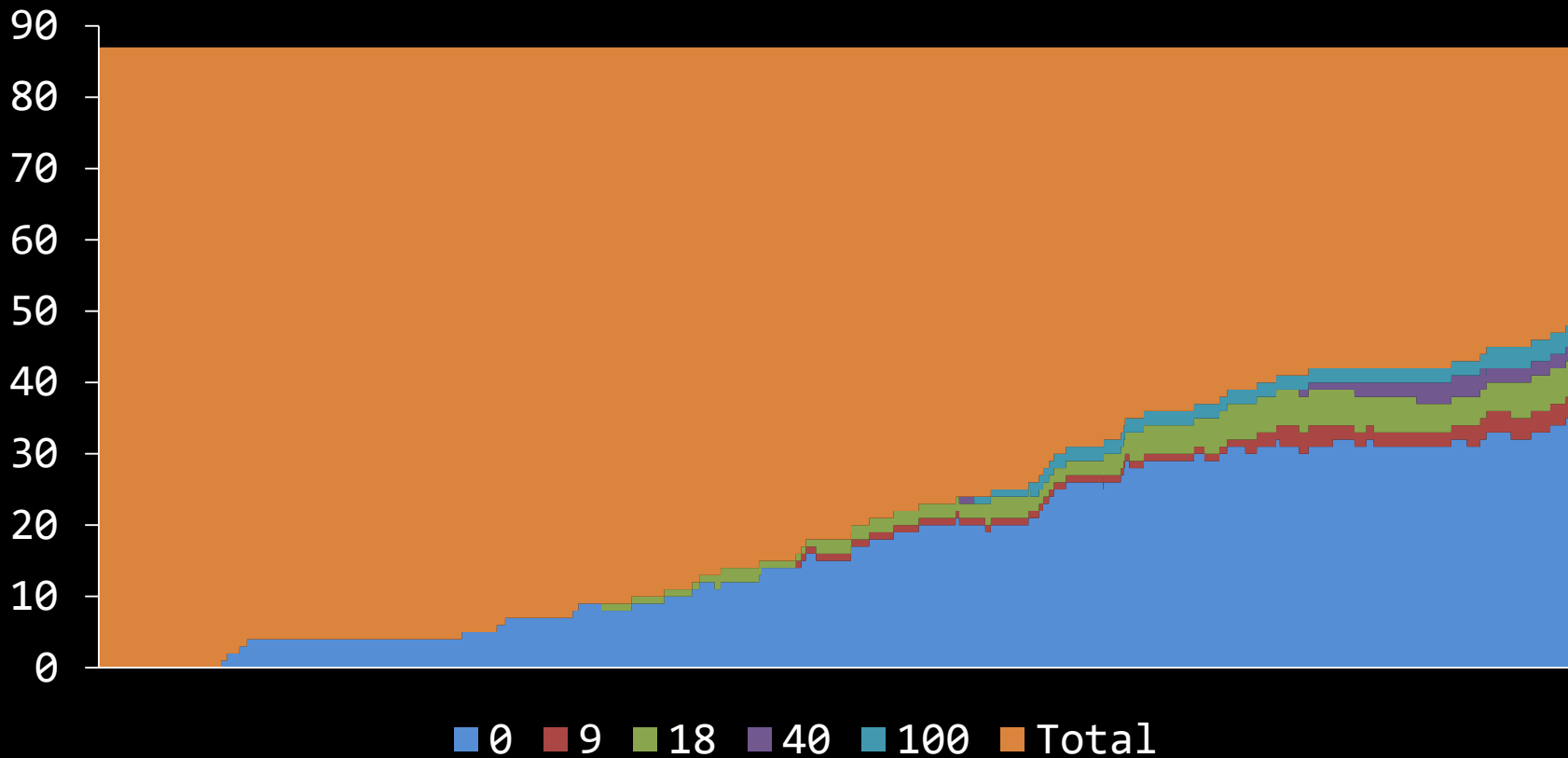


S173 Monster GO

Author: Anson Ho

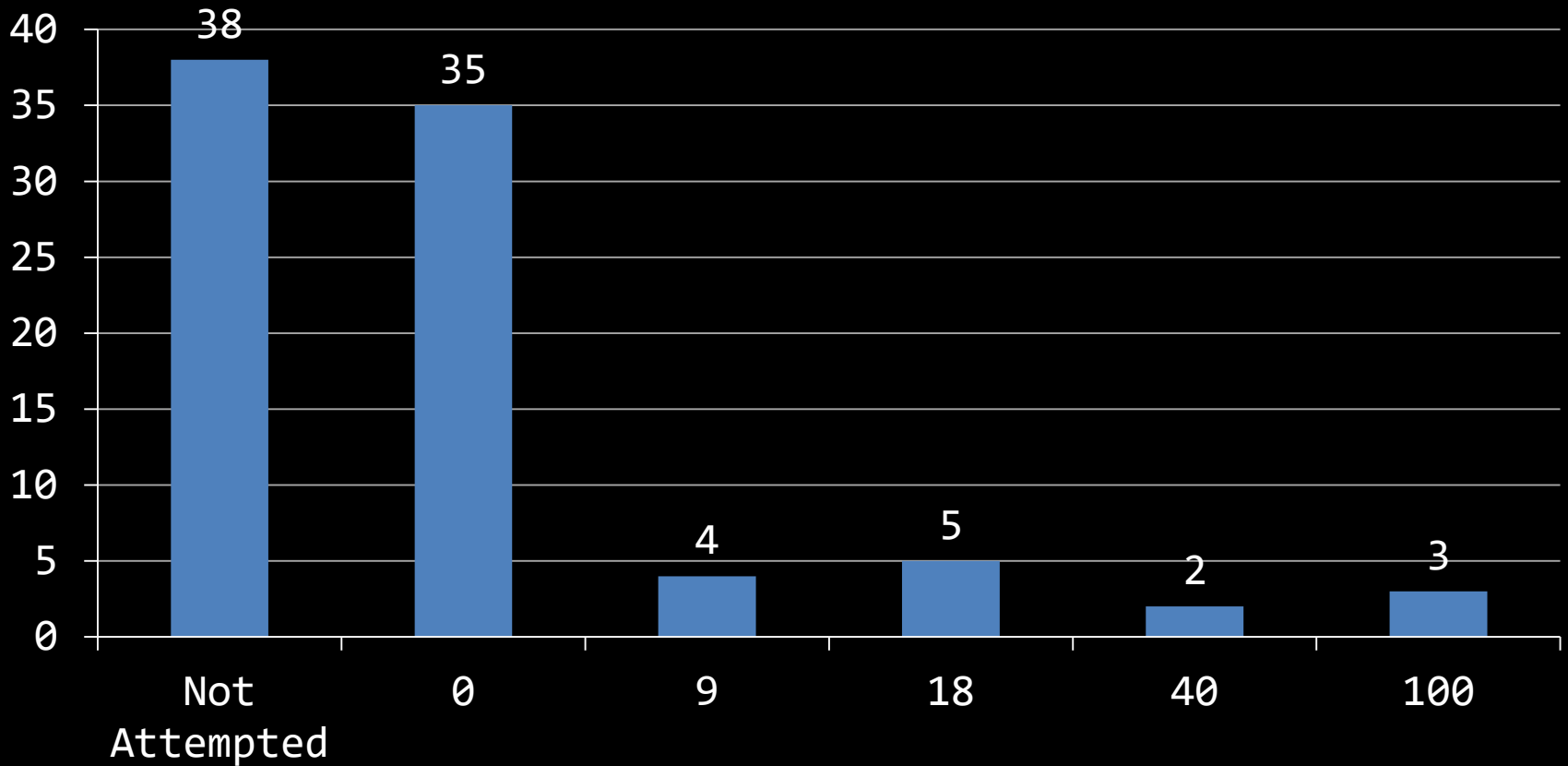
Statistic

S173 Monster GO

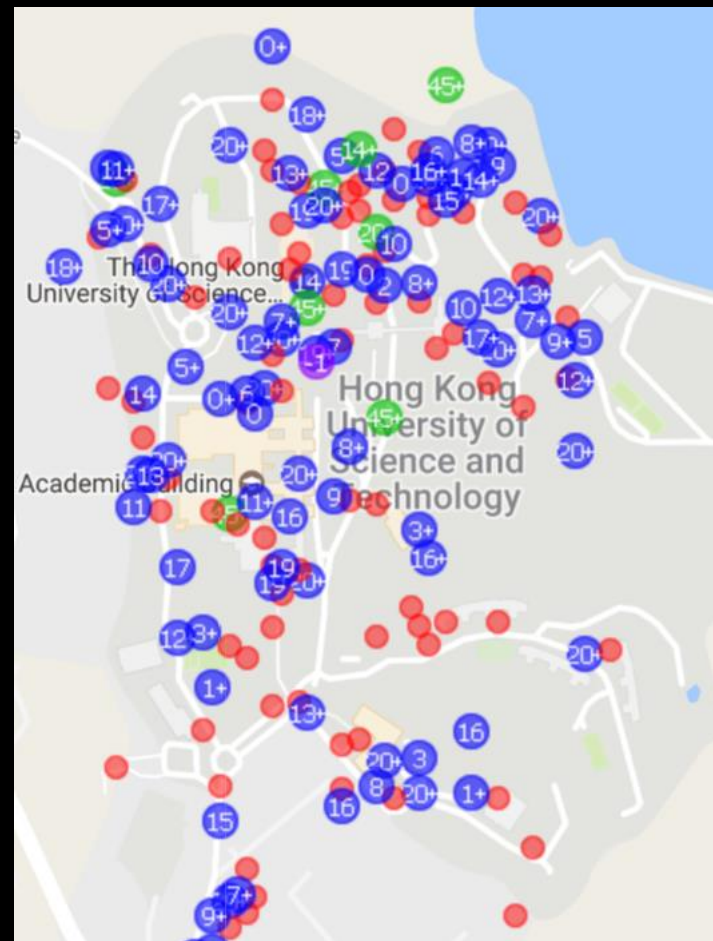


Statistic

S173 Monster GO



Background



Problem Statement

- N monsters live in N caves
- Different monsters live in different caves

Cave	1	2	3	4
Monster	C	A	B	D

Problem Statement

- Given a list of M radars
- Each radar have a given set of caves in range
- Each radar can show the monsters in the caves in range in an arbitrary order

Problem Statement

Cave	1	2	3	4
Monster	C	A	B	D

Radar	Cave 2	Cave 3	Cave 4
	2	3	4

- {A, B, D}
- {D, A, B}
- {B, A, D}

- {A, B}
- {C, A, B, D}
- {B, C, D}

Problem Statement

- Output the minimum number K such that the first K radars can determine the location of each monster
- \Leftrightarrow there is exactly one permutation of monster locations corresponding to the information of the K radars

Example

- $N = 2, M = 2$
- Range of radar 1 = {cave 1, cave 2}
- Range of radar 2 = {cave 2}

- Ans: $K = 2$

- Radar 1 actually gives no useful information
- The monster in cave 1 can be determined by method of elimination once the monster in cave 2 is known

Subtask 1

- $N, M \leq 2$
- When $N = 1$
- No need radar
- Output \emptyset
- Monster 1 always lives in cave 1

Subtask 1

- $N, M \leq 2$
- When $N = 2$
- Possible ranges of radars
- $\{1\}, \{2\}, \{1, 2\} = \{2, 1\}$
- $3 + 3^2$ (or $4 + 4^2$) possibilities
- Hardcode

Subtask 2

- $N \leq 2$, $NM \leq 2e5$
- When $N = 2$
- Possible ranges of radars
- $\{1\}$, $\{2\}$, $\{1, 2\} = \{2, 1\}$
- Done \Leftrightarrow $\{1\}$ or $\{2\}$ appear

Observation

- Cave i and Cave j are **distinguishable**
 - ↔ you can partition N monsters into two groups such that the monsters in cave i and cave j are in different groups
 - ↔ the sets of radars in range are different

Observation

- Denote the sets of radars in range by R_i and R_j for cave i and cave j respectively
- If $R_i = R_j$
Then the monsters in cave i and cave j will be both present or both absent in the result of each radar
→ Not distinguishable

Observation

- If $R_i \neq R_j$
Then there is one radar such that cave i is in range and cave j is not in range, or in the other way
→ Monsters can be partitioned by the result of that radar
→ **Distinguishable**
- Thus,
different set \Leftrightarrow **distinguishable**

Observation

- The radars are enough
 - ↔ the caves are pairwise **distinguishable**
 - ↔ the sets of radars in range are pairwise different

Subtask 3

- $N \leq 10$, $NM \leq 2e5$
- After a radar is added
- Set the pair of caves to be **distinguishable** if one is present and one is absent
- Check if every pair of caves is **distinguishable**
- $O(N^2M)$

Subtask 4

- $N, M \leq 2e5, NM \leq 2e5$
- “Not distinguishable” is an equivalence relation
- So the monsters can be partitioned into equivalence classes

Subtask 4

- $N, M \leq 2e5, NM \leq 2e5$
- ~~“Not distinguishable” is an equivalence relation~~
- ~~So the monsters can be partitioned into equivalence classes~~

Subtask 4

- $N, M \leq 2e5, NM \leq 2e5$
- If A, B are **not distinguishable** and B, C are **not distinguishable**
- Then A, C are **not distinguishable**
- Monsters can be partitioned into groups such that the members are pairwise **not distinguishable**

Subtask 4

- $N, M \leq 2e5, NM \leq 2e5$
- Reduce unnecessary checking
- Recall
Same set \Leftrightarrow **not distinguishable**
- How to efficiently check the equality of set?

Subtask 4

- For each cave, define a binary number where the i^{th} digit represents whether the cave is in the range of radar i

Subtask 4

- Radar 1 = {1}
 - Radar 2 = {1, 2}
 - Radar 3 = {2}
-
- Cave 1 = 110_2
 - Cave 2 = 011_2
 - Cave 3 = 000_2

Subtask 4

- If the N numbers are distinct
 - (can be checked by hashing)
 - Then it is done
-
- $O(NM \log N)$

Subtask 4

- If the N numbers are distinct
- (can be checked by hashing)
- Then it is done

- $O(NM \log N)$
- Not intended

Subtask 5

- $N, M \leq 2e5$, input size = $O(2e5)$
- Not to check every N numbers after a radar is added

Subtask 5

- $N, M \leq 2e5$, input size = $O(2e5)$
- Not to check every N numbers after a radar is added
- Else TLE

Subtask 5

- Method 1
- If the first x radars are enough
- Then the first y ($y > x$) radars are also enough
- Binary search
- $O(N \log N \log M)$

Subtask 5

- Method 1
- If the first x radars are enough
- Then the first y ($y > x$) radars are also enough
- Binary search
- $O(N \log N \log M)$
- Not intended

Subtask 5

- Method 2
- Notice that a group is split when the members are not all in range OR all not in range for the newly added radar

Subtask 5

- Method 2
- Implementation
- Define a group id for each group
- Split group → change id
- Only change id of those included in input

Subtask 5

- Method 2

Cave	1	2	3	...	N
Group id	1	1	1	2	K

- Radar = {1, 2}

Cave	1	2	3	...	N
Group id	K + 1	K + 1	1	2	K

Subtask 5

- Method 2

Cave	1	2	3	...	N
Group id	1	1	1	2	K

- Radar = {1, 2, 3}

Cave	1	2	3	...	N
Group id	1	1	1	2	K

Subtask 5

- Method 2

Cave	1	2	3	...	N
Group id	1	1	1	2	K

- Radar = {1, 2, 3}

Cave	1	2	3	...	N
Group id	K + 1	K + 1	K + 1	2	K

Subtask 5

- Method 2

Cave	1	2	3	...	N
Group id	1	1	1	2	K

- Radar = {6, 8, 9}

Cave	1	2	3	...	N
Group id	1	1	1	2	

Subtask 5

- Method 2

Cave	1	2	3	4	...	N
Group id	1	1	2	2	3	K

- Radar = {2, 3}

Cave	1	2	3	4	...	N
Group id	1	K + 1	K + 2	2	3	K

Subtask 5

- Method 2
- $O(\text{input size})$
- Expected score = 100 :D

Thank you