## Question 0 Enumeration (50 Marks)

```
Program Name: PROGRAMO.EXE
Input: Standard Input
Output: Standard Output
Maximum Execution Time: 1 second
```

Write a program to read an integer $N$ and output $N^{2}$ integers in the format specified below.

## Input

The input contains an integer $N(1 \leq N \leq 10)$.

## Output

The output consists of $N$ lines, each containing $N$ integers, separated by a single space. The $N^{\text {th }}$ number in the $i^{\text {th }}$ row is $N-i+1$. For the other integers, each of them is greater than the one on its right by 1 .

## Sample Input

4

Sample Output

```
7 6 5 4
6 5 4 3
54 3 2
4 3 2 1
```


## Question 1 WippiLeaks (100 Marks)

```
Program Name: PROGRAM1.EXE
Input: Standard Input
Output: Standard Output
Maximum Execution Time: 1 second
```

All of us know that sending data via the Internet without encryption could be very dangerous. However, Dr. Jones does not know that sending data with weak encryption is not safe as well. Here is his encryption method:

He defines a parameter $D$, which is explained below. For each character $C$ in the original message,

- If $C$ is a letter in upper case (' A ' to ' $Z$ '), change $C$ to the $D^{\text {th }}$ character after $C$. You can assume that the letter after ' $Z$ ' is ' $A$ ' again.
- If $C$ is a letter in lower case (' a ' to ' z '), change $C$ to the $D^{\text {th }}$ character after $C$. You can assume that the letter after ' $z$ ' is ' $a$ ' again.
- If $C$ is not the cases above, just keep $C$ remain unchanged.

You are now given the $k^{\text {th }}$ frequent character $X$ in lower case, along with the encrypted message, consisting of any printable character. Let the $k^{\text {th }}$ frequent character in the original message be $Y$, then $Y$ is the $D^{\text {th }}$ character after $X$.

As an assistant of Dr. Jones, to show him that the encryption method is not reliable, you are going to decrypt his encrypted message.

For example, Dr. Jones encrypted a message to
"Cngzkbkx ngvvktkj, ngvvktkj."
You also know that the $2^{\text {nd }}$ frequent character of the original message is ' p ' (character $X$ ) in the original message (including both ' p ' and ' P '). Since the $2^{\text {nd }}$ frequent character of the encrypted message is ' $v$ ' (character $Y$ ), ' $p$ ' is encrypted to be ' $v$ ' and you can decrypt the whole message after calculating the parameter $D=6$. The original message is

[^0]Here is the encryption scheme of Dr. Jones in the example above.

| Original | A | B | C | D | E | F | G | H | I | J | K | L | M |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Encrypted | G | H | I | J | K | L | M | N | O | P | Q | R | S |


| Original | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Encrypted | T | U | V | W | X | Y | Z | A | B | C | D | E | F |


| Original | a | b | c | d | e | f | g | h | i | j | k | l | m |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Encrypted | g | h | i | j | k | l | m | n | o | p | q | r | s |


| Original | n | o | p | q | r | s | t | u | v | w | x | y | z |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Encrypted | t | u | v | w | x | y | z | a | b | c | d | e | f |

## Input

The first line of input contains the integer $k$ and the character $X$ in lower case separated by a single space. The second line contains the encrypted message. There will not be any trailing space in the input.

## Output

Output the original message. There is always a unique solution.

## Sample Input

```
t
Proc yanbnwenm kh RbxujcN.
```


## Sample Output

```
Gift presented by IsolatE.
```


## Constraints

In all test cases,
> The length of the encrypted message ranges from 1 to 255.
> $1 \leq k \leq 26$

In $50 \%$ of all test cases,
$>$ The length of the encrypted message ranges from 1 to 50 .

$$
k=1
$$

## Question 2 Housekeeper (100 Marks)

```
Program Name: PROGRAM2.EXE
Input: Standard Input
Output: Standard Output
Maximum Execution Time:
1 second
```

"HouseKeeper Of the Imperial palace" (HKOI) is a title granted to the most diligent citizens in the kingdom. In return, they have to work in the imperial palace as housekeepers (Surprise!). Now the king would like to hold a ceremony in his palace. He would like to invite as many guests as possible to the ceremony. Tough time for the housekeepers.

The palace has $N$ rooms arranged in a line. A room is represented by a line segment along the line. Each room can be assigned to at most one person (guest or housekeeper). There are $M$ housekeepers in the palace who can serve the guests. A housekeeper can only serve the guests in the neighbouring rooms (rooms that share an end-point are neighbouring), so each room assigned to a guest must have at least one neighbouring room that is assigned by a housekeeper. For example, if the room is arranged as in the figure below, and $M=3$, then it is possible to invite 4 guests.


The king asks you to assign the guests and housekeepers to the rooms. Note that each housekeeper and invited guest has to be assigned to a room. Write a program to calculate the maximum number of guests that can be invited.

## Input

The first line contains two integers $N$ and $M(1 \leq M \leq N)$. Each of the following $N$ lines contains two integers $A_{i}, B_{i}\left(0 \leq A_{i}<B_{i} \leq 100,000\right)$, where $A_{i}$ is the starting position and $B_{i}$ is the ending position of the $i^{\text {th }}$ room. The rooms are given from left to right, i.e. increasing order of $A_{i}$. You may assume that no two rooms overlap.

## Output

The output contains one integer, the maximum number of guests that can be invited.

## Sample Input

$\square$

## Sample Output

4

## Constraints

In all test cases,
> $1 \leq N \leq 100,000$

In $50 \%$ of all test cases,
> $1 \leq N \leq 1,000$

## Question 3 Factor Factory (100 Marks)

```
Program Name: PROGRAM3.EXE
Input: Standard Input
Output: Standard Output
Maximum Execution Time: 1 second
```

It is so much fun for Victor, Dr. Jones' son, to learn factors in his primary school. He is trying to decompose some integers into prime factors, but he is so absent-minded that he always forgets what the prime factorization of an integer is. Therefore, he makes some cards for each integer.

Playing around with these cards, he finds that rearranging the cards can sometimes create a very big integer (in the view of a primary student). He tries to factorize every integer from 2 up to $N$, list their prime factors out, and then rearrange the prime factors to form an integer. Of these integers, he wonders what the biggest one is. Can you help him?

For example, 12 is factorized as 2,2 and 3 , which can be rearranged as 223,232 and 322 , of which 322 is the biggest. 34 is written as 2 and 17, which can form 217 and 172. Note that 1 is not a prime number.

## Input

A single integer $N$

## Output

One integer represents the biggest number Victor can create.

## Sample Input 1

## 12

## Sample Output 1

## Sample Input 2

20
Sample Output 2
2222

## Explanation of Sample 2

16 can be written as 2222 .

## Constraints

For all test data,
> $2 \leq N \leq 2 \times 10^{9}$

For some data worth $80 \%$ of the test scores,
> $2 \leq N \leq 10^{6}$

For some data worth $50 \%$ of the test scores,
> $2 \leq N \leq 10000$

For some data worth $20 \%$ of the test scores,
> $2 \leq N \leq 100$

## Question 4 Pyramid (100 Marks)

```
Program Name: PROGRAM4.EXE
Input: Standard Input
Output: Standard Output
Maximum Execution Time: 1 second
```

Pharaoh, the King of the Ancient Egypt, would like to build a great Pyramid with length $2 \mathrm{x} H-1$ (where $H$ is a positive integer) for his future use. After the consultation with his servants, a location for that Pyramid is chosen. What Pharaoh knows is the length of location, $2 \times H-1$, and the altitude of each unit length, $a_{1}, a_{2}, \ldots, a_{H}, \ldots a_{2 x H-2}, a_{2 x H-1}$. Assume the altitudes of the unit lengths of a Pyramid with height $H$ are $p_{l}, p_{2}, \ldots, p_{H}, \ldots p_{2 x H-2}, p_{2 x H-1}$, and they should follow the following 2 properties:

```
- p pi+1 = pi+1 for 1<=i<=H-1
- p pi+1 = pi-1 for }H<=i<=2\timesH-
```

For example, $[4,5,6,7,8,7,6,5,4]$ is a valid sequence of height of a Pyramid with length 9 , i.e. $H=5$.

Pharaoh needs to build a Pyramid at the location by changing the altitude. To change the original attitude of one unit length from $a_{i}$ to $p_{i}$, the cost is 1 no matter how large the difference of $a_{\mathrm{i}}$ and $p_{i}$ is. Now, Pharaoh wants to know the minimum total cost of doing so. If there are multiple Pyramids having the same minimum total cost, Pharaoh would like to have the highest Pyramid.

## Input

The first line of input contains a positive integer $H$.
There will be $2 * H-1$ integers $a_{1}, a_{2}, \ldots, a_{H}, \ldots a_{2 * H-2}, a_{2 * H-1}$ separated by space in the second line.

## Output

The first line of the output contains one integer $C$, the minimum total cost of building the Pyramid. The second line contains another integer, the height of the middle part of the Pyramid, ie. $p_{H}$.

## Sample Input

$\begin{array}{lllllllll}5 & & & & & & \\ 3 & 5 & 6 & 7 & 7 & 7 & 6 & 10 & 4\end{array}$

## Sample Output

3
8

## Constraints

In all test cases,
$>2 \leq H \leq 5000$
$>0 \leq a_{i} p_{i} \leq 100,000$

In $50 \%$ of all test cases,
> $2 \leq H \leq 500$
> $0 \leq a_{i}, p_{i} \leq 10,000$


[^0]:    "Whatever happened, happened."

