Hong Kong Olympiad in Informatics 2013
Heat Event (Senior Group)
Official Solution

Statistics (N = 185)
Full mark = 51. Maximum = 42. Median = 16. Advance to Final = 18 marks or above.

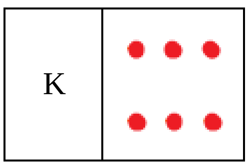
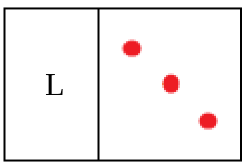
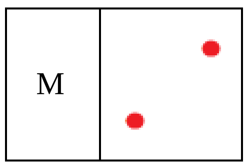
Section A

Q	A	Explanation
1	A	There are 8 possible configuration of the ring. Which are: (O represents black bend while X represents black bend) OOOOO XXXXX OOOOX XXXXO OOXOX XXOXO XXXOO OOOXX
2	B	It takes a long time to search for an element in specific in linked list but not sorted array. However insertion of elements in linked list takes only a little time (N/2 times faster than sorted array, where N is the number of element) Notice that linked list request additional memory for the pointers, so actually linked list takes more memory than a sorted array in case the number of element is known.
3	B	If Option A is correct, Option C must also be correct. If Option B is wrong, Option A must be correct (which is impossible), therefore only Option B is correct.
4	D	Memory size of each data type: `int` = 4 bytes, `char` = 1 byte. Therefore `node` = 20*4 + 40 = 120 bytes each. Answer = 1000*4 + 250*10 + 100*120 = 18500
5	C	x = 727650→363825→121275→24255→3465→385→35
6	A	In situation ii, there exists a winning strategy for the first player if the first player place an ‘O’ at the center for the next move.
7	C	Let the positions of A, B, C and D be *a*, *b*, *c* and *d* respectively. We have $c - a \le 5$, $d - b \le 9$ and $c - b \ge 3$, which means $b - c \le -3$. Distance between D and A: $$d - a = d - b + b - c + c - a = (d - b) + (b - c) + (c - a)$$ To maximize $d - a$, we maximize $d - b$, $b - c$ and $c - a$. $$d - a \le 5 + 9 + (-3) = 11$$

Q	A	Explanation
8	D	`random(x)` returns an integer between 0 and $x-1$ -2012 to 0 is obtainable by assuming the first `random(random(1011)*2)` to be 0. There are 2013 integers. Integers 2, 4, 6…, 2020 are obtainable by assuming the second random to be 0. There are 1009 other integers. Integers 1, 3, 5…, 2019 are obtainable by assuming the second random to be 1. There are 1009 other integers Thus, total possible integers generated = 2013+1009+1009=4031
9	B	Assuming we are trying to make Feb 1 larger than Jan 31, $2k+1>1k+31$ so $k>30$ You can try to prove that for other months $k>30$ is sufficient.
10	D	Here, `k = k * k;` would be executed if and only if `i == k` k = 2→4→16→256→65536 (as 256<2013)
11	C	The program implements bubble sort. Array `a` after the fourth iteration of `i`: 4,7,1,6,2,8,3,5→4,1,6,2,7,3,5,8→1,4,2,6,3,5,7,8→1,2,4,3,5,6,7,8→1,2,3,4,5,6,7,8 Question is asking for `ans = a[j] = a[7-i] = a[7-3] = a[4] = 5`
12	C	Let *x* and *y* be the values of wine X and wine Y (in thousands). $\begin{cases} \frac{1}{2}x = \frac{1}{3}y + (\frac{1}{2})(3) \\ \frac{2}{3}y = \frac{1}{4}x + (\frac{1}{4})(3) \\ \left(\frac{3}{4}\right)(3) = \frac{1}{4}x + \frac{1}{3}x \end{cases}$ $\begin{cases} 3x = 2y + 9 \\ 8y = 3x + 9 \\ 27 = 3x + 4y \end{cases}$ By solving the equations we get $x=5$ and $y=3$.
13	D	`a` = {4,2,9,10,1,3,7,5} Notice that variable `i` has been changed to 5 before function `sum5(1)` ends. As the two loop (one at `sum5` and one at main program) use the same global variable `i`, after we call `sum5(1)` at main program we actually set `b[5]=26` and exit the main program (as 5>4), therefore `b[1]=0` as `memset` function sets `b` = {0,0,0,0,0,0,0,0,0} initially.
14	B	`b` = {0,0,0,0,0,26,0,0,0}(0 based), therefore maximum number = 26
15	D	It is always possible that today is cloudy. Then, Ken feels sad and eats a lot of dinner. Thus, all three conditions are possible.
16	C	Precision error occurs in code segment iii. In code segment ii, `b = b - 0.5` would not result in precision error as both 2013.0 and 0.5 can be accurately stored using floating point data types.

Q	A	Explanation
17	B	P1 prints $N(N-1)(N-2)/6$ '*'s. For $N=4$, P1 prints 4 '*' s P2 prints N^2'*'s. For $N=4$, P2 prints 16 '*'s P3 prints fewer than $N(1+\lfloor\log_2 N\rfloor)$ '*'s. For $N=4$, P3 prints 12 '*' s P4 prints $2^{N-4}-1$ '*'s. For $N=4$, P1 prints 0 '*' s
18	A	P1 prints $N(N-1)(N-2)/6$ '*'s. For $N=9$, P1 prints 84 '*' s P2 prints N^2'*'s. For $N=9$, P2 prints 81 '*'s P3 prints fewer than $N(1+\lfloor\log_2 N\rfloor)$ '*'s. For $N=9$, P3 prints 33 '*' s P4 prints $2^{N-4}-1$ '*'s. For $N=9$, P1 prints 31 '*' s
19	C	We only need to compare P1 and P4 as P1 has the highest degree of *N*. For $N=11$, P1 prints 165 '*'s, P4 prints 127 '*'s. For $N=12$, P1 prints 220 '*'s, P4 prints 255 '*'s. (P2: 144, P3; 44)
20	D	7 '*'s are printed if the initial content of a is {1, -2, 7, -5, 1, -3, -3, -1}
21	C	Each integer from 0 to 4 is added 24 times.
22	A	Answer = $5C3\times 5^3=1250$
23	B	`f(n)` returns the number of '1's in the binary representation of `n`. From 0 to 31, '1' appears 16 times for each bits 0 - 4. Answer = 16*5 = 80
24	D	Let the length of road connecting A and B be *w*(A,B) and so on. It is given that *w*(A,C) + *w*(C,B) < *w*(A,B) Without losing generality we assume *w*(C,B) > *w*(A,C) , then *w*(C,B) = *w*(A,C) + *x*, where *x* is positive *w*(A,C) + *w*(C,B) < *w*(A,B) 2 *w*(A,C) + *x* < *w*(A,B). Therefore road between A and B is at least twice as long as the shortest road.
25	A	For each distinct prime factor *p* of *n*, `ans = ans / p * (p-1)` So f(997) = 997/997*996 = 996, f(1001) = 1001/7*6/11*10/13*12 = 720, f(1024)=1024/2*1 = 512, f(1089)=1089/3*2/11*10 = 660

Section B

	Answer and Explanation	
A	16 Try 1, 11, 111, 1111, etc. Using long division you will get you the answer. Note that there are only 17 possible remainders (0-16) when a number is divided by 17 so the maximum number of tries required is 17.	
B	108 Initial village = 4 choices, after first step each time we have 3 choices. Therefore number of paths with length 3 is $4 \times 3 \times 3 \times 3 = 108$.	
C	2916 Answer = $4 \times 3^6 = 2916$	
D	`peek(r)`	
E	`s,pop(r)` We want to be sure that when an element `x` is popped from `s`, the element will be positioned to appropriate location in `r` (assume that it is already sorted). To maintain the ascending order of `r`, we need to pop out all integers that are smaller than `x`, push `x` into `r` (as `x` now is the smallest element in `r`), and push back the popped element of `r`.	
F	14	54
G	`abs(x1-x2)+abs(x1-x2)` or `(x1-x2)*(x1-x2)+(y1-y2)*(y1-y2)` The function `dist` returns an `int`. The program fails to distinguish 1 (exactly 1) and $\sqrt{1+1} \approx 1.41$.	
F	Recommended: `(i+j)/2`. Answers between `i` and `j-1` inclusive are also acceptable.	
G	`max(0,k)` Replacing the first parameter 0 with `i` is acceptable. Replacing the second parameter `k` with the answer in F is also acceptable.	
J	`max(0,n-1)` The basic principle of the function is to find out the position of maximum element (using binary search), and set that element to negative so that the second maximum element become the maximum element.	
	   The fastest way to solve this problem is to tear out the question paper and fold a dice yourself!	