of a. $d = 10 - a$ then $10 - a + c = 5, c = a - 5$. Then $a - 5 + e = -3, e = 2 - a$. Then $b + 2 - a = 1, b = a - 1$. Then $a - 1 + f = 2, f = 3 - a$ Therefore, $a + f = a + 3 - a = 3$. $b + d = a - 1 + 10 - a = 9$. However, $d + e = 10 - a + 2 - a = 12 - 2a$, which is not a constant.
2	B	Precision error causes the first condition to return false. However, the sign of a floating point number is stored separately. Therefore, summing a floating point number and its negation always give 0.
3	B	We can choose 1, 6, 8, 9 for the first digit and 0, 1, 6, 8, 9 for the subsequent digits. The answer is a multiple of 10 so its units digit is 0.
4	C	The program tried to find the maximum sum of two consecutive numbers. But since all the sums are negative, the initial value of `max` (=0) is output.
5	C	For A, the overflow result of 2000000000 + 999999999 is a negative number. All other sums are negative. For B, the sums are very small. For C, the overflow result of -2000000000 + -1000000000 is a large positive number. For D, the only positive sum is 105000 which is very small.
6	A	First, two consecutive characters cannot be the same. The first and the third, the second and the fourth characters cannot be the same. Let the string be $s_1s_2s_3s_4$ s_1 can be arbitrarily chosen (26 choices). $s_2 \neq s_1$ (25 choices). $s_3 \neq s_1$ and $s_3 \neq s_2$ (24 choices). $s_4 \neq s_2$ and $s_4 \neq s_3$ (24 choices) Answer = $26 \times 25 \times 24 \times 24 = 374400$
7	C	Compressed files cannot be compressed again effectively.
8	A	The parent of a node is N / 2 (ignore remainder). Keep dividing 2338 and 2015 by 2, we get $2338 \to 1169 \to 584 \to 292 \to 146 \to 73 \to 36 \to 18 \to 9 \to 4 \to 2 \to 1$ $2015 \to 1007 \to 503 \to 251 \to 125 \to 62 \to 31 \to 15 \to 7 \to 3 \to 1$. The LCA is 1.

9	B	Let's try to put an `x` in the empty on the first row. The chessboard would become
		`xxoo`
		`ooxx`
		`xxoo`
		`ooxx`
		Let's also try to put an `o` instead. The chessboard would be similar to the given sample.
		In both ways the chessboard is uniquely determined. Therefore the answer is 2.

Q	A	Explanation
10	D	Initially, `t = 3` `i = 1: t = 6 ^ 2`, which gives 4. `i = 2: t = 8 ^ 9`, which gives 1. `i = 3: t = 2 ^ 5`, which gives 7.
11	D	The program computes the sum of digits of all numbers from 1 to 2003. Consider 1 to 1999 only. In the units, tens and hundreds digits, each number 0-9 appeared in each digit 200 times. In the thousands digit, 1 appeared 1000 times. Therefore the answer is $45 \times 200 \times 3 + 1000 + 2 + 3 + 4 + 5 = 28014$
12	C	Both players do not want to lose the whole game, so they need to be the first player in round N. To be the first player in round N, they need to lose in round N-1. However we do not know whether there is a losing strategy, so the answer is C.
13	D	Alice will choose to keep losing so that she can be the first player again and again. In the last round, Alice can use the winning strategy to win the whole game.
14	A	The loop is repeated 50 times. The 2^0 digit repeats in this pattern: $0 \to 1 \to 1 \to 0 \to \cdots$ The 2^1 digit repeats in this pattern: $0 \to 0 \to 1 \to 0 \to \cdots$ Since $50 \equiv 2 \pmod 4$, the answer modulo 4 should be 3.
15	C	If N has a prime factor `k` of any positive degree, the program will only `k` output once. For example when `n` = 8, the program will output `2 * 4` instead of `2 * 2 * 2`. $15 = 3 \times 5,\ 16 = 2 \times 2 \times 2 \times 2,\ 17 = 17, 18 = 2 \times 3 \times 3.$ Only 16 and 18 have a prime factor of degree greater than 1.
16	A	We try to place frequently accessed documents as near the end as possible. By placing document 1 and 5 at the ends, and document 4 in the middle. The minimum energy consumption is achieved. Answer = $5 + 6 + 2 \times 4 + 2 \times 2 + 3 \times 1 = 26$
17	B	In fact, we only need to try to count once for vertically and once horizontally. The counting should be symmetrical. We are looking for seats that have number 3 in both countings and their reflections. There are only 2 such seats (Row 1 & 4, Column 3).
18	C	We are interested in `tmp` modulo 5. Initially, $\text{tmp} \equiv 0 \pmod 5$ After first loop, $\text{tmp} \equiv 1 \pmod 5$ After second loop, $\text{tmp} \equiv 4 \pmod 5$ After third loop, $\text{tmp} \equiv 0 \pmod 5$ After forth loop, $\text{tmp} \equiv 4 \pmod 5$ After fifth loop, $\text{tmp} \equiv 0 \pmod 5$ You can see that tmp is a multiple of 5 two times per cycle. The loop is repeated 99 times so the answer is $19 \times 2 + 1$.

Q	A	Explanation
19	A	Let’s examine a[tx][ty] + tx modulo 3.

tx\ty	0	1	2	3	4
0	11+0: 2	3+0: 0	7+0: 1	15+0: 0	1+0: 1
1	16+1: 2	62+1: 0	53+1: 0	44+1: 0	37+1: 2
2	10+2: 0	12+2: 2	11+2: 1	31+2: 0	22+2: 0

Examine also a[tx][ty] + ty modulo 5

tx\ty	0	1	2	3	4
0	11+0: 1	3+1: 4	7+2: 4	15+3: 3	1+4: 0
1	16+0: 1	62+1: 3	53+2: 0	44+3: 2	37+4: 1
2	10+0: 0	12+1: 3	11+2: 3	31+3: 4	22+4: 1

$$(tx, ty) = (1,0) \rightarrow (2,1) \rightarrow (2,3) \rightarrow (0,4) \rightarrow (1,0)$$

The cycle length is 4. The loop is repeated 127 times so it will ends at (0, 4)

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20 D

Let *x* be the answer (expected number of flips)

$x = (H)(0.5) + (T)(0.5)$

$x = (1 + (HT)(0.5) + (HH)(0.5))(0.5) + (T)(0.5)$

$x = (1 + (1 + x)(0.5) + (1)(0.5))(0.5) + (1 + x)(0.5)$

$x = (1 + 0.5 + 0.5x + 0.5)(0.5) + (0.5 + 0.5x)$

$x = 1.5 + 0.75x$

$x = 6$

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21 A

Overflow may occur for i, ii, iv. Examples are

i: a = 2000000000, b = -1000000000

ii: a = 2147483647, b = 0

iv: a = 0, b = -2147483648

In these examples, `a > b` is true but the expression returns false

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22 A

For the jungler, there are 5 choices. For the top laner, there are 4 choices remaining. For the middle laner, there are 3 choices remaining.

Total number of combinations = $5 \times 4 \times 3 = 60$.

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23 C

Using the result obtained previously, there are now 4 choices for the role that has 2 players (instead of being restricted to jungler). Therefore the answer is $60 \times 4 = 240$.

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24 C

The program performs the following for each digit:

While the digit is not zero, deduct the digit by one, and add it to count if it is not 4.

= (6+5+3+2+1) + (3+2+1) + 1 + (8+7+6+5+3+2+1) + (5+3+2+1) + (2+1) + (3+2+1)

= 76

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25 C

The program rotates the array contents.

After the first rotation, `a = 2 4 3 5 1`. After the second rotation, `a = 4 3 5 1 2`.

The loop repeats 5102 times. $5102 \equiv 2 \pmod 5$

Section B

	Answer and Explanation	
A	`((AQB)Q(AQB))` The required result is A AND B, which is equivalent to NOT(A NAND B)	
B	`((AQB)Q((AQA)Q(BQB)))` The required result is A = B, which is equivalent to (A AND B) OR (NOT A AND NOT B) = NOT[NOT(A AND B) AND NOT(NOTA AND NOT B)] (DeMorgan's Theorem) = (A NAND B) NAND (NOT A NAND NOT B)	
C	`(i=j)or(i=8-j)`	`i==j||i==8-j`
	The diagonals are i=j and i=8-j	
D	`i*j mod 10=0`	`i*j%10==0`
	There is a * at `j`=5 when `i`=2, 4, 6 and 8. There are *s at `j`=2, 4, 6, 8 when `i` = 5. Also, the border * happens when `i` or `j` is 0 or 10. Therefore, the formula is related to even numbers and 5 and 10. The smallest such number is 10.	
E	1, 2, 5, 6, 8, 9	
F	`(r1+c1+r2+c2)mod 2=1`	`(r1+c1+r2+c2)%2==1`
	Note: `==1` is not acceptable if minus is used. `!=0` can be used instead. This checks whether the two bishops are placed on the same color (black / white). If the colors are the same, there exists a cell (of same color) that can be attacked by both bishops. If the colors are different, there is no such cell since bishops cannot move to a cell of different color.	
G	`f(2,0);`	
H	`f(5,1);` Notice that the first number printed is `x`. Therefore we only need to determine parameter `y`. `y` is stored into tmp which in turn is stored into `x`. This value is printed in the next iteration. Therefore the parameter `y` should be the second digit to be printed. Actually, array `a` is a "sum" bidirectional linked list: the left summand stores the index of the previous cell and the right summand stores the index of the next cell. To access the next element in the linked list, the index of previous element is required.	
I	`100-i div 4+i mod 4*4`	`100-i/4+i%4*4`
	Alternative answer: `100-i div 4*17+i*4`, `100-i/4*17+i*4` A character is 4 greater than the character on its left, so there should be a `i*4` term. Let's add a constant to make the first line is correct, we would have `100+i*4`. The output becomes `dhlp,tx??,????,????`. We need to change `t` to `c` (difference = 17) So we add the term `–i/4*17`.	
J	0 For integer division `n/d == -((-n)/d) == -(n/(-d))`, `n%d == -((-n)%d) == n%(-d)`	